Applied statistics: Coursework 1

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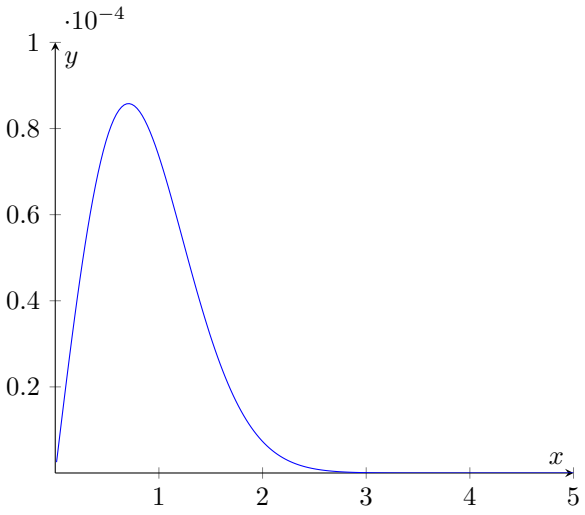
4.1 Part (1)

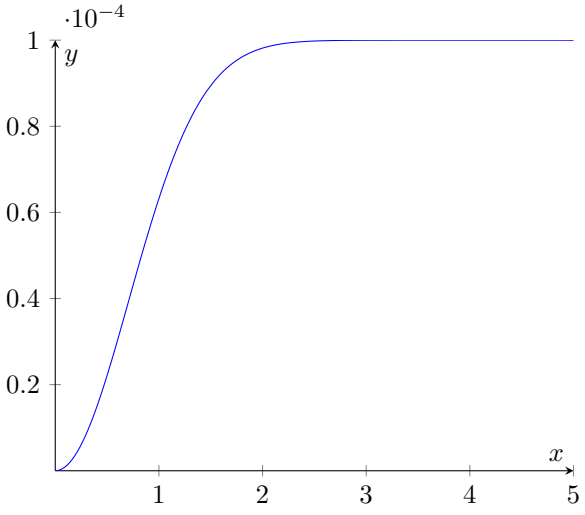
The probability density function $f(t)$ is

$$f(t) = \frac{2t \cdot \frac{\exp(-t^2)}{100}}{100} = \frac{t \cdot \exp(-t^2)}{5000}$$



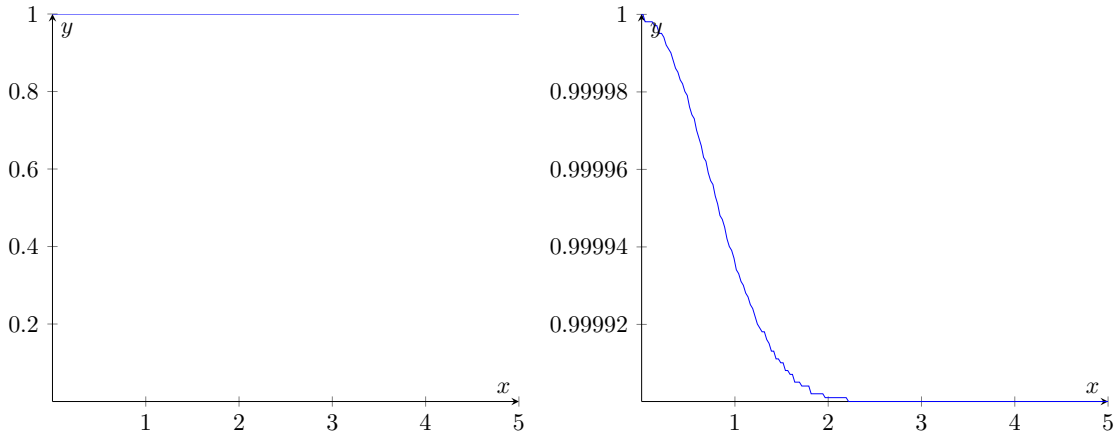
The cumulative distribution function $F(t)$ is then

$$\begin{aligned} F(t) &= \int_0^t f(\xi)\,\mathrm{d}\xi \\ &= \int_0^t \frac{\xi \cdot \exp(-\xi^2)}{5000}\,\mathrm{d}\xi \\ &= \frac{\exp(-t^2)\Big(\exp(t^2) - 1\Big)}{10000} \end{aligned}$$



