Assignment 9

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Question :

Players X and Y roll dice alternately starting with X. The player that rolls 11 wins. Show that the probability p that X wins equals $18/35$.

Solution :

Let the Random Variables A, B denote the following :
$A = 0$: The person who starts the game wins
$A = 1$: The person who starts the game loses
$B = 0$: 11 occurs at the first throw
$B = 1$: 11 does not occur at the first throw
Given,

$$\Pr(A = 0) = p \tag{1}$$
$$\Rightarrow \Pr(A = 1) = 1 - p \tag{2}$$
$$\Pr(B = 0) = \frac{2}{36} = \frac{1}{18} \tag{3}$$
$$\Rightarrow \Pr(B = 1) = \frac{34}{36} = \frac{17}{18} \tag{4}$$

The Events $B = 0$ and $B = 1$ form a partition to the Sample Space.

$$\Rightarrow \Pr(A = 0) = \Pr((A = 0)((B = 0) + (B = 1))) \tag{5}$$
$$\Rightarrow \Pr(A = 0) = \Pr((A = 0)(B = 0) + (A = 0)(B = 1)) \tag{6}$$

The Events $(A = 0)(B = 0)$ and $(A = 0)(B = 1)$ are mutually exclusive

$$\Rightarrow \Pr(A = 0) = \Pr((A = 0)(B = 0)) + \Pr((A = 0)(B = 1)) \tag{7}$$
$$\Rightarrow \Pr(A = 0) = \Pr(A = 0|B = 0)\Pr(B = 0) + \Pr(A = 0|B = 1)\Pr(B = 1) \tag{8}$$

$\Pr(A = 0|B = 0) = 1$ because if 11 occurs at first throw X wins.
Now the event $(A = 0|B = 1)$ is the case where X wins when 11 does not occur at first throw. So in this case if we consider the game from the second throw then Y throws first. But here we need the probability of the case where the person who starts the game loses i.e $\Pr(A = 1) = 1 - p$.
$\therefore \Pr(A = 0|B = 1) = 1 - p$

$$\Rightarrow p = 1 \times \frac{1}{18} + (1 - p) \times \frac{17}{18} \tag{9}$$
$$\Rightarrow p = \frac{1 + 17 - 17p}{18} \tag{10}$$
$$\Rightarrow 18p = 18 - 17p \tag{11}$$
$$\Rightarrow 35p = 18 \tag{12}$$
$$\Rightarrow p = \frac{18}{35} \tag{13}$$