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Preparation of Papers for IFAC Conferences & Symposia: Kalman Filter Based Control of Inverted Pendulum System

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Abstract: Stabilization problem of inverted pendulum system is one of the challenging issues to the researchers. To stabilize this unstable inverted pendulum system, linear quadratic regulator (LQR) controller has been designed along with the Kalman filter. Kalman filter estimates the state variables of the system and LQR controller takes these estimated state variables as inputs. The performance of this proposed controller has been tested for both stabilization and tracking problem of inverted pendulum system. Both the stabilization and tracking responses of the closed-loop system satisfies the required performance specifications. The responses of the closed loop system with Kalman filter and without Kalman filter also have been compared. Kalman filter based closed-loop system eliminates the effects of process and sensor noises that are present in the outputs of the system. The performance of this proposed controller is also compared with LMI based robust controller that had been earlier designed for the same system.

Keywords: Inverted pendulum, state estimation, Kalman filter, linear quadratic regulator control, robust control.

1. INTRODUCTION

The balancing control of inverted pendulum system has been extensively attacked by many researchers as a benchmark problem. Inverted pendulum (IP) system is used in different applications such as humanoid robot, rocket, satellites, aircraft landing etc.

Researchers have designed different types of controllers for stabilizing and tracking this highly unstable IP system. For the design of linear quadratic regulator (LQR) controller for stabilizing quadruple inverted pendulum system, a new algorithm called weighted artificial fish swarm proposed in (Tijani et. al., 2015). The weight matrices of LQR controller had been designed using this optimization technique and authors showed that proposed algorithm determines weight matrices efficiently than the other optimization techniques like Artificial Bee Colony etc. Yang et. al., 2018 introduced a control strategy for stabilizing the rotary inverted pendulum system based on nonlinear adaptive neural network and linear matrix inequality technique. They have also used trajectory planning scheme in order to solve the swing up case of the system and verified with experimental results. An observer-based state-feedback controller for inverted pendulum system has been designed in literature (Rawata et. al., 2018) and simulation has been done using LabVIEW. They showed that the observer-based controller estimates the state variables accurately. A high-gain observer-based sliding mode controller has been suggested by Liu et. al. in 2019 and stated the advantages of high gain observer over the global state observer. Mondal et. al., 2019 proposed a fractional order proportional-integral-derivative (PID) controller for robust stabilization of inverted pendulum system where the parameters of the controller had been designed using three evolutionary optimization technique. A state-feedback robust controller has been designed using linear matrix inequality technique in 2020 by Alam et. al. and presented that proposed controller gives better result than the LQR controller. Maneetham et. al., 2020 designed LQR controller for inverted pendulum and applied to the practical system using internet of things.

State estimation plays a vital role in case of state-feedback controller as accuracy of the controller depends on the measurements of the state variables of the system. The state variables of the system can be estimated using Kalman filter which was introduced by Kalman and Bucy in 1961 (Kalman et. al., 1961). Kalman filter is used in different system for estimation of the state variables (Auger et. al., 2013; Li et. al., 2015). Sen et. al., 2015 used Kalman filter for estimation of tire angles of vehicle. For estimation of distributed state, Wu et. al., 2016 proposed consensus-based Kalman filter. Valade et. al. (2018) applied Kalman filter to precisely estimate the physical value of a multi-sensor, multi-physical model. Grewal, 2019 implemented SigmaRho Kalman filter to estimate the parameters in satellite navigation receiver. Kalman filter also has been applied in biological field. Pascucci et. al., 2020 used self-tuning Kalman filter for modeling the brain network. Singh et. al., 2021 used Kalman filter to estimate the future spread of Covid-19.

In this study, Kalman filter based linear quadratic regulator controller has been designed for stabilizing and tracking inverted pendulum system. The main contribution of the present work is given below.

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- Kalman filter has been designed to estimate the state variables of servo motor driven inverted pendulum system to minimize the effects of sensor and process noises on the outputs of the system.
- Linear quadratic regulator controller has been designed to stabilize the system and to get better tracking performance from the system.

