⇒ Suppose U and W are 5-dimensional subspaces of $\mathbb{R}^9$. Show that $U \cap W \neq \{0\}$

$$\dim U = \dim W = 5$$

$$\dim(U \cap W) = 5 + 5 - \dim(U+W)$$

$$= 10 - \dim(U+W)$$

$$\dim(U+W) \leq \dim(\mathbb{R}^9)$$

$$\leq 9$$

$$\therefore \dim(U \cap W) \geq 1$$

$$\therefore U \cap W \neq \{0\}$$