

Consider the following network structure. You can assume the initial weights. Assume bias to be zero for easier computations. Given that  $\langle x_1, x_2, \hat{y}_1, \hat{y}_2 \rangle = \langle 1, 1, 0, 1 \rangle$  where  $\hat{y}$  is the target. Assume  $\alpha = 0.9$  and  $\beta = 0.01$ .

draw the mlp and solve below

- Compute the forward propagation and generate the output. Use Relu for hidden layers and Sigmoid activation function for output layer. [2]
- Compute the Softmax loss function for both outputs. [1]
- Let the initial weights that assumed be the weights at time  $(t-1)$ . Compute the weights  $w_{11}$ ,  $w_{12}$  and  $w_{22}$  at time  $t$  using SGD. [1.5]
- Let the weight at time  $t$  be the ones computed in part (c). Compute the weights  $w_{11}$ ,  $w_{12}$  and  $w_{22}$  at  $(t+1)$  when momentum is used. [1.5]

### ChatGPT

Given the input features  $x_1$  and  $x_2$  with initial weights and biases assumed as follows:

- $w_{11} = 0.2, w_{12} = 0.3, w_{21} = 0.4, w_{22} = 0.5$
- $v_{11} = 0.1, v_{12} = 0.2, v_{21} = 0.3, v_{22} = 0.4$
- $b_1 = 0, b_2 = 0$

Given input:  $x_1 = 1, x_2 = 1$ , target output:  $\hat{y}_1 = 0, \hat{y}_2 = 1$

Learning rate:  $\alpha = 0.01$ , momentum term:  $\beta = 0.9$

(a) Forward Propagation:

- The ReLU activation function is applied to the hidden layer, and the sigmoid activation function is applied to the output layer.

Forward propagation equations:

$$z_1 = w_{11}x_1 + w_{21}x_2 + b_1$$

$$z_2 = w_{12}x_1 + w_{22}x_2 + b_2$$

$$h_1 = \text{ReLU}(z_1)$$

$$h_2 = \text{ReLU}(z_2)$$

$$o_1 = \sigma(v_{11}h_1 + v_{21}h_2 + b_1)$$

$$o_2 = \sigma(v_{12}h_1 + v_{22}h_2 + b_2)$$

Let's calculate the forward propagation.

(b) Softmax Loss Function:

- The softmax loss function computes the loss based on the predicted output probabilities and the target labels.

Softmax loss function equation for each output:

$$L = -\sum_i y_i \log(\hat{y}_i)$$

Let's compute the softmax loss function for both outputs.

(c) Updating Weights using Stochastic Gradient Descent (SGD):

- Weights are updated using the gradient descent algorithm with the learning rate.

Weight update equation for each weight:

$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \alpha \frac{\partial L}{\partial w_{ij}}$$

Let's compute the updated weights  $w_{11}$ ,  $w_{12}$ , and  $w_{22}$  using SGD.

(d) Updating Weights with Momentum:

- Weights are updated using momentum, which incorporates the previous weight update direction.

Weight update equation with momentum for each weight:

$$\Delta w_{ij}^{(t)} = \beta \Delta w_{ij}^{(t-1)} + (1 - \beta) \frac{\partial L}{\partial w_{ij}}$$
$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \alpha \Delta w_{ij}^{(t)}$$

Let's compute the updated weights  $w_{21}$ ,  $w_{12}$ , and  $w_{22}$  using momentum.