AI1110 Assignment-6

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L^AT_EX

Outline

Question

In a factory which manufactures bolts, machines A, B and C manufacture respectively 25% , 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

Solution

Let the random variable X denote the following :

$X = 0$: the bolt is manufactured by machine A

$X = 1$: the bolt is manufactured by machine B

$X = 2$: the bolt is manufactured by machine C

A bolt must be manufactured from exactly one of the machines A,B,C. Therefore $X = 0, X = 1, X = 2$ are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space.

Let the random variable Y denote the following :

$Y = 0$: the bolt drawn at random is defective

$Y = 1$: the bolt drawn at random is not defective

Given that

$$\Pr(X = 0) = 25\% = 0.25 \tag{1}$$
$$\Pr(X = 1) = 35\% = 0.35 \tag{2}$$
$$\Pr(X = 2) = 40\% = 0.4 \tag{3}$$

And

$$\Pr(Y = 0|X = 0) = 5\% = 0.05 \tag{4}$$
$$\Pr(Y = 0|X = 1) = 4\% = 0.04 \tag{5}$$
$$\Pr(Y = 0|X = 2) = 2\% = 0.02 \tag{6}$$

From Bayes Theorem ,

$$\Pr(X = 1|Y = 0) = \frac{\Pr(X = 1)\Pr(Y = 0|X = 1)}{\Pr(X = 0)\Pr(Y = 0|X = 0) + \Pr(X = 1)\Pr(Y = 0|X = 1) \atop + \Pr(X = 2)\Pr(Y = 0|X = 2)} \tag{7}$$

$$\Rightarrow \Pr\left(X=1|Y=0\right) = \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02} \tag{8}$$

$$\Rightarrow \Pr\left(X=1|Y=0\right) = \frac{0.014}{0.0125 + 0.014 + 0.008} \tag{9}$$

$$\Rightarrow \Pr\left(X=1|Y=0\right) = \frac{0.014}{0.0345} \tag{10}$$

$$\Rightarrow \Pr\left(X=1|Y=0\right) = \frac{28}{69} \tag{11}$$
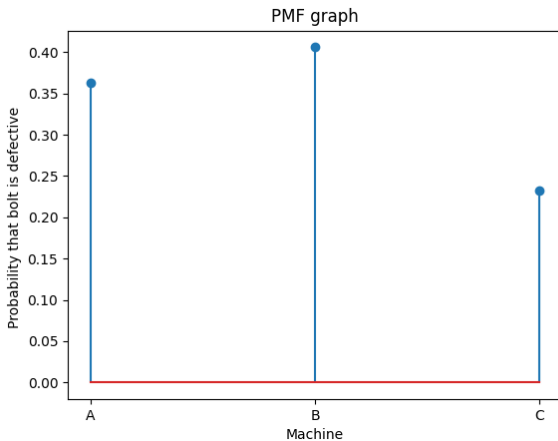
$$\therefore \Pr\left(X=1|Y=0\right) = \frac{28}{69} = 0.4058 \tag{12}$$

Graph

Figure: Probability Mass Function(PMF) graph