$\Rightarrow$ Suppose $b, c \in \mathbb{R}$. Define $T: \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(x, y, z) = (2x - 4y + 3z + b,\ 6x + cxyz).$$

Show that T is linear iff $b = c = 0$

For T be a linear map, it should satisfy additivity and homogeneity.

- $T(u+v) = Tu + Tv$
- $T(\lambda u) = \lambda T u$

Consider the 2nd property

$$T(\lambda u) = T(\lambda x, \lambda y, \lambda z) = (2\lambda x - 4\lambda y + 3\lambda z + b,\ 6\lambda x + c\lambda^3 xyz)$$

$$\lambda T u = \lambda T(x, y, z) = (2\lambda x - 4\lambda y + 3\lambda z + b\lambda,\ 6\lambda x + c\lambda xyz)$$

Clearly, $T(\lambda u) \neq \lambda T u$ unless $b = c = 0$.

QED.