Inverse Filtering of Room Acoustics

Signal Processing Final Report

TEAM 35- Plugged In

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PROJECT ABSTRACT:

This project proposes a method to compute the exact inverse filtering of room acoustics. This project uses the MINT principle, which is a proposed method to find the exact inverse of room acoustics. MINT principle (Multiple Input/ output Inverse Theorem), as the name suggests, uses multiple inputs (speakers), and\or multiple outputs (microphones), to exactly inverse the room acoustics and remove noise to obtain the transmitted signal in the original form.

Exact inverse is not achievable using the previous methods like LSE (Least Squares Error), due to non-minimum phase of room impulse response. With applications of matrix algebra, and FIR filter computations, we can achieve the exact inverse, and cite the possibility of exact replication of signals, without noise.

INTRODUCTION:

Generally, signals transmitted in a room suffer distortions due to reflections by the walls, absorption of different objects, etc. So, there is a need to develop an inverse filter to remove distortions and obtain the same (desired) output. This can be done by finding the response to a known signal and building a system which inverses the distortions. Using only one transmitter and receiver, it is not possible to find the exact inverse of the system, as the matrix equation doesn't have exact solutions. We use MINT theorem to resolve this issue, by taking multiple transmitters and\or multiple receivers. This makes the matrix equation into a linear combination with multiple solutions, which are exactly inverse filters for cancelling the distortions.

BACKGROUND THEORY:

The LSE (Least Squares Error) method is characterized by the below given block diagram. This direct method of inversing, containing single input and output has the equation as:

$$d(k) = h(k) \circledast g(k)$$

where, $\circledast$ is defined as the linear convolution of $h(k)$ and $g(k)$, and $d(k)$ is defined as:

$$d(k) = \begin{cases} 1, & if \;\; k = 1 \\ 0, & if \; k = 2,3,\ldots \end{cases}$$

Expressing in matrix form, we have the expression:

