

1. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.
Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \{\frac{\pi}{2}\}$.
2. If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$ and $b = \tan^{-1}(\sqrt{3}) + \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$, then find the value of $a + b$.
3. Find: $\int \cos^3 x \cdot e^{\log(\sin x)} dx$
4. Find: $\int \frac{1}{5+4x-x^2} dx$
5. Sand is pouring from a pipe at the rate of $15\text{cm}^3/\text{minute}$. The falling sand forms a cone on the ground such that the height of the cone is always one-third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4cm
6. Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the co-ordinate axes.
7. Verify whether the function f defined by $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not.
8. Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.
9. Find the particular solution of the differential equation :
 $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}; y(0) = 5$
10. Solve the following differential equation:
 $x^2dy + y(x+y)dx = 0$
11. Find the values of a and b so that the following function is differentiable for all values of x $f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \leq -1 \end{cases}$
12. Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.
13. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.
14. Evaluate: $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx = 0$
15. Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = 3\hat{i} - \hat{k}$ and $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and $\vec{c} \cdot \vec{d} = 3$.
16. Bag I contains 3 red and black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

17. Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
Also, find the perpendicular distance of the given point from the line.
18. Find the shortest distance between the lines L_1 & L_2 given below:
 L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$
 L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$
19. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$ then find A^{-1} and use it to solve the following system of equations :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

20. Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{bmatrix}$ $\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$, then find AB and use it to solve the system of linear equations :

$$x - 2y = 3$$

$$2x - y - z = 2$$

$$-2y + z = 3$$

21. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$. using integration.
22. Solve the following Linear Programming problem graphically:
 Maximise $Z = 300x + 600y$
 Subject to

$$x + 2y \leq 12$$

$$2x + y \leq 12$$

$$x + \frac{5}{4}y \geq 5$$

$$x \geq 0, y \geq 0.$$

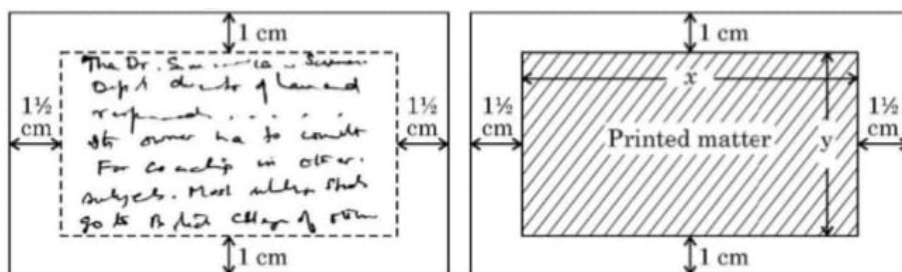
23. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

- (i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.
Find $P(E_1), P(E_2)$
 - (ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.
 - (iii) Find the probability of customer paying second month's bill in time.
 - (iv) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.
24. Students of a school are taken to a railway museum to learn about railways heritage and its history
An exhibit in the museum depicted many rail lines on the track near the



- railway station. let L be the set of all rail lines on the railway track and R be the relation on L defined by
 $R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$
 On the basis of the above information, answer the following questions:
- (a) Find whether the relation R is symmetric or not.
 - (b) Find whether the relation R is transitive or not.
 - (c) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$ then find the set of rail lines in R related to it.
25. Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.
26. A rectangular visiting card is to contain 24 sq. cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be cm as shown below:



On the basis of the above information, answer the following questions:

- Write the expression for the area of the visiting card in terms of x .
- Obtain the dimensions of the card of minimum area.