2. INVERTED PENDULUM SYSTEM

Three basic components of IP system are a pendulum, a cart and a D.C. servomotor. The pendulum rod is attached to the cart which is driven by the D.C. servomotor along the horizontal direction. The control objective is to keep the pendulum rod at the upright position in presence of external disturbances and when the cart moves from one position to another along the track.

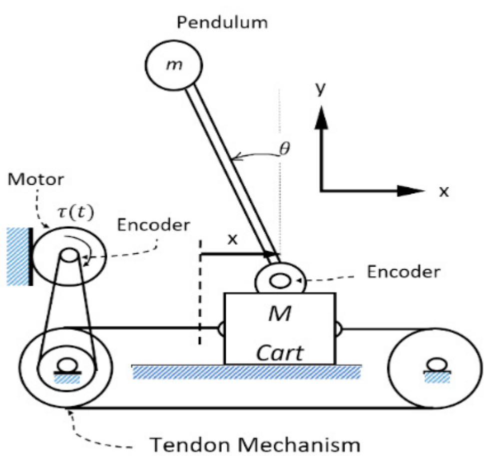

Figure 1. Servo-driven inverted pendulum system

The servo-driven IP system is shown in Figure 1 and values of the parameters of the system (Single Inverted Pendulum (SIP), User Manual, 3rd ed., Quanser Consulting, Inc.) are given in Table 1. Here, the angular position of the pendulum rod with respect to vertical position is considered as θ and the cart position is denoted by x_c. For the system D.C. servomotor voltage(u) is the input and x_c, θ are the outputs of the system. The system is highly nonlinear system and is linearized considering θ is very small i.e., pendulum rod is very close to vertical axis. The differential equation representing IP system (Single Inverted Pendulum (SIP), User Manual, 3rd ed., Quanser Consulting, Inc.) are

$$\ddot{x}_c = \frac{1}{J_T}(-(J_P + m_P l_P^2)B_{eq}\dot{x}_c - m_P l_P \dot{\theta} + m_P^2 l_P^2 g\theta + (J_P + m_P l_P^2)F_c)$$

$$\ddot{\theta} = \frac{1}{J_T}(-(m_P l_P B_{eq})\dot{x}_c - (J_{eq} + M_P)B_P\dot{\theta} + (J_{eq} + M_P)M_P l_P g\theta + M_P l_P F_c)$$

where $J_T = J_{eq}J_P + M_P J_P + J_{eq}M_P l_P^2$; $J_{eq} = M_c + \frac{\eta_g K_g^2 J_m}{r_{mp}^2}$

The state variables of the system are $x_1 = x_c, x_2 = \theta, x_3 = \dot{x}_c$ & $x_4 = \dot{\theta}$. The linearized state-space model of the servo-driven IP system is given below.

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\dot{x}_4\end{bmatrix} = \frac{1}{J_T}\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\0 & m_P^2 l_P^2 g & W & -m_P l_P B_P\\0 & (J_C + m_P)m_P l_P g & X & -(J_C + m_P)B_P\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \frac{1}{J_T}\begin{bmatrix}0\\0\\Y\\Z\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}u$$

where, $W = (J_P + m_P l_P^2)\left[-B_{eq} - \frac{\eta_g K_g^2 \eta_m K_T K_m R_m}{r_{mp}^2}\right]$;

$$X = m_P l_P\left[-B_{eq} - \frac{\eta_g K_g^2 \eta_m K_T K_m R_m}{r_{mp}^2}\right];$$

$$Y = (J_P + m_P l_P^2)\frac{\eta_g K_g \eta_m K_T R_m}{r_{mp} J_T};\ Z = m_P l_P \frac{\eta_g K_g \eta_m K_T R_m}{r_{mp}}.$$