$$D = GH$$

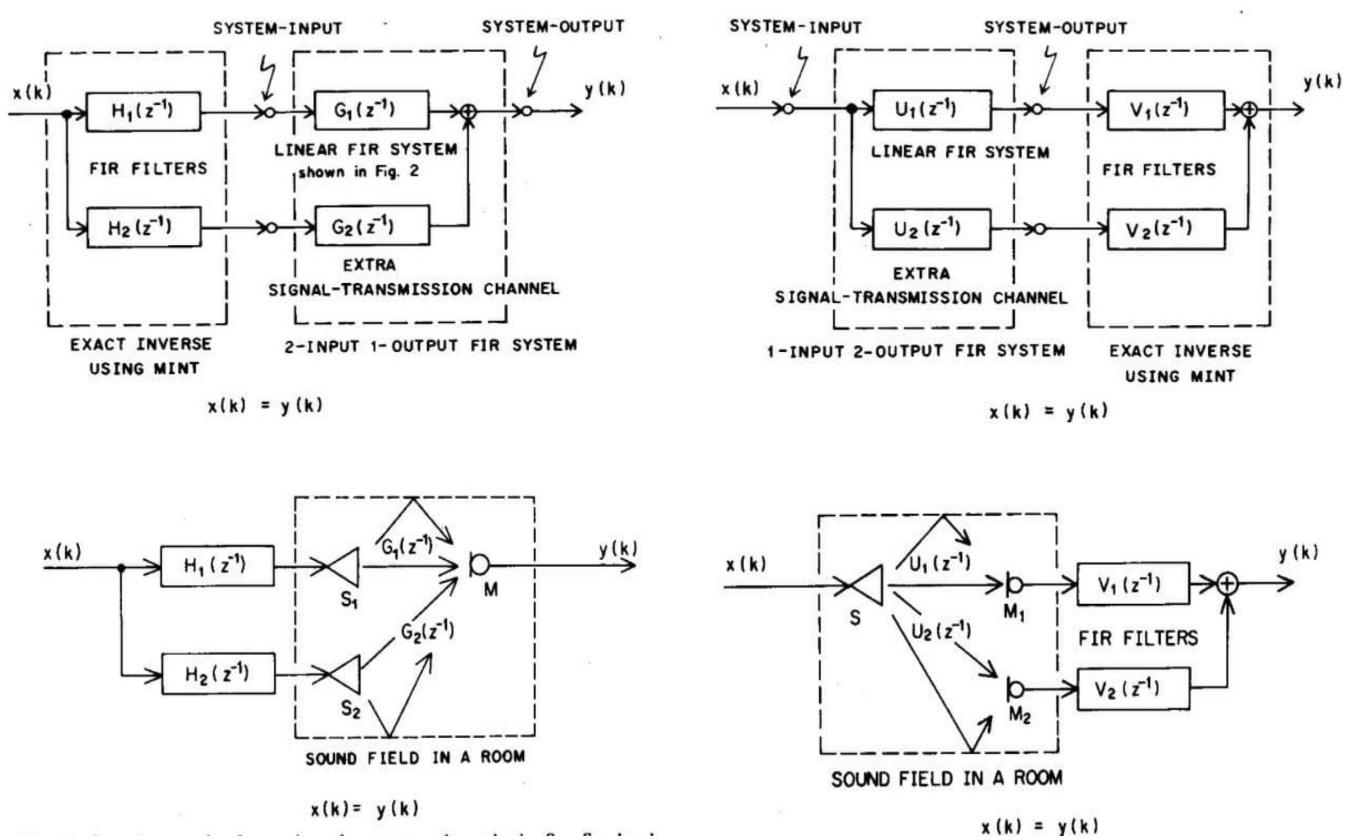
This system doesn't have a solution because the number of columns is less than the number of rows in G. As G is not a square matrix, it doesn't possess an inverse. Therefore, we have to resort to alternative algorithms like trimming the matrix, to add in null rows (rows completely filled with 0s), etc., to make it a square matrix. By doing so, the obtained inverse for the system isn't exact and possesses distortions to a certain extent, and the level of distortion depends on how strong the algorithm used is, to convert the matrix to a square matrix.

PROBLEM STATEMENT:

From the conventional method, we use only one transmission channel, that is a single input – single output system. This is a major drawback and is the basis over which the multi – channel system is built. Many of the modern-day transmission systems can be implemented on the basis of scalability (using multiple transmission channels and equipment). The problem to be emphasized upon is the exact inversing without any approximation or making any algorithm efficient enough to reduce the errors. Therefore, our objective is to use multiple channels (multiple input – single output, single – input multiple output and multiple input – multiple output) to devise an exact inverse filter for room acoustics, using the proposed MINT theorem.

SOLUTION APPROACH:

BASIC PRINCIPLE



Starting off with a basic example of a two input – single output system, assuming the two channels to be denoted by the system functionals $G_1(z^{-1}), G_2(z^{-1})$, we must calculate the two FIR filters $H_1(z^{-1}),\ H_2(z^{-1})$, are the inverses for $G_1(z^{-1}), G_2(z^{-1})$, placed in cascade with their respective blocks, to remove distortions. The relation they must satisfy for exact inverse filtering is:

$$D(z^{-1}) \ = \ 1 \ = \ G_1(z^{-1})H_1(z^{-1}) \ + \ G_2(z^{-1})H_2(z^{-1})$$

Here, $D(z^{-1})$ is the Z – transform of $d(k)$, given as:

$$d(k) = \begin{cases} 1, & if \;\; k = 1 \\ 0, & if \; k = 2,3,\dots \end{cases}, \qquad D(z^{-1}) = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \; of \; length \; (L+1)$$

