Prove that if T is a linear map from F^4 to F^2 such that

$$\text{null } T = \{ (x_1, x_2, x_3, x_4) \in F^4 : x_1 = 5x_2 \text{ and } x_3 = 7x_4 \}$$

Then T is surjective.

Proof:

Let $v = (x_1, x_2, x_3, x_4) \in \text{null } T$

$$v = (5x_2, x_2, 7x_4, x_4)$$

$$= x_2(5, 1, 0, 0) + x_4(0, 0, 7, 1)$$

$(5, 1, 0, 0)$ & $(0, 0, 7, 1)$

Span null T & are linearly independent

$\therefore$ They form basis of null T

$\therefore$ dim null $T = 2$

$$\dim T = \dim \text{null } T + \dim \text{range } T$$

$$4 = 2 + \dim \text{range } T$$

$$\dim \text{range } T = 2 \quad \text{---(1)}$$

we know, range $T \subseteq F^2$

but dim range $T = 2 = \dim F^2$

$\therefore$ range $T = F^2$

$\therefore$ T is surjective.