$\Rightarrow$ Let $-\infty$ and ∞ be distinct objects.

and let $V = \mathbb{R} \cup \{\infty\} \cup \{-\infty\}$

The sum of two reals (and product) is as usual.

for any $t \in \mathbb{R}$,

$$t\infty = \begin{cases} -\infty, & t < 0 \\ 0, & t = 0 \\ \infty, & t > 0 \end{cases}$$

$$t(-\infty) = \begin{cases} \infty, & t < 0 \\ 0, & t = 0 \\ -\infty, & t > 0 \end{cases}$$

$$t + \infty = \infty + t = \infty$$

$$t + (-\infty) = (-\infty) + t = -\infty$$

$$\infty + \infty = \infty, \quad (-\infty) + (-\infty) = -\infty, \quad \infty + (-\infty) = 0$$

is V a vector space over $\mathbb{R}$?

<u>Soln:</u> The following properties of addition & scalar multiplication must hold for V to be a vector space...

- commutativity
  true for $\mathbb{R}$.
  true for V from above identities
- associativity.
  Similarly...
- additive identity
  true for reals. for ∞, 0 and for $-\infty$, 0.
- additive inverse
  true for $\mathbb{R}$. ∞, $-\infty$ are inverses of each other.
- multiplicative identity
  true for $\mathbb{R}$. $t = 1$ & identities above
- distributive properties : true for $\mathbb{R}$ & for V using the identities