AI1110 Assignment-7

Gollapudi Sasank CS21BTECH11019

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Outline

Question

Suppose box 1 contains a white balls and b black balls , and box 2 contains c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball?

Solution

If no ball is transferred from the first box into the second box, the probability of obtaining a white ball from the second one is simply $\frac{c}{c+d}$. In the present case, a ball is first transferred from box 1 to box 2 and there are only two mutually exclusive possibilities for this event-the transferred ball is either a white ball or a black ball. Let the random variable X denote the following :

$X = 0$: transferred ball is white.

$X = 1$: transferred ball is black.

The events $X = 0$ and $X = 1$ together form a partition S ,

$S = (X = 0) + (X = 1)$

From the given data

$$\Pr(X = 0) = \frac{a}{a + b} \tag{1}$$

$$\Pr(X = 1) = \frac{b}{a + b} \tag{2}$$

After one ball is transferred at random from box 1 to box 2 Let the random variable Y denote the following :
$Y = 0$: The ball drawn at random from box 2 is white.
$Y = 1$: the ball drawn at random from box 2 is black.

$$\Pr(Y = 0|X = 0) = \frac{c+1}{c+d+1} \tag{3}$$

$$\Pr(Y = 0|X = 1) = \frac{c}{c+d+1} \tag{4}$$

$$\Pr(Y = 1|X = 0) = \frac{d}{c+d+1} \tag{5}$$

$$\Pr(Y = 1|X = 1) = \frac{d+1}{c+d+1} \tag{6}$$

We have to find the value of $\Pr(Y=0)$

$$\Pr(Y=0) = \Pr((Y=0)(S)) \tag{7}$$
$$\Rightarrow \Pr(Y=0) = \Pr((Y=0)((X=0)+(X=1))) \tag{8}$$
$$\Rightarrow \Pr(Y=0) = \Pr(((Y=0)(X=0))+((Y=0)(X=1))) \tag{9}$$

Clearly the two events $(Y=0)(X=0)$ and $(Y=0)(X=1)$ are mutually exclusive because when we transfer one ball from box 1 to box 2 it must be either WHITE or BLACK.

$$\Rightarrow \Pr(Y=0) = \Pr((Y=0)(X=0)) + \Pr((Y=0)(X=1)) \tag{10}$$

$$\Rightarrow \Pr(Y=0) = \Pr(X=0)\Pr(Y=0|X=0) + \Pr(X=1)\Pr(Y=0|X=1) \quad (11)$$

$$\Rightarrow \Pr(Y=0) = \frac{a(c+1)}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \quad (12)$$

$$\Rightarrow \Pr(Y=0) = \frac{ac+a}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \quad (13)$$

$$\Rightarrow \Pr(Y=0) = \frac{ac+bc+a}{(a+b)(c+d+1)} \quad (14)$$