Assignment 12

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QUESTION :

In a production process the number of defective units per hour is a Poisson distributed random variable x with parameter $\lambda = 5$. A new process is is introduced and it is observed that the hourly defectives in a 22-hour period are
$x_i = 3, 0, 5, 4, 2, 6, 4, 1, 5, 3, 7, 4, 0, 8, 3, 2, 4, 3, 6, 5, 6, 9$
Test the hypothesis $\lambda = 5$ against $\lambda < 5$ with $\alpha = 0.05$

SOLUTION :

We shall use the sum of x_i as Test Static (q)
$q = x_1 + x_2 + ... + x_n$
Here q is also a poisson random variable with parameter $\eta_q = n\lambda$
we need to test the hypothesis H_0 ($\lambda = 5$)
Under Hypothesis H_0 , $\lambda = \lambda_0 = 5$
The critical region of the hypothesis is $q < q_\alpha$, where
$q = x_1 + x_2 + ... + x_n = 90$
To find q_α we use the normal approximation method with $\alpha = 0.05$

$$q_\alpha = n\lambda_0 + z_\alpha\sqrt{n\lambda_0} \tag{1}$$
$$\tag{2}$$

Here $n = 22, \lambda_0 = 5, \alpha = 0.05$,

$$\Rightarrow z_\alpha = z_{0.05} \tag{3}$$
$$\Rightarrow z_\alpha = -z_{1-0.05} \tag{4}$$
$$\Rightarrow z_\alpha = -z_{0.95} \tag{5}$$
$$\Rightarrow z_\alpha = -1.645 \tag{6}$$

$$q_\alpha = 110 - (1.645)(\sqrt{110}) \tag{7}$$
$$\Rightarrow q_\alpha = 110 - 17.25 \tag{8}$$
$$\Rightarrow q_\alpha = 92.75 \tag{9}$$

Here the Hypothesis H_1 is $\lambda < \lambda_0$
We accept H_0 iff $q > q_\alpha$.
Here $q < q_\alpha$.
So we reject the hypothesis H_0.