

Assignment 1

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ICSE 2018 Question 6 (a)

(a) Using properties of proportion, solve for x . Given that x is positive:

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

Solution:

From the properties of proportion

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \quad - (1)$$

The Given Equation is :

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = \frac{4}{1}$$

From (1)

$$\frac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \frac{4 + 1}{4 - 1}$$

$$\Rightarrow \frac{4x}{2\sqrt{4x^2 - 1}} = \frac{5}{3}$$

$$\Rightarrow \frac{6x}{5} = \sqrt{4x^2 - 1}$$

$$\Rightarrow \frac{36x^2}{25} = 4x^2 - 1$$

$$\Rightarrow 1 = \left(4 - \frac{36}{25}\right)x^2$$

$$\Rightarrow x^2 = \frac{25}{64}$$

$$\Rightarrow x = \pm \frac{5}{8}$$

But Given x is positive

$$\Rightarrow x = \frac{5}{8}$$

Verification with Graph:

If we draw the graph of

$$y = f(x) = \frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}}$$

and find point of intersection with $y = 4$, we get $x = 0.625 = \frac{5}{8}$

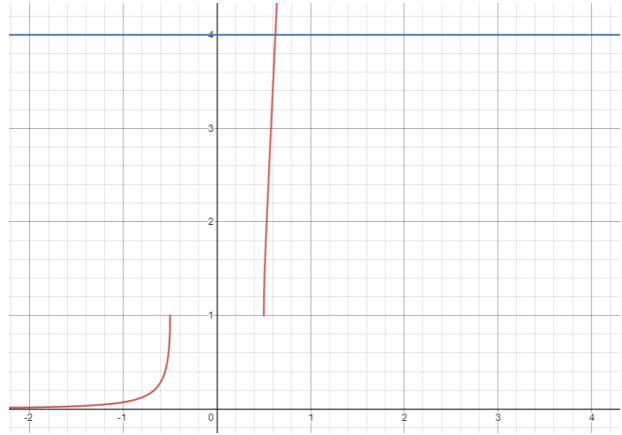


Figure 1: graph of $y = f(x)$

$$\therefore x = \frac{5}{8} = 0.625$$