The matrices G and H are assumed to be:

$$G = [G_1 \; G_2] = \begin{bmatrix} g_1(0) & & & g_2(0) & & \\ g_1(1) & \ddots & 0 & g_2(1) & \ddots & 0 \\ \vdots & & & \vdots & & \\ \vdots & & g_1(0) & g_2(n) & & \\ g_1(m) & & g_1(1) & & \ddots & g_2(0) \\ & \ddots & \vdots & & & g_2(1) \\ 0 & & \vdots & 0 & & \vdots \\ & & g_1(m) & & & g_2(n) \end{bmatrix}$$

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} h_1(0) \\ h_1(1) \\ \vdots \\ \vdots \\ h_1(i) \\ h_2(0) \\ \vdots \\ h_2(j) \end{bmatrix}$$

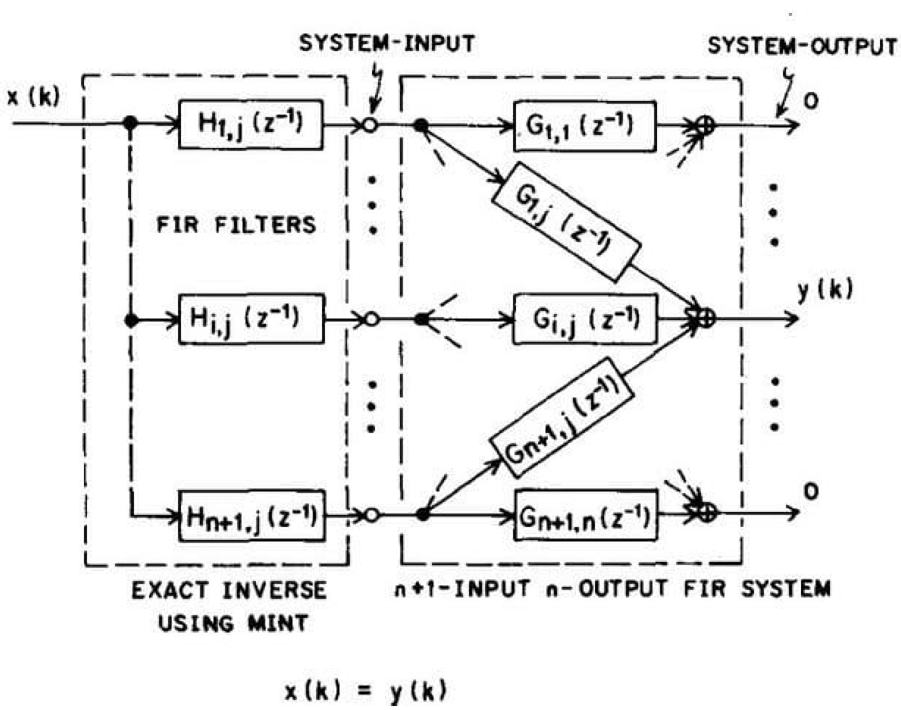
Here, we see that the G matrix has a dimension $(L+1) \times (i+j+2)$ and the H matrix has a dimension $(i+j+2) \times 1$, therefore, their product will give a matrix of dimension $(L+1) \times 1$, which matches with the dimension of matrix D. To find the solution $H_1 \; and \; H_2$, we use the following procedure:

$$D = [G_1 G_2] \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$

$$\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = [G_1 G_2]^{-1} D$$

The matrices G_1, G_2 are given, and the matrix D is also defined above. So as all the quantities on the RHS of the equation are known values, we can say that this system finds a valid solution for H_1, H_2, assuming G has is invertible (non – singular).