Table 1. Description and values of the parameters of IP system

Description of the parameters	Value
Mass of the cart M_c	0.57kg
Mass of the pendulum m_p	0.127kg
Pendulum rod length l_p	0.1778m
Moment of inertia of pendulum J_P	1.1988×10^{-3} kg.m^2
Rotor inertia of servo motor J_m	3.9×10^{-7} kg.m^2
Pinion radius of motor r_{mp}	6.35×10^{-3}m
Planetary gearbox gear ratio K_g	3.71
Equivalent viscous damping coefficient B_{eq}	5.4Nms/rad
Armature resistance of servo motor R_m	2.6Ω
Back emf constant of servo motor K_m	7.68×10^{-3}N m/A
Torque constant of servo motor K_t	7.68×10^{-3}N m/A
Acceleration due to gravity g	9.81 m/s^2
Motor efficiency η_m	7.68×10^{-3}N m/A
Planetary gearbox efficiency η_g	1

3. STATE ESTIMATION WITH KALMAN FILTER

Kalman filter estimates the state variables of the system based on the information of input and output of the system. In this study, the linearized state space model of nonlinear IP system has been taken for analysis and design of the controller. The model could have many unmodelled uncertainties. The output of the system also has been measured using sensors. Kalman filter estimates the state variables of the system accurately in presence of process noise and sensor noise. The recursive predictor-corrector algorithm of Kalman filter (Kalman et. al., 1961) is given in Figure 2.

Consider a linear time-invariant, discrete-time system model as

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (1)$$
$$y_k = Cx_k + v_k \quad (2)$$

Here w_k and v_k are process and sensor noise respectively which are considered as random Gaussian noise which has a finite covariance and zero mean.

$w(k) \approx N(0, Q_w)$ and $v(k) \approx N(0, Q_v)$

where Q_w and Q_v denotes the error covariance of process and sensor noise respectively. The process noise covariance Q_w depends on the uncertainty present in the model of the system. But in practical system, Q_w can be determined from the noise level of the steady-state state variables of the system. Sensor noise covariance Q_v can be determined from the sensor sensitivity which can be found from the data sheet of the manufacturer.

Consider initial value of state and error covariance matrices as $\hat{x}_0$ and P_0 respectively.

Predictor equations (Kalman et. al., 1961) are-

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \quad (3)$$
$$P_{k+1|k} = AP_{k|k}A^T + Q_w \quad (4)$$

The measurement update equations are given below.

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}[y_{k+1|k} - C\hat{x}_{k+1|k}] \quad (5)$$
$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k} \quad (6)$$

The Kalman filter gain is given by-

$$K_{k+1} = P_{k+1|k}C^T(Q_v + CP_{k+1|k}C^T)^{-1}$$

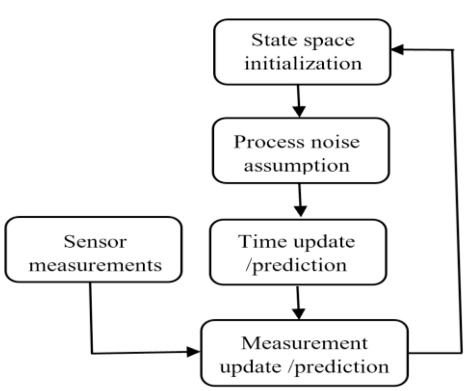

Figure 2. Recursive algorithm of Kalman filter

4. LQR CONTROLLER

Linear quadratic regulator (LQR) controller is one of the popular and efficient state-feedback controllers whose gain is determined by optimizing a quadratic cost function (Naidu, D. S., 2002). Consider a linear time-invariant, continuous-time, stabilizable system whose state equation is given by

$$\dot{x}(t) = Ax(t) + Bu(t); x(0) = x_0 \quad (7)$$

where $x(t)$ and $u(t)$ are state vector and control input respectively.

The control input is given by $u(t) = -Kx(t)$ where K is gain of the controller and is obtained by minimizing the performance index (PI) for infinite time

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt$$

where Q is a positive semi-definite matrix and R is positive definite matrix.

The gain of the LQR controller is $K = R^{-1}B^TP$ and P is the solution of algebraic Riccati equation

$$A^TP + PA - (PB)R^{-1}(B^TP) + Q = 0 \quad (8)$$

Here Q and R are weight matrices and have great influences on system performance. In this study, Q and R have been selected using trial and error method.

