ENSE622, Spring 2018:
Homework 3: More Programming Techniques
SOLUTIONS - Rev 1
(2/26/18 – jem)

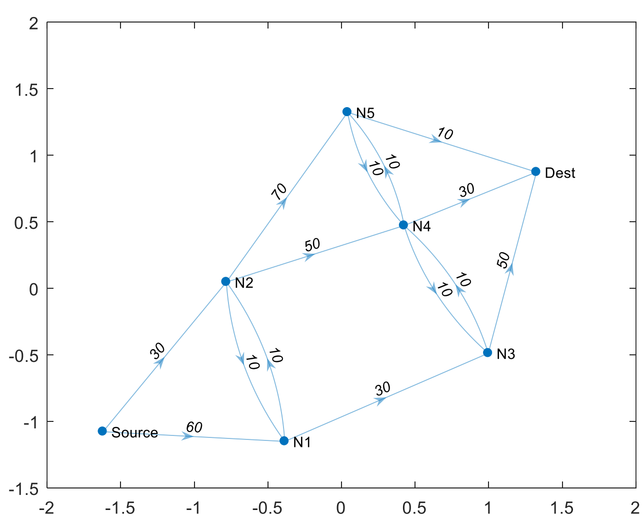
This Homework has a maximum point value of 100 points (10 points of extra credit). Note that you must show your work to get full credit.

1. **Problem 1 (20 pts) – Networks & Shortest Path:**

   Suppose you need to write a routing algorithm for a communications network that will route traffic through an internetwork of networks (Ni) at the lowest possible cost. You manager wants to know the least cost path between Source (So) and Sink (Si). Consider a communications network that is characterized by the following link costs.

Link	Cost ($/Tb)		Link	Cost ($/Tb)		Link	Cost ($/Tb)
So-N1	60		N2-N4	50		N5-N4	10
So-N2	30		N2-N5	70		N3 -Si	50
N1-N2	10		N3-N4	10		N4-Si	30
N2-N1	10		N4-N3	10		N5-Si	10
N1-N3	30		N4-N5	10			

(a) (4) Draw the network showing node names, the links, and the cost of each link.



(b) (8) Develop and submit a MATLAB program that:

- Accepts the number the cost of each link.
- Provides the following output:
  i. A graph of the network showing node names and link costs.

ii. The lowest cost path between the two specified nodes.
iii. The resulting Minimum Cost.

(See MATLAB program HW_3_1_Short.m)

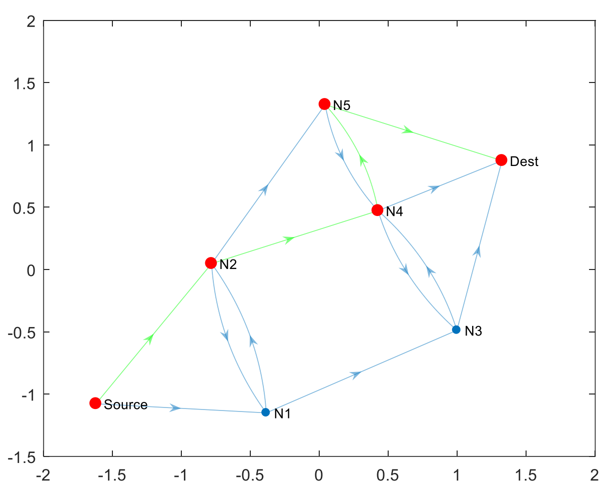
(c) (8) Run the program to find the minimum cost path between nodes So and Si and provide your output below.

- Following is command line output of MATLAB program.
- Graphical output is provided in answer to (a) and (d).

```
p =
  GraphPlot with properties:
   ...
  Show all properties
shortPath =
  1×5 cell array
  Columns 1 through 5
    'Source'    'N2'    'N4'    'N5'    'Dest'
Length =
   100
>>
```

(d) (5) How would you report the results of your analysis to your manager?

