

AI1110 Assignment-10

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Outline

1 Question

2 Solution

Question

Show that if the random variables x, y , and z are jointly normal and independent in pairs, they are independent.

Solution

First here we assume $\eta_x = 0, \eta_y = 0, \eta_z = 0$ where η_x, η_y, η_z are the mean values of the random variables x, y, z respectively.

Given x, y, z are jointly normal and pairwise independent.

$$x = N(0, \sigma_x), y = N(0, \sigma_y), z = N(0, \sigma_z)$$

$$f_x(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-(x-\eta_x)^2/2\sigma_x^2} \quad (1)$$

$$\Rightarrow f_x(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-x^2/2\sigma_x^2} \quad (2)$$

Similarly

$$f_y(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{-y^2/2\sigma_y^2} \quad (3)$$

$$f_z(z) = \frac{1}{\sigma_z \sqrt{2\pi}} e^{-z^2/2\sigma_z^2} \quad (4)$$

We know that if the random variables x_i for $i = 1, 2, 3 \dots n$ are jointly normal

$$f(x_1, x_2, \dots, x_n) = \frac{1}{\sigma_1 \sigma_2 \dots \sigma_n \sqrt{(2\pi)^n}} e^{-\frac{1}{2} \left(\frac{x_1^2}{\sigma_1^2} + \dots + \frac{x_n^2}{\sigma_n^2} \right)} \quad (5)$$

$$\Rightarrow f(x, y, z) = \frac{1}{\sigma_x \sigma_y \sigma_z \sqrt{(2\pi)^3}} e^{-\frac{1}{2} \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2} \right)} \quad (6)$$

$$\Rightarrow f(x, y, z) = \left(\frac{e^{-x^2/2\sigma_x^2}}{\sigma_x \sqrt{2\pi}} \right) \left(\frac{e^{-y^2/2\sigma_y^2}}{\sigma_y \sqrt{2\pi}} \right) \left(\frac{e^{-z^2/2\sigma_z^2}}{\sigma_z \sqrt{2\pi}} \right) \quad (7)$$

$$\Rightarrow f(x, y, z) = f_x(x) f_y(y) f_z(z) \quad (8)$$

$\therefore x, y, z$ are independent.