

AI1110 Assignment-12

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Outline

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Question

In a production process the number of defective units per hour is a Poisson distributed random variable x with parameter $\lambda = 5$. A new process is introduced and it is observed that the hourly defectives in a 22-hour period are

$$x_i = 3, 0, 5, 4, 2, 6, 4, 1, 5, 3, 7, 4, 0, 8, 3, 2, 4, 3, 6, 5, 6, 9$$

Test the hypothesis $\lambda = 5$ against $\lambda < 5$ with $\alpha = 0.05$

Solution

We shall use the sum of x_i as Test Static (q)

$$q = x_1 + x_2 + \dots + x_n$$

Here q is also a poisson random variable with parameter $\eta_q = n\lambda$

we need to test the hypothesis H_0 ($\lambda = 5$)

Under Hypothesis H_0 , $\lambda = \lambda_0 = 5$

The critical region of the hypothesis is $q < q_\alpha$, where

$$q = x_1 + x_2 + \dots + x_n = 90$$

To find q_α we use the normal approximation method with $\alpha = 0.05$

$$q_\alpha = n\lambda_0 + z_\alpha \sqrt{n\lambda_0} \quad (1)$$

(2)

Here $n = 22, \lambda_0 = 5, \alpha = 0.05$,

$$\Rightarrow z_{\alpha} = z_{0.05} \quad (3)$$

$$\Rightarrow z_{\alpha} = -z_{1-0.05} \quad (4)$$

$$\Rightarrow z_{\alpha} = -z_{0.95} \quad (5)$$

$$\Rightarrow z_{\alpha} = -1.645 \quad (6)$$

$$q_{\alpha} = 110 - (1.645)(\sqrt{110}) \quad (7)$$

$$\Rightarrow q_{\alpha} = 110 - 17.25 \quad (8)$$

$$\Rightarrow q_{\alpha} = 92.75 \quad (9)$$

Here the Hypothesis H_1 is $\lambda < \lambda_0$

We accept H_0 iff $q > q_{\alpha}$.

Here $q < q_{\alpha}$.

So we reject the hypothesis H_0 .