

# Assignment 7

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QUESTION :

Suppose box 1 contains  $a$  white balls and  $b$  black balls , and box 2 contains  $c$  white balls and  $d$  black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball?

SOLUTION :

If no ball is transferred from the first box into the second box, the probability of obtaining a white ball from the second one is simply  $\frac{c}{c+d}$ . In the present case, a ball is first transferred from box 1 to box 2 and there are only two mutually exclusive possibilities for this event-the transferred ball is either a white ball or a black ball. Let the random variable  $X$  denote the following :

$X = 0$  : transferred ball is white.

$X = 1$  : transferred ball is black.

The events  $X = 0$  and  $X = 1$  together form a partition  $S$  ,  $S = (X = 0) + (X = 1)$

From the given data

$$\Pr(X = 0) = \frac{a}{a+b} \quad (1)$$

$$\Pr(X = 1) = \frac{b}{a+b} \quad (2)$$

After one ball is transferred at random from box 1 to box 2 Let the random variable  $Y$  denote the following :

$Y = 0$  : The ball drawn at random from box 2 is white.

$Y = 1$  : the ball drawn at random from box 2 is black.

$$\Pr(Y = 0|X = 0) = \frac{c+1}{c+d+1} \quad (3)$$

$$\Pr(Y = 0|X = 1) = \frac{c}{c+d+1} \quad (4)$$

$$\Pr(Y = 1|X = 0) = \frac{d}{c+d+1} \quad (5)$$

$$\Pr(Y = 1|X = 1) = \frac{d+1}{c+d+1} \quad (6)$$

We have to find the value of  $\Pr(Y = 0)$

$$\Pr(Y = 0) = \Pr((Y = 0)(S)) \quad (7)$$

$$\Rightarrow \Pr(Y = 0) = \Pr((Y = 0)((X = 0) + (X = 1))) \quad (8)$$

$$\Rightarrow \Pr(Y = 0) = \Pr(((Y = 0)(X = 0)) + ((Y = 0)(X = 1))) \quad (9)$$

Clearly the two events  $(Y = 0)(X = 0)$  and  $(Y = 0)(X = 1)$  are mutually exclusive because when we transfer one ball

from box 1 to box 2 it must be either WHITE or BLACK.

$$\Rightarrow \Pr(Y = 0) = \Pr((Y = 0)(X = 0)) + \Pr((Y = 0)(X = 1)) \quad (10)$$

$$\Rightarrow \Pr(Y = 0) = \Pr(X = 0) \Pr(Y = 0|X = 0) + \Pr(X = 1) \Pr(Y = 0|X = 1) \quad (11)$$

$$\Rightarrow \Pr(Y = 0) = \frac{a(c+1)}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \quad (12)$$

$$\Rightarrow \Pr(Y = 0) = \frac{ac+a}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \quad (13)$$

$$\Rightarrow \Pr(Y = 0) = \frac{ac+bc+a}{(a+b)(c+d+1)} \quad (14)$$