$\Rightarrow$ $L(V, W)$ is the set of all linear maps from V to W

addition in $L(V, W)$

$$(S+T)(v) = Sv + Tv$$

multiplication in $L(V, W)$

$$(\lambda T)\, v = \lambda(Tv)$$

$$S, T \in L(V, W), \quad v \in V, \quad \lambda \in F$$

Show that $L(V, W)$ is a vector space.

It should satisfy the following properties

Commutativity

$$(S+T)v = Sv + Tv = (T+S)v$$

associativity

$$((S+T)+U)\, v = Sv + Tv + Uv = (S + (T+U))\, v$$

additive identity

$$0: V \to W \quad \text{s.t.} \quad 0v = 0$$

additive inverse

for any $S \in L$, let $-S$ be s.t. $(-S)v = -(Sv)$

multiplicative identity

$$(1 \cdot T) \cdot v = 1 \cdot (Tv) = Tv$$

distributive

$$T(u_1 + u_2) = Tu_1 + Tu_2$$

$$T(\lambda u) = \lambda Tu$$ ∵ ~~bc~~ T is a linear map.