EXTENSION OF THE PRINCIPLE:



The above-mentioned example is for two input – single output systems. This can be implemented for multiple input – multiple output systems too, that is, it is applicable for a input – b output system, where both a and b are positive integers and at least one of a or b is greater than 1 which implies the valid choices of the pair (a,b) are (1,2), (2,1), (2,2), etc., while (0,2), (1,0), (1,1) are invalid. In such a multiple input (a>1) and multiple output (b>1), and assuming a = b+1, and b = 1,2,3..., we have the matrix relation:

$$R_j = G H_j$$

Where,

$$R_j(k) = \begin{cases} 1, & if\ k = j \\ 0, & if\ k \neq j \end{cases}$$

$$G = [G_1\ G_2 \dots G_{n+1}]$$

$$H_j = inverse\ of\ j^{th}\ filter\ (G_j)$$

Where, G_i is the collection of all acoustics that affect microphone j.

SOME IMPORTANT PROPERTIES AND RESTRICTIONS ON MATRICES:

For a two input – single output system, the conditions on $G_1(z^{-1})$, $G_2(z^{-1})$, $H_1(z^{-1})$, $H_2(z^{-1})$ are:

1. The solutions for equation $D(z^{-1}) = 1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$ exist if and only if are $G_1(z^{-1}),\ G_2(z^{-1})$ relatively prime, that is, $G_1(z^{-1}),\ G_2(z^{-1})$, have no zeros in common.

   <u>PROOF:</u>
   <u>If condition</u>: The solutions for equation

   $$D(z^{-1}) = 1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$$

   exist if $G_1(z^{-1}),\ G_2(z^{-1})$ are relatively prime.

   <u>Proof by contradiction</u> – Assume they are not relatively prime, and have a common zero at $z^{-1} = a$. Then, in the equation, substituting $z^{-1} = a$, we get:

   As $G_1(a) = G_2(a) = 0$, we obtain the relation $1 = 0$, which is never true. Therefore, by contradiction, we can say that $G_1(z^{-1}),\ G_2(z^{-1})$ must be relatively prime.

   <u>Only if condition</u>: The solutions for equation

   $$D(z^{-1}) = 1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$$

   exist only if $G_1(z^{-1}),\ G_2(z^{-1})$ are relatively prime.

   Assume that the equation $D(z^{-1}) = 1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$, has a solution. If $G_1(z^{-1})$ has a zero at $z^{-1} = a$, substituting in the equation, when $z^{-1} = a$, we have:

   $$1 = G_1(a)H_1(a) + G_2(a)H_2(a) = G_2(a)H_2(a)$$

   As solutions exist, $G_2(a)H_2(a) \neq 0$ Therefore, $G_2(a) \neq 0$, implies they are co-prime.

   Hence, we have proved that the solutions for equation $D(z^{-1}) = 1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$ exist if and only if $G_1(z^{-1}),\ G_2(z^{-1})$ are relatively prime, that is, $G_1(z^{-1}),\ G_2(z^{-1})$, have no zeros in common.

2. The equation $D(z^{-1}) = 1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$ has unique solutions if the orders of $H_1(z^{-1}),\ H_2(z^{-1})$ are less than the orders of $G_2(z^{-1}),\ G_1(z^{-1})$ respectively.

   <u>PROOF</u>:
   According to the Euclidean algorithm, for obtaining a general solution of $1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$, we can say that:

   $$deg\left(H_1\left(z^{-1}\right)\right) < deg\left(G_2\left(z^{-1}\right)\right)$$

$$deg\left|H_2\left|z^{-1}\right|\right| < deg\left(G_1\left(z^{-1}\right)\right)$$

Assume that $H_{11}(z^{-1}),\ H_{21}(z^{-1})$ are solutions to the equation. Assuming another set of solutions $H_{12}(z^{-1}),\ H_{22}(z^{-1})$ for the same equation, we obtain the relation that:

$$deg\left|H_1\left|z^{-1}\right|\right| = deg\left|G_2\left|z^{-1}\right|K\left|z^{-1}\right|\right| > deg\left|G_2\left|z^{-1}\right|\right|$$

$$deg\left|H_2\left|z^{-1}\right|\right| = deg\left|G_1\left|z^{-1}\right|K\left|z^{-1}\right|\right| > deg\left|G_1\left|z^{-1}\right|\right|$$

Which contradict the results from Euclidean algorithm. Hence, we can conclude that the solutions to the equation $1 = G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})$ are unique when orders of $H_1(z^{-1}),\ H_2(z^{-1})$ are less than the orders of $G_2(z^{-1}),\ G_1(z^{-1})$ respectively.