For the survival function we get

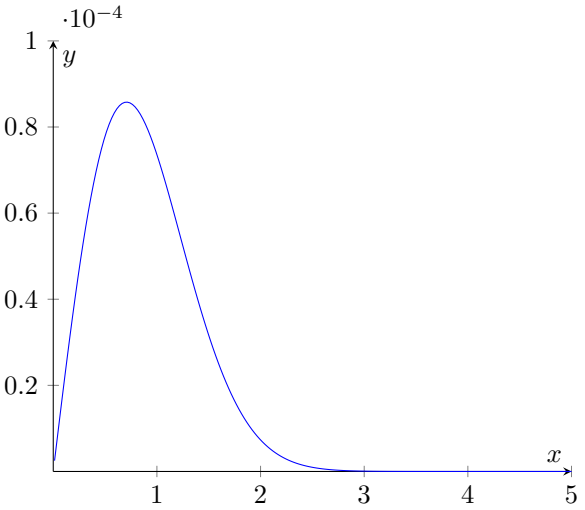
$$\begin{aligned} R(t) &= 1 - F(t) \\ &= \frac{\exp(-t^2) + 9999}{10000} \end{aligned}$$



To get the reliability of the component at $t = 7$ we simply evaluate $R(7)$ which is 0.9999.

The hazard function is defined as

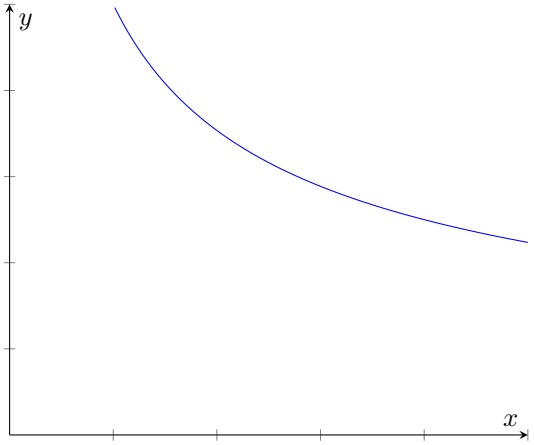
$$\begin{aligned} h(t) &= \frac{f(t)}{1 - F(t)} \\ &= \frac{2t}{9999 \cdot \exp(t^2) + 1} \end{aligned}$$



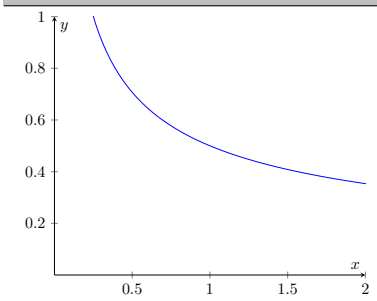
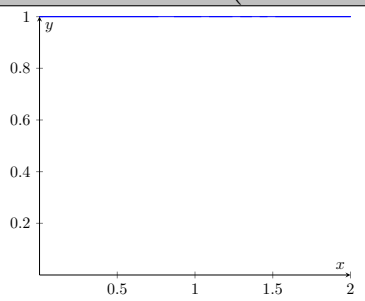
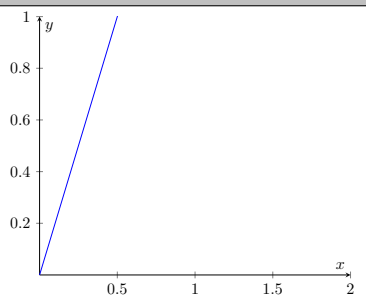
The hazard function describes how an item ages where t affects the risk of failure. It is the frequency with which the item fails, expressed in failures per unit of time.

4.2 Part (2)

Given $h(x) \sim (\sqrt{x})^{-1}$ we will try to find out the *shape*-parameter of the WEIBULL distribution first.



Comparing this graph to graphs of the hazard function with different *shape*-parameters we see that $shape = 0.5$ fits best.

$shape = 0.5$	$shape = 1$	$shape = 2$
Hazard function $\left(h = \frac{\text{PDF}}{1-\text{CDF}}\right)$		
		

To get the *scale*-parameter of the distribution we use the other provided information:

$$\begin{aligned} 5 &= \mu \\ &= scale \cdot \Gamma\left(1 + \frac{1}{shape}\right) \\ &= scale \cdot \Gamma(3) \\ \Rightarrow scale &= \frac{5}{2} \end{aligned}$$

Let's build the survival function:

$$\begin{aligned} R(t) &= 1 - \left(1 - \exp\left(-\sqrt{\frac{x}{5/2}}\right)\right) \\ &= \exp(-\sqrt{x} \cdot \sqrt{2.5}) \end{aligned}$$

That mean that the probability of surviving 6 years (30 years) is $R(6) = 0.0208$ ($R(30) = 0.0002$).