- The least cost routing path from the Source node to the Destination is:
  Source -> Net 1 -> Net 3 -> Net 4 -> Destination



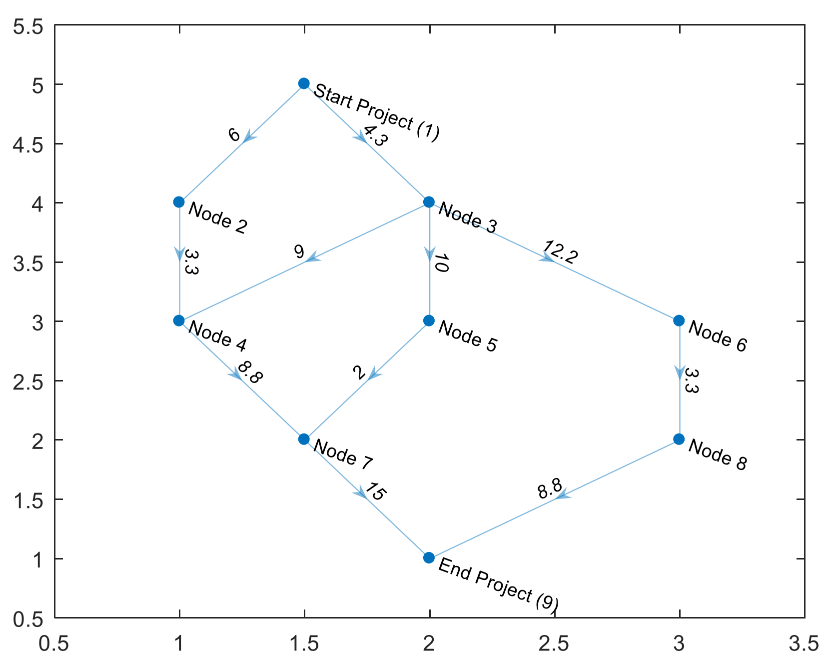
- It has a total cost of $100.

2. **Problem 2 (30 pts) – PERT Schedule Network, Project Cost, & Critical Path:**

Consider a Project that is characterized by the following activity edge table. The table indicates: 1) the activity names; 2) the associated event node names (numbers); 3) the minimum, maximum, and best estimate activity duration times (in days) for each activity; and 4) the minimum, maximum, and best estimate hourly labor costs for each activity. Assume two people are required to complete each task and that each works 8 hours a day.

Activity	Nodes	Duration Est (days)				Labor Cost ($/h)		
		Min	Max	Exp		Min	Max	Exp
A	1,2	4	8	6		$ 100	$ 150	$ 120
B	1,3	2	8	4		$ 100	$ 150	$ 120
C	2,4	1	7	3		$ 100	$ 150	$ 120
D	3,4	6	12	9		$ 100	$ 150	$ 120
F	3,5	5	15	10		$ 100	$ 150	$ 120
I	3,6	7	18	12		$ 90	$ 110	$ 100
J	4,7	5	12	9		$ 90	$ 110	$ 100
K	5,7	1	3	2		$ 90	$ 110	$ 100
L	6,8	2	6	3		$ 90	$ 110	$ 100
M	7,9	10	20	15		$ 90	$ 110	$ 100
N	8,9	6	11	9		$ 90	$ 110	$ 100

(a) (3) Draw the network showing node names, the links, and best guess duration of each link.



(b) (4) Develop a table that indicates the expected mean duration and standard deviation for each task and the expected mean cost and standard deviation for each task. Explain how you calculated these.

a. Assume a Triangular or Beta Distribution (since max & min and not symmetric). Assuming a Beta Distribution for duration and labor cost:
   i. Mdi = Mean Duration (Task i) = (Min + 4 Likely + Max)/6
   ii. SDdi = SD Duration (Task i) = Sqrt((Max-Min)^2/36)
   iii. Mlci = Mean Labor Cost (Task i) = (Min + 4 Likely + Max)/6
   iv. SDlci = SD Labor Cost (Task i) = Sqrt((Max-Min)^2/36)

b. The following tables summarize this (note that use of triangular distribution should give similar results):

