Assignment 4

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Question :

In a factory which manufactures bolts, machines A, B and C manufacture respectively 25% , 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

Solution :

Let events B_1, B_2, B_3 be the following :
B_1 : the bolt is manufactured by machine A
B_2 : the bolt is manufactured by machine B
B_3 : the bolt is manufactured by machine C
A bolt must be manufactured from exactly one of the machines A,B,C.
Therefore B_1, B_2, B_3 are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space. Let the event E be 'the bolt is defective'.
The event E occurs with B_1 or with B_2 or with B_3.
Given that

$$\Pr(B_1) = 25\% = 0.25 \tag{1}$$
$$\Pr(B_2) = 35\% = 0.35 \tag{2}$$
$$\Pr(B_3) = 40\% = 0.4 \tag{3}$$

And also $\Pr(E|B_1)$ = Probability that the bolt drawn is defective given that the bolt is manufactured from machine A $= 5\% = 0.05$
Similarly

$$\Pr(E|B_1) = 5\% = 0.05 \tag{4}$$
$$\Pr(E|B_2) = 4\% = 0.04 \tag{5}$$
$$\Pr(E|B_3) = 2\% = 0.02 \tag{6}$$

We need to find the Probability that bolt is manufactured by B_2, Given that the bolt is defective i.e the value of $\Pr(B_2|E)$
From Bayes Theorem ,

$$\Pr(B_2|E) = \frac{\Pr(B_2)\Pr(E|B_2)}{\Pr(B_1)\Pr(E|B_1) + \Pr(B_2)\Pr(E|B_2) + \Pr(B_3)\Pr(E|B_3)} \tag{7}$$

$$\Rightarrow \Pr(B_2|E) = \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02} \tag{8}$$

$$\Rightarrow \Pr(B_2|E) = \frac{0.014}{0.0125 + 0.014 + 0.008} \tag{9}$$

$$\Rightarrow \Pr(B_2|E) = \frac{0.014}{0.0345} \tag{10}$$

$$\Rightarrow \Pr(B_2|E) = \frac{28}{69} \tag{11}$$

$$\therefore \Pr(B_2|E) = \frac{28}{69} = 0.4058 \tag{12}$$

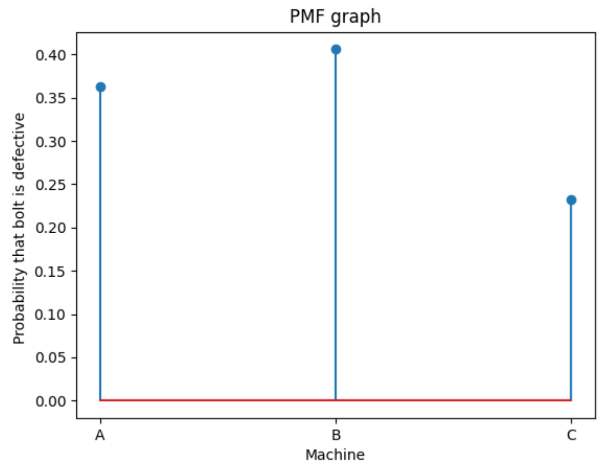

Fig. 1. PMF graph