$\Rightarrow$ Suppose V is finite-dimensional and U is a subspace of V and $\dim U = \dim V$. Show that $U = V$

For any $v \in V$ we'll show $v \in U$

Let $\{u_1\ u_2\ u_3 \ldots u_m\}$ is the basis for U

$m = \dim U = \dim V$

$\{u_1\ u_2\ u_3 \ldots u_m\ v\}$ is linearly dependent

where, $v \in V$

$\therefore\ a_1 u_1 + a_2 u_2 + a_3 u_3 + \ldots + a_m u_m + bv = 0$

$\hookrightarrow$ not all zero

$\beta \neq 0$ since $\{u_1 \ldots u_m\}$ are linearly independent.

$$\therefore\ v = -\left(\frac{a_1}{b} u_1 + \frac{a_2}{b} u_2 + \frac{a_3}{b} u_3 + \ldots + \frac{a_m}{b} u_m \right)$$

$v \in U$

QED.