For a general case of multiple input – multiple output (n+1 - n), the properties and restrictions are:

1. The j[th] output of the system can be inverted using FIR filters $H_{ij}(z^{-1}),\ (i = 1, 2, ..., n+1)$ independently of the other outputs.
2. Solutions for $H_{ij}(z^{-1})$ exist when the Smith canonical form of G can be represented as matrix $[I_n\ 0]$, where I_n denotes the $n \times n$ unit matrix and 0 is an $n \times 1$ column vector with all zero elements. The Smith canonical form of any matrix is given as:

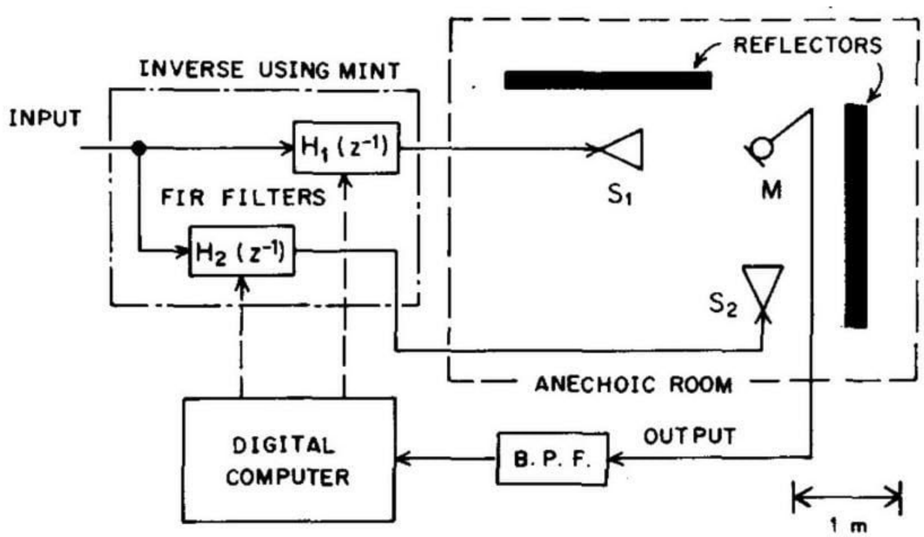
$$D = \begin{bmatrix} f_1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & f_2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & f_r & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}$$

Where, $f_i \in F[x]$, which is the field of values to which the diagonal entries belong.

INVERSE FILTERING EXPERIMENT IN A SOUND FIELD:

In general, using the conventional methods will not help in finding the exact inverse filtering of room acoustics as mentioned earlier. This is due to the fact that the room acoustics (distortions due to wall reflections, etc.) have a non-minimum phase. By definition, a non-minimum phase system is one which is causal and stable, but its inverse is causal, but unstable. We overcome this issue by implementing the MINT principle.

Therefore, taking a real-life example of transmitting sound waves in a room. The sound waves can be produced by two different sources S_1 and S_2, and can be received by a single microphone M. This setup produces a very close inverse filtering, a lot more effective and efficient inverse than the conventional methods, except for certain positions of the reflectors placed, specifically in symmetric positions, such that the impulse responses $g_1(k),\ g_2(k)$ are observed to have common zeros.



