Assignment 11

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QUESTION :

Show that if the random variables x, y, and z are jointly normal and independent in pairs, they are independent.

SOLUTION :

First here we assume $\eta_x = 0, \eta_y = 0, \eta_z = 0$ where η_x, η_y, η_z are the mean values of the random variables x, y, z respectively.
Given x, y, z are jointly normal and pairwise independent.
$x = N(0, \sigma_x), y = N(0, \sigma_y), z = N(0, \sigma_z)$

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-(x-\eta_x)^2/2{\sigma_x}^2} \tag{1}$$

$$\Rightarrow f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-x^2/2{\sigma_x}^2} \tag{2}$$

Similarly

$$f_y(y) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{-y^2/2{\sigma_y}^2} \tag{3}$$

$$f_z(z) = \frac{1}{\sigma_z\sqrt{2\pi}} e^{-z^2/2{\sigma_z}^2} \tag{4}$$

We know that if the random variables x_i for $i = 1, 2, 3...n$ are jointly normal

$$f(x_1, x_2, ..., x_n) = \frac{1}{\sigma_1\sigma_2......\sigma_n\sqrt{(2\pi)^n}} e^{-\frac{1}{2}\left(\frac{x_1^2}{\sigma_1^2} + ... \frac{x_n^2}{\sigma_n^2}\right)} \tag{5}$$

$$\Rightarrow f(x, y, z) = \frac{1}{\sigma_x\sigma_y\sigma_z\sqrt{(2\pi)^3}} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)} \tag{6}$$

$$\Rightarrow f(x, y, z) = \left(\frac{e^{-x^2/2{\sigma_x}^2}}{\sigma_x\sqrt{2\pi}}\right)\left(\frac{e^{-y^2/2{\sigma_y}^2}}{\sigma_y\sqrt{2\pi}}\right)\left(\frac{e^{-z^2/2{\sigma_z}^2}}{\sigma_z\sqrt{2\pi}}\right) \tag{7}$$

$$\Rightarrow f(x, y, z) = f_x(x)f_y(y)f_z(z) \tag{8}$$

$\therefore x, y, z$ are independent.