⇒ for each given subset of F^3, determine if it's a subspace...

a) $\{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 0\}$

b) $\{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 4\}$

c) $\{(x_1, x_2, x_3) \in F^3 : x_1 x_2 x_3 = 0\}$

d) $\{(x_1, x_2, x_3) \in F^3 : x_1 = 5x_3^9\}$

soln: Need to check for add. identity, closure under addn, scalar multiplication

a) Let V

$x_1 + 2x_2 + 3x_3 = 0$

(i) $0 \in V$ ✓

(ii) $(x_1, 2x_2, 3x_3) + (y_1, 2y_2 + 3y_3)$

$= ((x_1 + y_1), 2(x_2 + y_2), 3(x_3 + y_3))$

$= (z_1, 2z_2, 3z_3)$

$\in V$ ✓

(iii) $k \cdot (x_1, 2x_2, 3x_3)$

$(kx_1, 2kx_2 + 3kx_3)$ also $(kx_1 + 2kx_2 + 3kx_3 = 0 \cdot k = 0)$ ✓

↳ $\in V$ ✓

V is a subspace of F^3

b) Let V

(i) $0 \notin V$

not a subspace

$0 + 2(0) + 3(0) \neq 4$

c) Let V

(i) $0 \in V$

$0 \cdot 0 \cdot 0 = 0$

(ii)

$(x_1, x_2, x_3) + (y_1, y_2, y_3)$

$= (x_1+y_1, x_2+y_2, x_3+y_3)$

$x_1 x_2 x_3 = 0, \quad y_1 y_2 y_3 = 0$ ↓ not always in V

$x_1 = 0 \quad y_2 = 0$

$x_2 \neq x_3 \neq y_1 \neq y_3 \neq 0$

↳ counter example(s)

* V is not a subspace

d) Let V

(i) $0 \in V$

$0 = 5 \cdot 0$ ✓

(ii) $(x_1+y_1, x_2+y_2, x_3+y_3) \in V$?

$x_1 = 5 x_3$

$y_1 = 5 y_3$

$x_1+y_1 = 5(x_3+y_3)$ ✓

(iii) (x_1, x_2, x_3)

(kx_1, kx_2, kx_3)

$x_1 = 5 x_3$

$(kx_1) = 5(kx_3)$ ✓

V is a subspace