Chapter 4
Adaptive Rejection Sampling Methods

Abstract This chapter is devoted to describing the class of the adaptive rejection sampling (ARS) schemes. These (theoretically) universal methods are very efficient samplers that update the proposal density whenever a generated sample is rejected in the RS test. In this way, they can produce i.i.d. samples from the target with an increasing acceptance rate that can converge to 1. As a by-product, these techniques also generate a sequence of proposal pdfs converging to the true shape of the target density. Another advantage of the ARS samplers is that, when they can be applied, the user only has to select a set of initial conditions. After the initialization, they are completely automatic, self-tuning algorithms (i.e., no parameters need to be adjusted by the user) regardless of the specific target density. However, the need to construct a suitable sequence of proposal densities restricts the practical applicability of this methodology. As a consequence, ARS schemes are often tailored to specific classes of target distributions. Indeed, the construction of the proposal is particularly hard in multidimensional spaces. Hence, ARS algorithms are usually designed only for drawing from univariate densities.

In this chapter we discuss the basic adaptive structure shared by all ARS algorithms. Then we look into the performance of the method, characterized by the acceptance probability (which increases as the proposal is adapted), and describe various extensions of the standard ARS approach which are aimed either at improving the efficiency of the method or at covering a broader class of target pdfs. Finally, we consider a hybrid method that combines the ARS and Metropolis-Hastings schemes.

4.1 Introduction

The main limitation of RS methods is the difficulty of finding a proposal function $\pi(x)$ and a bound $L \geq p(x)/\pi(x)$, such that the envelope function, $L\pi(x) \geq p(x)$, is actually "close" enough to the target density, which is required in order to attain good acceptance rates. One way to tackle this difficulty is by constructing the proposal $\pi(x)$ adaptively.

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