⇒ Prove or give counterexample: if $(v_1, v_2 \cdots v_m)$
is a linearly independent list of vectors in V
then $(5v_1 - 4v_2, v_2 \; v_3 \cdots v_m)$ is linearly
independent.

Let
$$a_1(5v_1 - 4v_2) + a_2 v_2 + a_3 v_3 + \cdots + a_m v_m = 0$$

$$a_1, a_2 \cdots a_m \in \mathbb{F}$$

Rearranging, we get

$$5a_1 v_1 + (a_2 - 4a_1)v_2 + a_3 v_3 + \cdots + a_m v_m = 0$$

$$\Rightarrow 5a_1 = a_2 - 4a_1 = a_3 = a_4 = \cdots = a_m = 0$$

$$5a_1 = 0$$
$$a_1 = 0$$

$$a_2 - 4a_1 = 0$$
$$a_2 = 0$$

∴ $(5v_1 - 4v_2, v_2, v_3 \cdots v_m)$ are
linearly independent. QED.