The system is set up in such a way that - two reflectors were placed in an L-shaped arrangement in an anechoic room. Another reflector was also placed on the floor. Microphone M was placed 1 m from loudspeakers S_1 and S_2. The output of M was fed through a band-pass filter (BPF), intended for the frequency band 315-3150 Hz. Then, the inverse filters $H_1(z^{-1}),\ H_2(z^{-1})$ are calculated and fed to the system in cascade and the observations are note down as follows:

1. We observe that the plot of error of the LSE method with increasing frequency is largely dependent on the nature of the acoustic channel used, which means the method is not generalized for all acoustics and works efficiently (less than MINT) only in certain cases.
2. The error obtained in the proposed MINT method is 40dB lesser than that obtained from LSE method, and works the same for all acoustics, that is, is not influenced by the nature of acoustic signal-transmission channel in use.

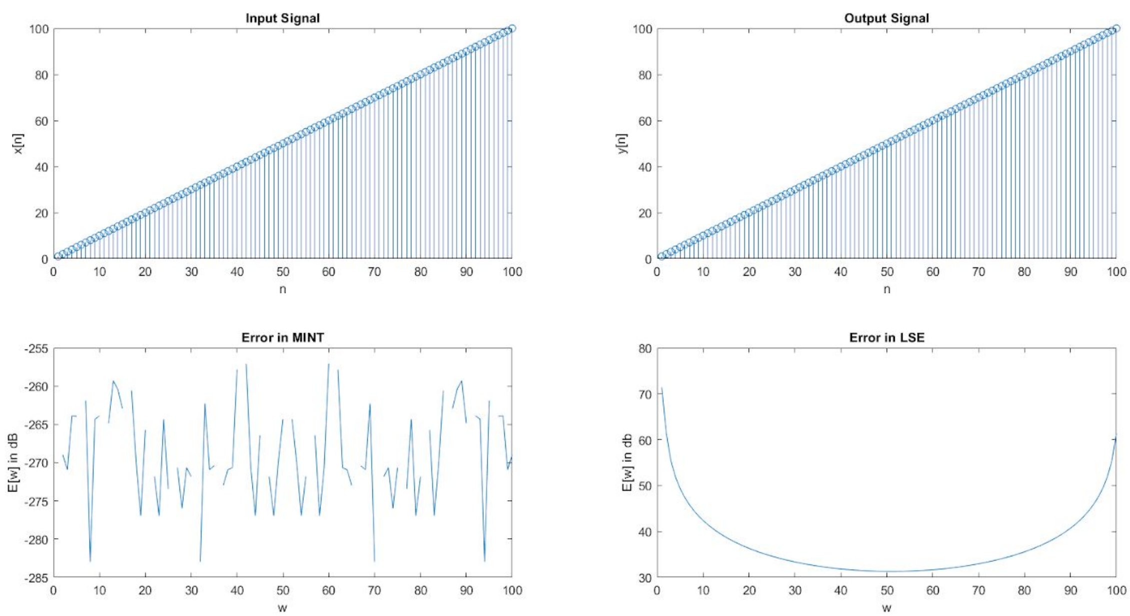
The error obtained is assumed to be due to the accuracy limits of the computer used for the inverse computations.