Activity	Nodes	Duration Est (days)				Task Duration (days)		
		Min	Max	Likely		Mean	SD	VAR
A	1,2	4	8	6		6.0	0.7	0.4
B	1,3	2	8	4		4.3	1.0	1.0
C	2,4	1	7	3		3.3	1.0	1.0
D	3,4	6	12	9		9.0	1.0	1.0
F	3,5	5	15	10		10.0	1.7	2.8
I	3,6	7	18	12		12.2	1.8	3.4
J	4,7	5	12	9		8.8	1.2	1.4
K	5,7	1	3	2		2.0	0.3	0.1
L	6,8	2	6	3		3.3	0.7	0.4
M	7,9	10	20	15		15.0	1.7	2.8
N	8,9	6	11	9		8.8	0.8	0.7

Activity	Nodes	Labor Cost ($/h)				Task Labor Costs ($/hr)		
		Min	Max	Likely		Mean	SD	VAR
A	1,2	$ 100	$ 150	$ 120		$ 121.7	$ 8.3	$ 69.4
B	1,3	$ 100	$ 150	$ 120		$ 121.7	$ 8.3	$ 69.4
C	2,4	$ 100	$ 150	$ 120		$ 121.7	$ 8.3	$ 69.4
D	3,4	$ 100	$ 150	$ 120		$ 121.7	$ 8.3	$ 69.4
F	3,5	$ 100	$ 150	$ 120		$ 121.7	$ 8.3	$ 69.4
I	3,6	$ 90	$ 110	$ 100		$ 100.0	$ 3.3	$ 11.1
J	4,7	$ 90	$ 110	$ 100		$ 100.0	$ 3.3	$ 11.1
K	5,7	$ 90	$ 110	$ 100		$ 100.0	$ 3.3	$ 11.1
L	6,8	$ 90	$ 110	$ 100		$ 100.0	$ 3.3	$ 11.1
M	7,9	$ 90	$ 110	$ 100		$ 100.0	$ 3.3	$ 11.1
N	8,9	$ 90	$ 110	$ 100		$ 100.0	$ 3.3	$ 11.1

(c) (3) What would you report to your manager as your estimate of project total cost?

a. From calculations provided in (d) below, **the total cost of the project is expected to be <u>$143,900 +/- $450 (SD)</u>**, i.e., there is a about a 17% chance that the project cost would exceed $144,400.

(d) (3) Briefly explain you got this result. Be sure to indicate how you determined the uncertainty associated with your estimate.

a. The total cost and associated uncertainty is obtained by calculating the Mean and SD cost for each task and then the Mean Total Project Cost and SD as follows:

i. MCi = Mean Task (i) Cost = Mdi * 16 hr/day * Mlci

ii. SDCi = Task (i) Cost SD = Sqrt(Mdi^2*SDlci^2+Mlci^2*SDdi^2 +(SDdi*SDlci)^2)

iii. Mean Total Project Cost = Sum of MCi

iv. SD Total Project Cost = Sqrt(Sum of SDi^2)

b. Calculations are provided in the following table:

Activity	Nodes	Task Cost		
		Mean	SD	VAR
A	1,2	$ 11,680.0	$ 95.3	$ 9,090.1
B	1,3	$ 8,435.6	$ 127.0	$ 16,123.5
C	2,4	$ 6,488.9	$ 124.9	$ 15,591.0
D	3,4	$ 17,520.0	$ 143.0	$ 20,444.4
F	3,5	$ 19,466.7	$ 219.3	$ 48,091.0
I	3,6	$ 19,466.7	$ 187.8	$ 35,268.1
J	4,7	$ 14,133.3	$ 120.4	$ 14,485.9
K	5,7	$ 3,200.0	$ 34.0	$ 1,157.8
L	6,8	$ 5,333.3	$ 67.6	$ 4,572.3
M	7,9	$ 24,000.0	$ 174.0	$ 30,288.9
N	8,9	$ 14,133.3	$ 88.4	$ 7,817.0
Total Project Cost =		$ 143,857.8	$ 450.5	$ 202,930.1

(e) (5) Develop and submit a MATLAB program that:

a. Accepts the expected mean duration for each task.

b. Provides the following output:

i. A graph of the network showing node names and link durations.

ii. The critical path (in event names)

iii. The resulting expected mean project duration.

