Assignment 7

Gollapudi Sasank CS21BTECH11019

QUESTION :

Suppose box 1 contains a white balls and b black balls , and box 2 contains c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball?

SOLUTION :

If no ball is transferred from the first box into the second box, the probability of obtaining a white ball from the second one is simply $\frac{c}{c+d}$. In the present case, a ball is first transferred from box 1 to box 2 and there are only two mutually exclusive possibilities for this event-the transferred ball is either a white ball or a black ball. Let the random variable X denote the following :
$X = 0$: transferred ball is white.
$X = 1$: transferred ball is black.
The events $X = 0$ and $X = 1$ together form a partition of sample space S, $S = (X = 0) + (X = 1)$
From the given data

$$\Pr(X = 0) = \frac{a}{a+b} \tag{1}$$

$$\Pr(X = 1) = \frac{b}{a+b} \tag{2}$$

After one ball is transferred at random from box 1 to box 2 Let the random variable Y denote the following :
$Y = 0$: The ball drawn at random from box 2 is white.
$Y = 1$: the ball drawn at random from box 2 is black.

$$\Pr(Y = 0|X = 0) = \frac{c+1}{c+d+1} \tag{3}$$

$$\Pr(Y = 0|X = 1) = \frac{c}{c+d+1} \tag{4}$$

$$\Pr(Y = 1|X = 0) = \frac{d}{c+d+1} \tag{5}$$

$$\Pr(Y = 1|X = 1) = \frac{d+1}{c+d+1} \tag{6}$$

We have to find the value of $\Pr(Y = 0)$

$$\Pr(Y = 0) = \Pr((Y = 0)(S)) \tag{7}$$

$$\Rightarrow \Pr(Y = 0) = \Pr((Y = 0)((X = 0) + (X = 1))) \tag{8}$$

$$\Rightarrow \Pr(Y = 0) = \Pr(((Y = 0)(X = 0)) + ((Y = 0)(X = 1))) \tag{9}$$

Clearly the two events $(Y = 0)(X = 0)$ and $(Y = 0)(X = 1)$ are mutually exclusive because when we transfer one ball from box 1 to box 2 it must be either WHITE or BLACK.

$$\Rightarrow \Pr(Y = 0) = \Pr((Y = 0)(X = 0)) + \Pr((Y = 0)(X = 1)) \tag{10}$$

$$\Rightarrow \Pr(Y = 0) = \Pr(X = 0)\Pr(Y = 0|X = 0) + \Pr(X = 1)\Pr(Y = 0|X = 1) \tag{11}$$

$$\Rightarrow \Pr(Y = 0) = \frac{a(c+1)}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \tag{12}$$

$$\Rightarrow \Pr(Y = 0) = \frac{ac+a}{(a+b)(c+d+1)} + \frac{bc}{(a+b)(c+d+1)} \tag{13}$$

$$\Rightarrow \Pr(Y = 0) = \frac{ac+bc+a}{(a+b)(c+d+1)} \tag{14}$$