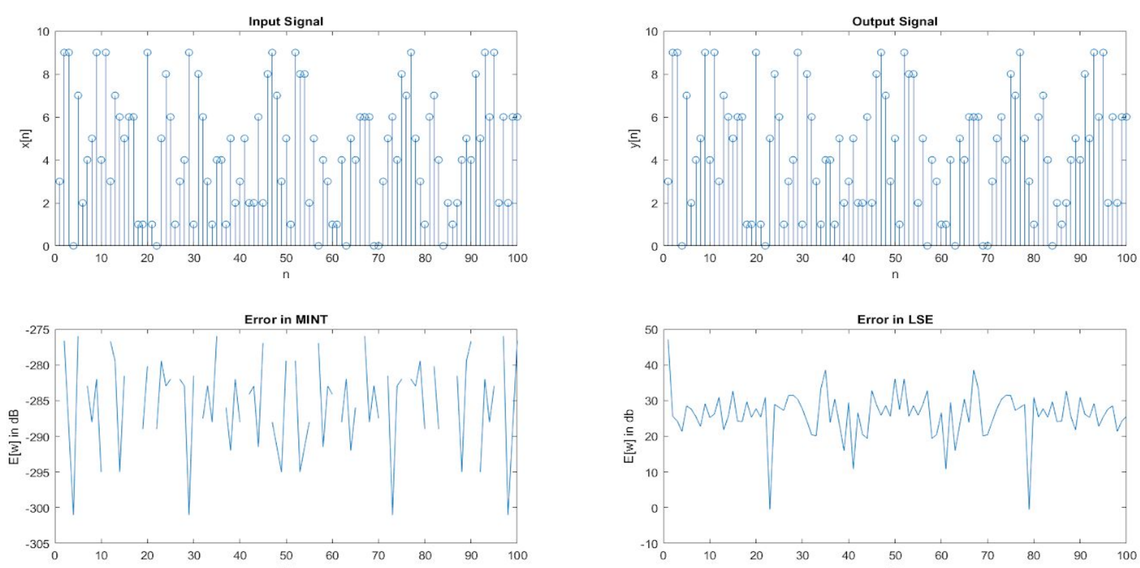
EXPERIMENT WITH A SOUND FILE - DISTORTIONS AND INVERSING:

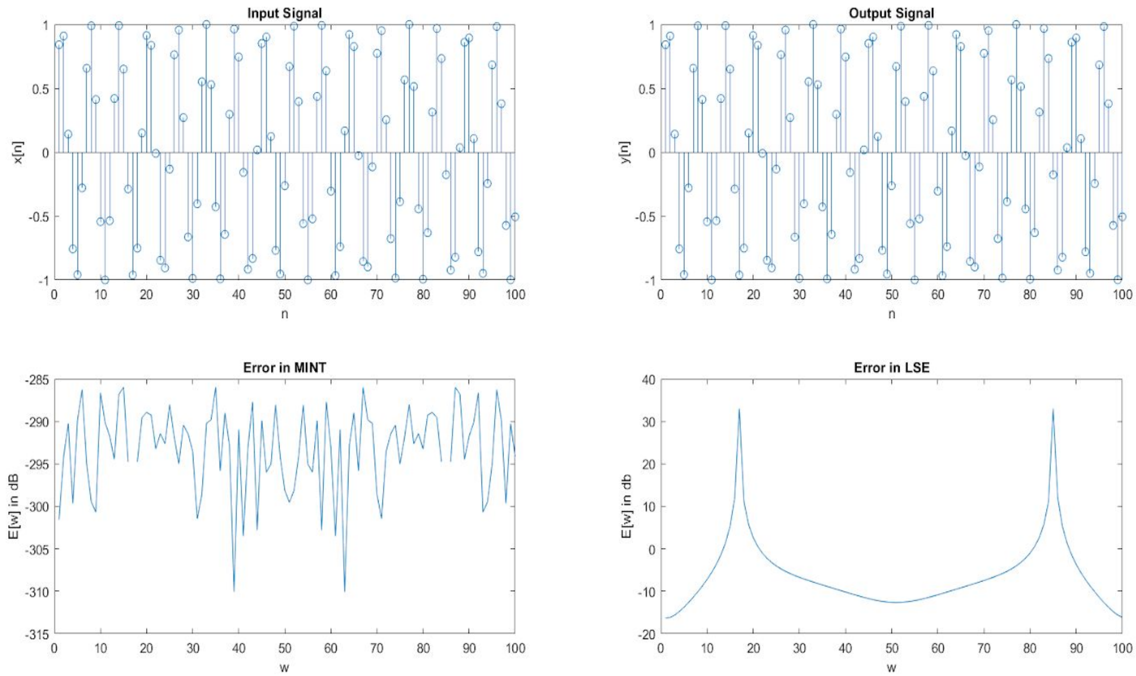
An experiment is conducted on MATLAB, with the input ($x(k)$) is any of the inbuilt MATLAB .wav files, or can be any other recorded file, and can be converted to .wav format. By the sound() command in MATLAB, we can listen to the sound initially, then pass it through a randomly generated room acoustics filter, again realize the sound and its distortions, and then pass it through the inverse filter obtained through MINT principle and comparing with the original sound obtained.