(See MATLAB program HW_3_2_CPM_Rev_1.m)

(f) (5) Run the program to find the minimum cost path between nodes So and Si and provide your output below.

CLP =

0

6.0000

4.3000

13.3000

14.3000
16.5000
22.1000
19.8000
37.1000

Path =
1
1
3
3
3
4
6
7
9
>>

(g) (3) What would you report to your manager as your estimate of project duration?

a. The critical path is from 1 to 3 to 4 to 7 to 9.
b. From calculations **the total duration of the project is expected to be <u>37.1 d +/- 2.5 d (SD)</u>**, i.e., there is a about a 17% chance that the project duration would exceed 40 days.

(h) (4) Briefly explain you got this result. Be sure to indicate how you determined the uncertainty associated with your estimate.

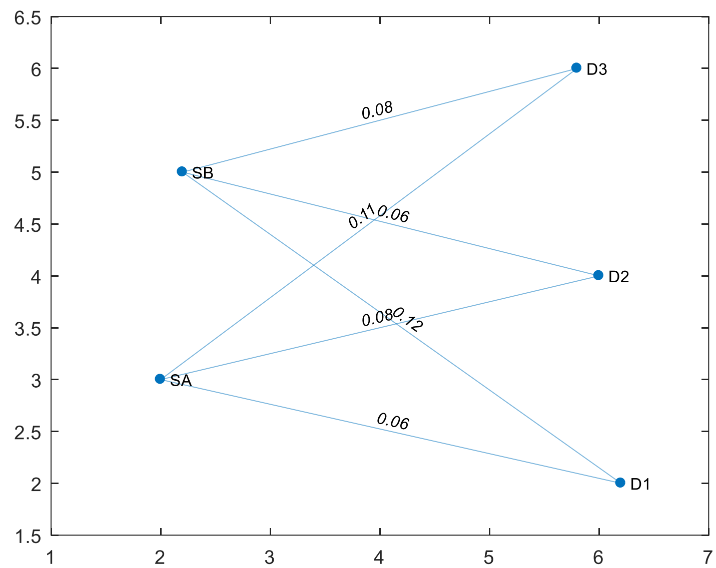
a. The Mean Total Project Duration and SD are determined as follows:
   i. Mean Total Project Duration = Sum of Mean Durations of the CP Tasks
   ii. SD Total Project Duration = Sqrt(Sum of SDi^2 of the CP Task durations).
b. Calculations are provided in the following table:

CP Activities	Nodes	Task Duration (days)		
		Mean	SD	VAR
B	1,3	4.33	1.00	1.0
D	3,4	9.00	1.00	1.0
J	4,7	8.83	1.17	1.4
M	7,9	15.00	1.67	2.8
Total Duration =		37.17	2.5	6.1

3. **Problem 3 (20 pts) – Transportation/Production Problem:**

See problem 8.12 in Bronson. Simplify the problem by deleting Chain 4 from the problem and using 2,000 and 1,500 as the production capabilities for Plant A and B.

(a) (3) Draw a schematic (network) diagram of the system.



(b) (2) Write down the objective function. Be sure to clearly define the decision variables (factors).

a. Profit Z = (0.39*X1+0.37*X2+0.4* X3) - (0. 23 * Xa + 0.25* Xb) – (0.06*Xa1+0.08*Xa2+0.11*Xa3 + 0.12*Xb1+0.06*Xb2+0.08*Xb3)

(c) (3) Express the problem in standard form.

a. Maximize Z
b. Subject to:
   i. X1 <= 1800
   ii. X2 <= 2300
   iii. X3 <= 550
   iv. Xa <= 2000
   v. Xb <= 1500
   vi. Xa1 + Xa2 + Xa3 – Xa = 0 (Conservation of production)
   vii. Xb1 + Xb2 + Xb3 – Xb = 0
   viii. Xa1 + Xb1 – X1 = 0 (Conservation of demand)
   ix. Xa2 + Xb2 – X2 = 0
   x. Xa3 + Xb3 – X3 = 0
c. Xi, Xj, and Xij are integers >= 0

(d) (4) Develop and submit a MATLAB program that:

- Accepts the input values for the production limits, product costs, demand limits and demand prices, and shipment costs.
- Provides the following output:
  i. A graph of the network showing node names.
  ii. The solution to the problem (number of loaves produced by each plant, the number of loaves shipped from each plant to each chain, and the total profit.