5. SIMULATION RESULTS

Servo-driven IP system is 4th order, single-input multi-output system. Servo-motor voltage (u) is the input and cart position (x_C), pendulum angle (θ) are the outputs. The transfer función of the open-loop system between input u and output x_C is given below.

$$\frac{X_C(s)}{U(s)} = \frac{1.5662(s + 6.753)(s - 6.292)}{s(s - 6.385)(s + 6.385)(s + 12.42)}$$

The open-loop transfer function shows that system is unstable and non-minimum phase system. In this study, Kalman filter has been designed to estimate the state variables of system and based on the estimated states LQR controller has been designed in order to stabilize this unstable system.

5.1 Design of Kalman filter

Kalman filter identifies the state variables of the system in presence of model and sensor noises. In this problem, Kalman filter (KF) has been designed using MATLAB. The process noise covariance Q_W and sensor noise covariance Q_v had been determined using trial and error method and chosen as $Q_W =$ diag([0.0001 0.00001 0.0001 0.00001]); $Q_v = diag$([0.0001 0.00001]).
The process and sensor noises are taken as band limited white noise. The estimated and actual state variables of the open-loop system are shown in Figure 3 and Error covariance matrix becomes

$$P = \begin{bmatrix} 0.0001003 & 1.759e^{-7} & 3.502e^{-7} & -2.593e^{-6} \\ 1.759e^{-7} & 0.0001295 & 5.048e^{-6} & 0.0008337 \\ 3.502e^{-7} & 5.048e^{-6} & 4.438e^{-6} & 2.071e^{-5} \\ -2.593e^{-6} & 0.0008337 & 2.071e^{-5} & 0.005485 \end{bmatrix}$$

From the response and matrix P, it can be seen that KF identifies the state variables of the system accurately and also it eliminates the effect of sensor and process noise. As the system is unstable so the state variables are here unbounded signals.

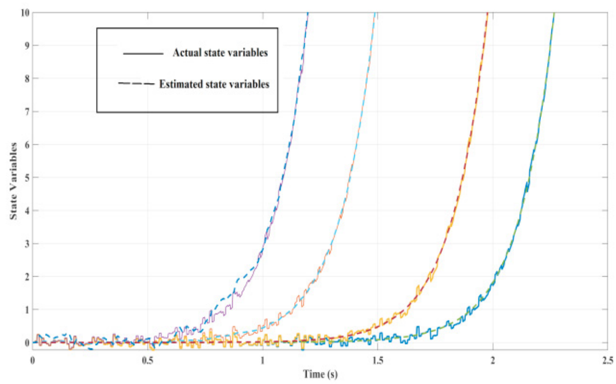

Figure 3. Actual and estimated state variables of the system

5.2 Design of Kalman filter based LQR controller

To stabilize the highly nonlinear system, LQR controller has been designed and the required specifications for the closed-loop system are

- The settling time should be less than 5s with minimum overshoot, undershoot and zero steady-state error for the horizontal cart movement on the track.
- The controller should give satisfactory performance in presence of external disturbances.

For the design of state feedback LQR controller, the weight matrices are chosen as
$Q = diag([120\ 100\ 0.1\ 0.1])$; $R = 1$.

The LQR controller gain which is determined using MATLAB is $K = [-10.9545 \quad 47.112 \quad -19.7811 \quad 6.7336]$. The SIMULINK block dia. of the closed-loop system with KF and LQR controller is given in Figure 4. Voltage rating of D.C. servomotor is ±24V. Hence in this block, saturation block is taken to limit the control input voltage limit.

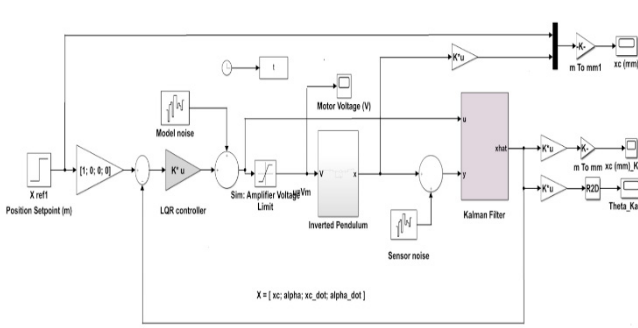

Figure 4. SIMULINK block diagram of closed-loop system with KF and LQR controller

