Assignment 6

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QUESTION :

In a factory which manufactures bolts, machines A, B and C manufacture respectively 25% , 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

SOLUTION :

Let the random variable X denote the following :
$X = 0$: the bolt is manufactured by machine A
$X = 1$: the bolt is manufactured by machine B
$X = 2$: the bolt is manufactured by machine C
A bolt must be manufactured from exactly one of the machines A,B,C.
Therefore $X = 0, X = 1, X = 2$ are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space.
Let the random variable Y denote the following :
$Y = 0$: the bolt drawn at random is defective
$Y = 1$: the bolt drawn at random is not defective
Given that

$$\Pr(X = 0) = 25\% = 0.25 \tag{1}$$
$$\Pr(X = 1) = 35\% = 0.35 \tag{2}$$
$$\Pr(X = 2) = 40\% = 0.4 \tag{3}$$

And

$$\Pr(Y = 0|X = 0) = 5\% = 0.05 \tag{4}$$
$$\Pr(Y = 0|X = 1) = 4\% = 0.04 \tag{5}$$
$$\Pr(Y = 0|X = 2) = 2\% = 0.02 \tag{6}$$

From Bayes Theorem ,

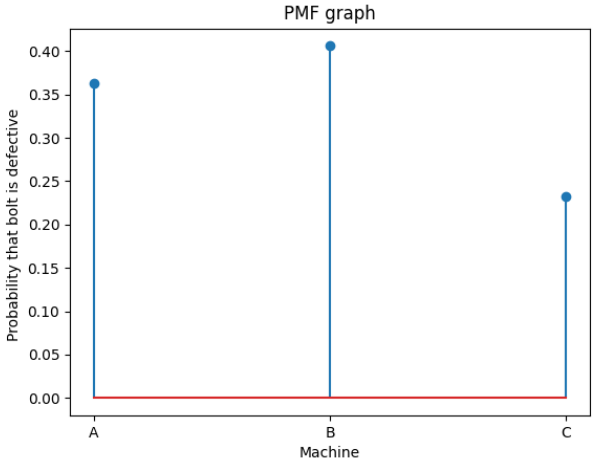

Fig. 1. PMF graph

$$\Pr(X = 1|Y = 0) = \frac{\Pr(X = 1)\Pr(Y = 0|X = 1)}{\Pr(X = 0)\Pr(Y = 0|X = 0) + \Pr(X = 1)\Pr(Y = 0|X = 1) + \Pr(X = 2)\Pr(Y = 0|X = 2)} \tag{7}$$

$$\Rightarrow \Pr(X = 1|Y = 0) = \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02} \tag{8}$$

$$\Rightarrow \Pr(X = 1|Y = 0) = \frac{0.014}{0.0125 + 0.014 + 0.008} \tag{9}$$

$$\Rightarrow \Pr(X = 1|Y = 0) = \frac{0.014}{0.0345} \tag{10}$$

$$\Rightarrow \Pr(X = 1|Y = 0) = \frac{28}{69} \tag{11}$$

$$\therefore \Pr(X = 1|Y = 0) = \frac{28}{69} = 0.4058 \tag{12}$$