(See MATLAB code HW_3_3_Transport.m)

(e) (4) Run the program and provide your output below.

- Following is command line output of MATLAB program.
- Graphical output is provided in answer to (a).
- Parenthetical elements in green are manual additions (indicating the variable associated with the solution output.

```
Optimization terminated.

Solution =
  1.0e+03 *

   1.8000      (X1)
   1.1500
   0.5500
   2.0000      (Xa)
   1.5000      (Xb)
   1.8000      (Xa1)
   0.2000      (Xa2)
   0.0000
   0.0000
   0.9500      (Xb2)
   0.5500      (Xb3)

Profit =
  287.5000

AProd =
  2.0000e+03

BProd =
  1.5000e+03
```

>>

(f) (10) Suppose you are uncertain in the transportation costs Ca1, Ca2, & Ca3 by +/- 20%. Perform a sensitivity analysis.

a. (4) Provide a tornado diagram indicating the impact of these uncertainties on profit.
b. (2) Identify the uncertainty that has the greatest impact on profit.
    i. Changing Ca1 has the greatest impact on profit
c. (4) How do these uncertainties affect the recommended solution with respect to shipping plans?
    i. Increasing Ca2 by 20% and decreasing Ca3 by 20% both change the shipping plans. Shipping moves 200 loaves from Xa2 to Xa3, and 200 loaves from Xb3 to Xb2.

(g) (4) How would you report the results of your analysis to your manager?

a. To obtain the optimal solution (profit = $287.50):
    i. Produce 2000 loaves at Plant A and ship 1800 loaves to Chain 1 and 200 loaves to Chain 2.
    ii. Produce 1500 loaves at Plant B and ship 960 loaves to Chain 2 and 550 loaves to Chain 3.
b. There is an uncertainty in the expected profit and shipping solution due to a 20% uncertainty in shipping costs from A. Changes in these shipping costs could result in:
    i. The profit being as low as $265.9 or as high as $309.1
    ii. As well as changes in the desired shipping schedules.

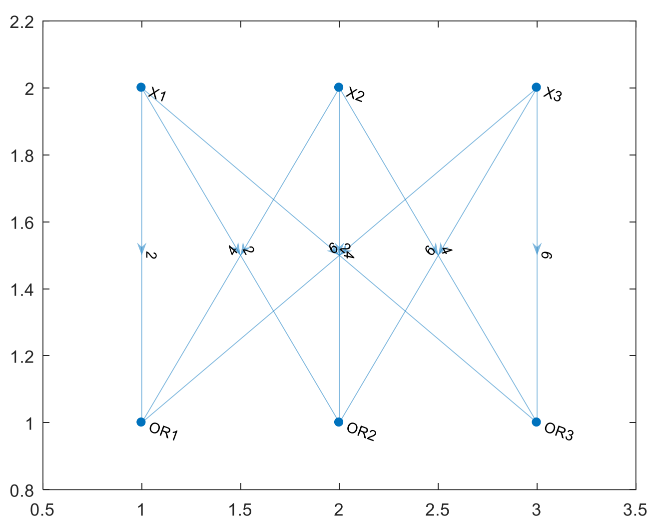
4. Problem 4 (20 pts) – Scheduling Problem:

Consider a hospital system consisting of three operating rooms, each operating room only permits 12 hours of operation per day. Suppose you have the following information:

Patient Type	Demand (Patients/day)	OR Time (hrs)	Profit/ operation
1	12	2	\$9,000
2	6	4	\$20,000
3	3	6	\$32,000

Let X1j = number of Type 1 patients scheduled for surgery in OR j, X2j = number of Type 2 patients scheduled for surgery in OR j, and X3 = number of Type 1 patients scheduled for surgery in OR j.