- *Stabilization problem*

In order to test stabilization, the system is considered as regulator problem and excited by initial condition $[0.01 \quad 0 \quad 0 \quad 0]^T$ without external noises. The closed-loop responses of the regulator system such as cart position, pendulum angle and control input (servomotor voltage) are given in Figure 5, Figure 6 and Figure 7 respectively. From the responses (Figure 5 and Figure 6), it can be observed that the settling time is approximately 4s. The proposed controller stabilizes the nonlinear system perfectly. But when initial condition of the pendulum rod angle (θ) becomes high ($\theta > 60^0$), the controller will not be able to stabilize this unstable system.

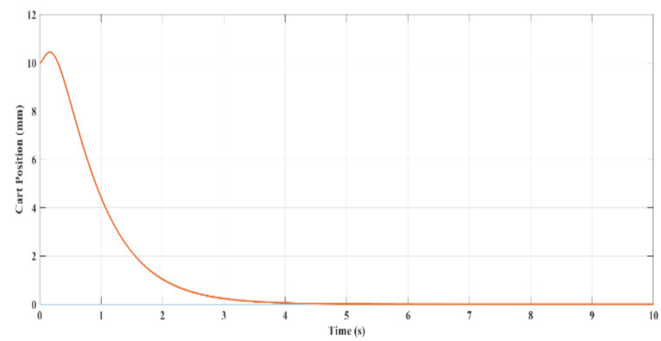

Figure 5. Cart position for stabilization problem

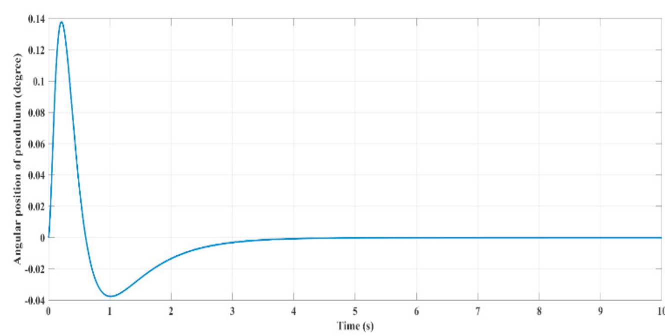

Figure 6. Pendulum angular position for stabilization problem

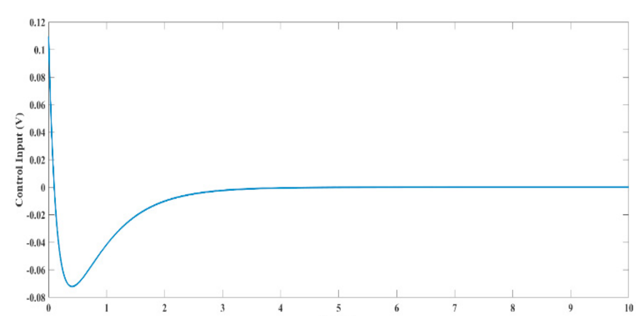

Figure 7. Control input for stabilization problem

- *Tracking problema in presence of process and sensor noises*

For tracking problem, 10mm cart position is taken as the reference input and the performance of the controller is tested in presence of process and sensor noise. The responses of the closed-loop system with KF and without KF are given from

Figure 8 – 10. The responses show that KF eliminates the effects of process and sensor noises on the outputs of the system. The transient response specifications of closed-loop system are given in Table 2.

