

\Rightarrow Suppose U and W are 5-dimensional subspaces of \mathbb{R}^9 . Show that $U \cap W \neq \{0\}$.

$$\dim U = \dim W = 5$$

$$\begin{aligned}\dim(U \cap W) &= 5 + 5 - \dim(U + W) \\ &= 10 - \dim(U + W)\end{aligned}$$

$$\begin{aligned}\dim(U + W) &\leq \dim(\mathbb{R}^9) \\ &\leq 9\end{aligned}$$

$$\therefore \dim(U \cap W) \geq 1$$

$$\therefore U \cap W \neq \{0\}$$