(a) (3) Draw a schematic (network) diagram of the system.



(b) (5) Develop and submit a MATLAB program that will help you solve problems 4.(c)-(d) below.

(See MATLAB code HW_3_4_Sched.m)

(c) (7) Suppose your customer's objective is to perform the maximum number of surgeries.

a. (2) Write down the objective function.

$Z = \sum \sum Xij$

b. (2) Express the problem in standard form.

- Maximize $Z = \sum \sum Xij$
- Subject to:

        - $X14 + X15 + X16 <= 12$
        - $X24 + X25 + X26 <= 6$
        - $X34 + X35 + X36 <= 3$
        - $2*X14 + 4*X24 + 6*X34 <= 12$
        - $2*X15 + 4*X25 + 6*X35 <= 12$
        - $2*X16 + 4*X26 + 6*X36 <= 12$
    - X_{ij} are **integers** >= 0
  c. (3) Find the solution (number of surgeries of each type performed in each OR and the total number of surgeries performed).
    - MATLAB solution is:

      Xns = [6 6 0 0 0 3 0 0 0]

      Ns = 15
    - **=> Max number of surgeries of 15 is obtain from:**
        - **OR 1 doing 6 Type 1 surgeries**
        - **OR 2 doing 6 Type 1 surgeries**
        - **OR 3 doing 3 Type 2 surgeries.**

(d) (7) Suppose your customer's objective is to obtain maximum profit.
  a. (2) Write down the objective function.
    - $Z = \sum (\$9{,}000\ X1j + \$20{,}000\ X2j + 32{,}000\ X3j)$
  b. (2) Express the problem in standard form.
    - Maximize $Z = \sum \$9{,}000 X_{ij}$
    - Subject to: Same constraints as in (c)
  c. (3) Find the solution (number of surgeries of each type performed in each OR and the total profit).
    - MATLAB solution is:

      Xpft = [0 1.00 0 3.00 1.00 0 0 1.00 2.00]

      Profit = 185.0000
    - **=> Max profit of $185,000 is obtained from:**
        - **OR1 doing 3 Type 2 surgeries,**
        - **OR 2 doing 1 Type 1 surgery, 1 Type 2 surgery and 1 Type 3 surgery**
        - **OR 3 doing 2 Type 3 surgeries.**

(e) (3) What are some problems with each of these strategies?
  - Under the maximize number of surgeries objective, no Type 3 surgeries are performed.
  - Under the maximize profit strategy, very few Type 1 surgeries are performed.

(f) (2) How might you use multi-objective value/utility function to address some of these problems?

- One might develop a MAVF that addresses the importance of metrics other than profit (e.g., live saved, meeting the needs of the community, etc.).

(g) (3) What did you learn from this problem?

- Accept whatever is provided.