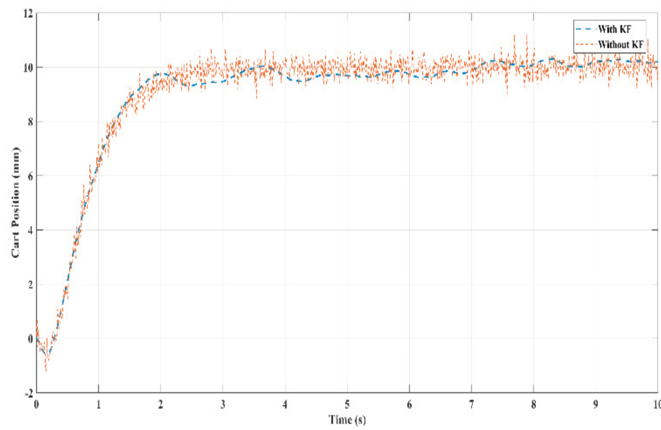

Figure 8. Cart position of closed-loop system in presence of noise

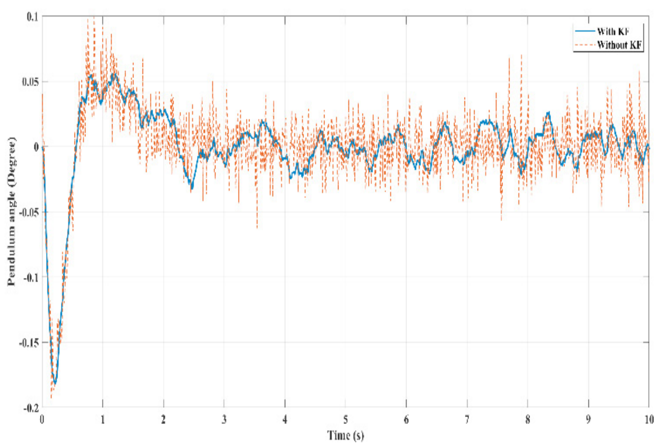

Figure 9. Pendulum angle of closed-loop system in presence of noise

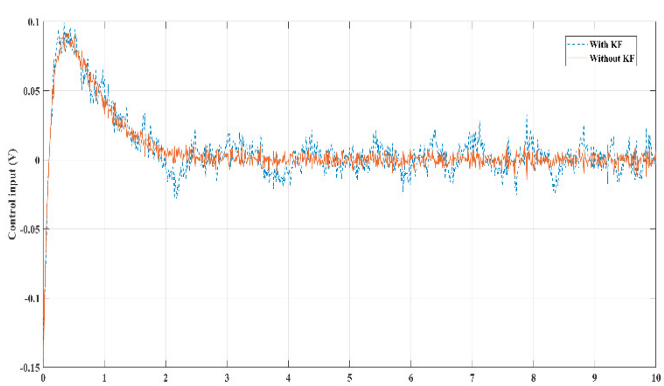

Figure 10. Control input of closed-loop system in presence of noise

Table 2. Transient response specifications for tracking problem

Transient response specifications	Value
Settling time	4s
Maximum overshoot	0
Maximum undershoot	0.5

The tracking performance of this KF based LQR controller has been compared with the performance of LMI based robust controller (Alam et. al., 2020). Cart positions of both controllers are shown in Figure 11 for reference cart position 20mm. Responses show that both controllers eliminate the sensor and process noises efficiently.

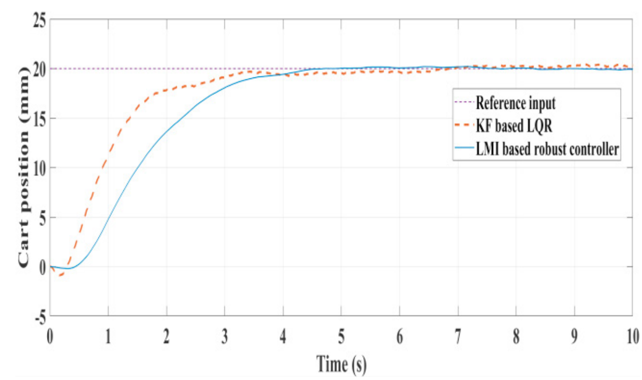

Figure 11. Cart position of proposed controller and LMI based robust controller in presence of noise

6. CONCLUSIONS

In present study, Kalman filter (KF) has been designed to estimate the state variables of the inverted pendulum system efficiently and for the control of the system LQR controller has been designed. The KF based controller gives better result in presence of process and sensor noise than without KF controller. The proposed controller gives robust performance in presence of noise. In order to estimate, the state variables of the system more accurately extended Kalman filter (EKF) may be designed in future.

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