

# Assignment 8

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QUESTION :

A fair coin is tossed twice , and let the random variable  $X$  represent the number of heads. Find  $F_X(x)$  (Cumulative Distribution Function of  $X$ )

SOLUTION :

Let us denote Head by  $H$  and tail by  $T$ .  
Sample Space( $S$ ) =  $\{HH, HT, TH, TT\}$ .  
 $X$  : Number of heads obtained

$$X(HH) = 2 \quad (1)$$

$$X(HT) = 1 \quad (2)$$

$$X(TH) = 1 \quad (3)$$

$$X(TT) = 0 \quad (4)$$

(5)

$$\Pr(X = 0) = \frac{1}{4} \quad (6)$$

$$\Pr(X = 1) = \frac{1}{2} \quad (7)$$

$$\Pr(X = 2) = \frac{1}{4} \quad (8)$$

From the definition of Cumulative Distribution Function(CDF) of a random variable  $X$

$$F_X(x) = \Pr(X \leq x) \quad (9)$$

1) For  $x < 0$  ,  $X \leq x \Rightarrow X < 0$

$$F_X(x) = \Pr(X < 0) \quad (10)$$

$$F_X(x) = 0 \quad (11)$$

2) For  $0 \leq x < 1$  ,  $X \leq x \Rightarrow X < 1$

$$F_X(x) = \Pr(X < 1) \quad (12)$$

$$F_X(x) = \Pr(X = 0) \quad (13)$$

$$F_X(x) = \frac{1}{4} \quad (14)$$

3) For  $1 \leq x < 2$  ,  $X \leq x \Rightarrow X < 2$

$$F_X(x) = \Pr(X < 2) \quad (15)$$

$$F_X(x) = \Pr(X = 0) + \Pr(X = 1) \quad (16)$$

$$F_X(x) = \frac{1}{4} + \frac{1}{2} \quad (17)$$

$$F_X(x) = \frac{3}{4} \quad (18)$$

4) For  $x \geq 2$  ,  $X \leq x \Rightarrow X < \infty$

$$F_X(x) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) \quad (19)$$

$$F_X(x) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} \quad (20)$$

$$F_X(x) = 1 \quad (21)$$

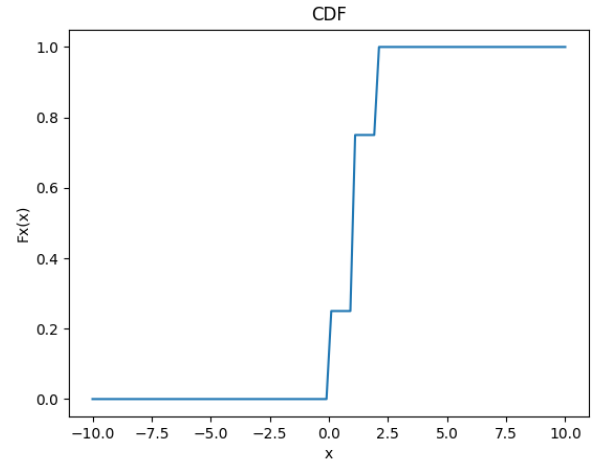


Fig. 1. CDF graph