5. **Problem 5 (20 pts) – Transshipment/Production Problem:**

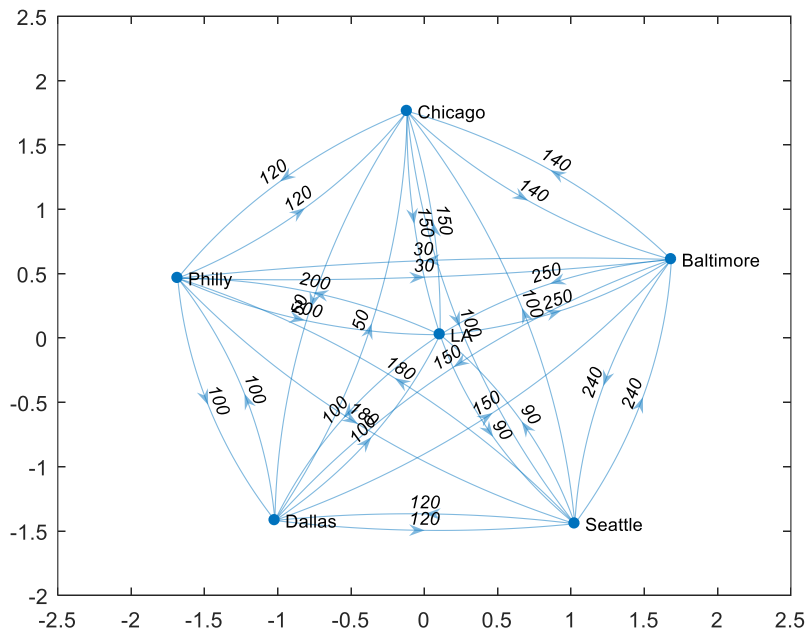
Suppose that you have two plants that manufacture drones, one in Chicago and one in Dallas. Suppose the Chicago plant can produce a maximum of 100 drones per week and the Dallas Plant can produce 200 drones/week. Suppose there are two major markets you serve, one in LA and the other in Baltimore. Suppose that the demand for drones in LA is 130 and the demand in Baltimore is 140 (drones per week).

Suppose there are the following per drone costs associated with shipping drones from one city to another (assume costs are symmetric (i.e. Cost i->j = Cost j->i):

	To					
From	**LA**	**Seattle**	**Chicago**	**Dallas**	**Philadelphia**	**Baltimore**
LA	-	\$90	\$150	\$100	\$200	\$250
Seattle		-	\$100	\$120	\$180	\$240
Chicago			-	\$50	\$120	\$140
Dallas				-	\$100	\$150
Philadelphia					-	\$30
Baltimore						-

Your manager wants to know the production and shipping schedule that will meet demand at the least cost.

(a) (4) Draw a schematic (network) diagram of the system.



(b) (2) Write down the objective function. Be sure to clearly define the decision variables (factors).

$Z = \sum\sum Xij$

Xij = cost of shipping from i to j

Let 1 = LA, 2 = Seattle, ... 6 = Baltimore

(c) (5) Express the problem in standard form.

- Minimize Z
- Subject to:
  - -(X31 + X32 + X34 + X35 + X36) >= -100
    - (Node 3 Flow: Supply of Chicago drones)
  - -(X41 + X42 + X43 + X45 + X46) >= -200
    - (Node 4 Flow: Supply of Chicago drones)

  - X21 + X31 + X41 + X51 + X61 = 130
    - (Node 1 Flow: Demand in LA)
  - (X21+X23+X24+X25+X26) – (X12+X32+X42+X52+X62) = 0
    - (Node 2 Flow: Out = In)
  - (X51+X52+X53+X54+X56) – (X15+X25+X35+X45+X65) = 0
    - (Node 5 Flow: Out = In)
  - X16 + X26 + X36 + X46 + X56 = 140
    - (Node 6 Flow: Demand in Baltimore)
- Xij are integers >= 0.

(d) (5) Develop and submit a MATLAB program that:

- Accepts the input values for the production constraints, demand, and transportation costs.
- Provides the following output:
  i. A graph of the network showing node names and costs.
  ii. The solution to the problem (number of drones shipped over each link and the resulting total cost).

(See MATLAB code HW_3_5_Transship.m)

(e) (5) Run the program and provide your output below.

- Following is command line output of MATLAB program.
- Graphical output is provided in answer to (a).
- Parenthetical elements in green are manual additions (indicating the variable associated with the solution output.

LP: Optimal objective value is 31900.000000.

Optimal solution found.

...

Soln =

```
    0    (= X12)
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
   70    (= X36)
  130    (= X41)
    0
    0
   70   (=X45)
    0
    0
    0
    0
    0
   70   (=X56)
    0
    0
    0
    0
    0
```

TotCost =

```
       31900
```

>>

(h) (4) How would you report the results of your analysis to your manager?

- The least expensive option is $32,900. For this option:
    - Produce **70 drones** in Chicago and ship them directly to Baltimore

- Produce **200 drones in Dallas** and
    - Ship 130 directly to LA
    - Ship 70 to Baltimore via Philadelphia.