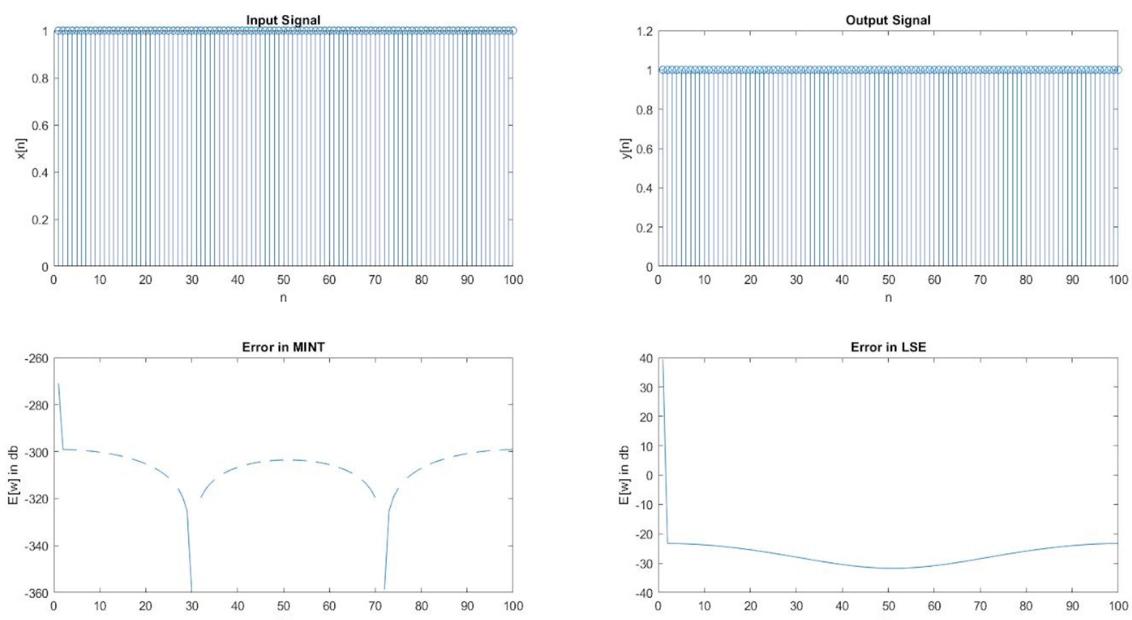
It is observed that the original sound gets distorted when passed through the room acoustics filter, which is evidently audible as random amplifications and worsening of quality of sound, which when again passed through the inverse filter obtained by MINT, is set back to the original sound, which shows the exact inversing of the MINT principle.

SIMULATIONS AND PLOTS:









These plots refer to the input, output and error, through the system, with the acoustics and inverse filtering. The observed error is very low (-200 - -300dB), which is due to MATLAB computational limitations.

CONCLUSION:

A method has been developed for finding the exact inverse of room acoustics. Due to the non-minimum phase, the conventional methods fail to obtain a stable and exact inverse for the room acoustics. This method employs the use of MINT principle, which makes use of multiple inputs (speakers) and\or multiple outputs (microphones).

Using the basic principles of matrix algebra and concept of circular matrix representations of linear convolution, we arrive at the exact inverse for the room acoustics for any scenarios of wall reflections and distortions.

An experiment conducted also inferred that this system gave more efficient inversing than conventional methods (LSE). Therefore, this proposed method proves to be valid, for impulse responses, which have no common zeros in their Z-transforms.

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