→ Suppose $(v_1\ v_2\ v_3\ v_4)$ is a basis of V

Prove that $(v_1+v_2,\ v_2+v_3,\ v_3+v_4,\ v_4)$ is also a basis of V.

$v_1\ v_2\ v_3\ v_4$ are linearly independent

$v_1+v_2,\ v_2+v_3,\ v_3+v_4,\ v_4$?

$$a_1(v_1+v_2)+a_2(v_2+v_3)+a_3(v_3+v_4)+a_4(v_4)=0$$

Rearranging,

$$a_1v_1+(a_1+a_2)v_2+(a_2+a_3)v_3+(a_3+a_4)v_4=0$$

$$a_1=a_1+a_2=a_2+a_3=a_3+a_4=0$$

$a_1=0$

$a_2=0$

$a_3=0$

$a_3=0$

$\therefore$ $(v_1+v_2,\ v_2+v_3,\ v_3+v_4,\ v_4)$ are linearly independent.

(v_1, v_2, v_3, v_4) spans V

for any $v \in V$

$$\cancel{v = a_1v_1+a_2v_2+a_3v_3+a_4v_4}$$

$$\cancel{a_1v_1+(a_1+a_2)v_2+(a_2+a_3)v_3+(a_3+a_4)v_4}$$

$$V = a_1v_1+a_2v_2+a_3v_3+a_4v_4$$

Let $a_2 = b_2+a_1$

$a_3 = b_2+b_3$

$a_4 = b_3+b_4$