

AI1110 Assignment-7

Gollapudi Sasank CS21BTECH11019

May 19, 2022

Outline

1 Question

2 Solution

Question

Suppose box 1 contains a white balls and b black balls, and box 2 contains c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball?

Solution

If no ball is transferred from the first box into the second box, the probability of obtaining a white ball from the second one is simply $\frac{c}{c+d}$. In the present case, a ball is first transferred from box 1 to box 2 and there are only two mutually exclusive possibilities for this event-the transferred ball is either a white ball or a black ball. Let the random variable X denote the following :

$X = 0$: transferred ball is white.

$X = 1$: transferred ball is black.

The events $X = 0$ and $X = 1$ together form a partition of sample space S ,
 $S = (X = 0) + (X = 1)$

From the given data

$$\Pr(X = 0) = \frac{a}{a+b} \quad (1)$$

$$\Pr(X = 1) = \frac{b}{a+b} \quad (2)$$

After one ball is transferred at random from box 1 to box 2 Let the random variable Y denote the following :

$Y = 0$: The ball drawn at random from box 2 is white.

$Y = 1$: the ball drawn at random from box 2 is black.

$$\Pr(Y = 0|X = 0) = \frac{c + 1}{c + d + 1} \quad (3)$$

$$\Pr(Y = 0|X = 1) = \frac{c}{c + d + 1} \quad (4)$$

$$\Pr(Y = 1|X = 0) = \frac{d}{c + d + 1} \quad (5)$$

$$\Pr(Y = 1|X = 1) = \frac{d + 1}{c + d + 1} \quad (6)$$

We have to find the value of $\Pr(Y = 0)$

$$\Pr(Y = 0) = \Pr((Y = 0)(S)) \quad (7)$$

$$\Rightarrow \Pr(Y = 0) = \Pr((Y = 0)((X = 0) + (X = 1))) \quad (8)$$

$$\Rightarrow \Pr(Y = 0) = \Pr(((Y = 0)(X = 0)) + ((Y = 0)(X = 1))) \quad (9)$$

Clearly the two events $(Y = 0)(X = 0)$ and $(Y = 0)(X = 1)$ are mutually exclusive because when we transfer one ball from box 1 to box 2 it must be either WHITE or BLACK.

$$\Rightarrow \Pr(Y = 0) = \Pr((Y = 0)(X = 0)) + \Pr((Y = 0)(X = 1)) \quad (10)$$

$$\Rightarrow \Pr(Y = 0) = \Pr(X = 0)\Pr(Y = 0|X = 0) \\ + \Pr(X = 1)\Pr(Y = 0|X = 1) \quad (11)$$

$$\Rightarrow \Pr(Y = 0) = \frac{a(c+1)}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \quad (12)$$

$$\Rightarrow \Pr(Y = 0) = \frac{ac+a}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \quad (13)$$

$$\Rightarrow \Pr(Y = 0) = \frac{ac+bc+a}{(a+b)(c+d+1)} \quad (14)$$