Applied statistics: Coursework 1

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Contents

1 Task 1

1.1 Part (1)

In the given data were two out of 26 data points with an Al/Be ratio of more than 4.5. That means

$$\hat{p} = \frac{2}{26} = \frac{1}{13}$$

1.2 Part (2)

Using the following formula from the lecture we get the 95% confidence interval:

$$\hat{p} \pm 2 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
$$\frac{1}{13} \pm \underbrace{2 \cdot \sqrt{\frac{\frac{1}{13} \cdot \frac{12}{13}}{26}}}_{0.1045}$$

Our 95% confidence interval is [-0.0276,0.1814] which means that we are 95% sure that the true proportion lies between -0.0276 and 0.1814.

1.3 Part (3)

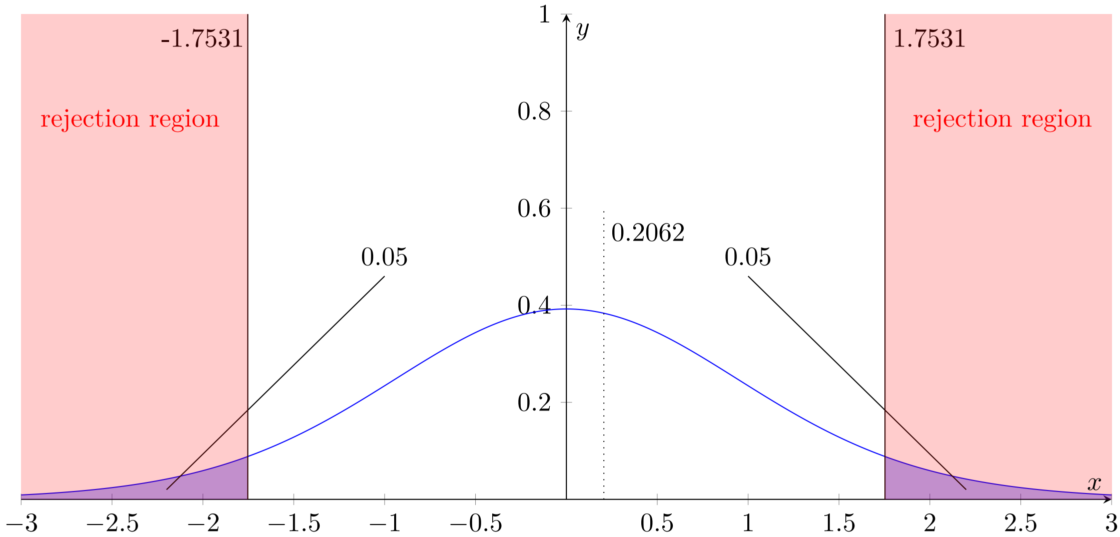
1.4 Part (4)

2 Task 2

2.1 Part (1)

```
x = [-4.5, -1, -0.5, -0.15, 0, 0.01, 0.02, 0.05, ...
0.15, 0.2, 0.5, 0.5, 1, 2, 3];
m = mean(x);
s = std(x);
```

null hypothesis	H_0: $\mu = 0$
alternative hypothesis	H_A: $\mu \neq 0$
t-test for μ	$t = \frac{m-0}{\frac{s}{\sqrt{15}}} = \frac{0.0853}{\frac{1.6031}{\sqrt{15}}} = 0.2062$
rejection region	`tinv(0.05,15)` = -1.7531
conclusion	t lies not in the rejection region so H_0 is accepted at the 10% significance level.



2.2 Part (2)

If we reduce the significance level our rejection region gets smaller. With $\alpha = 0.05$ the rejection region will start at `tinv(0.025,15)` = -2.1314. The t calculated in part (1) won't change $\Rightarrow$ our decision won't change too.

To get the type 2 error we use the MATLAB function `sampsizepwr` and $type\,2\,error = 1 - power$.

```
testtype = 't';
p0 = [0 1.6031];
p1 = 0.0853;
n = 15;
power = sampsizepwr(testtype,p0,p1,[],n)
```

This gives $power = 0.0542 \Rightarrow type\,2\,error = 0.9458$. This is the probability of wrongly accepting H_0 when it is false.

2.3 Part (3)

3 Task 3

3.1 Part (1)

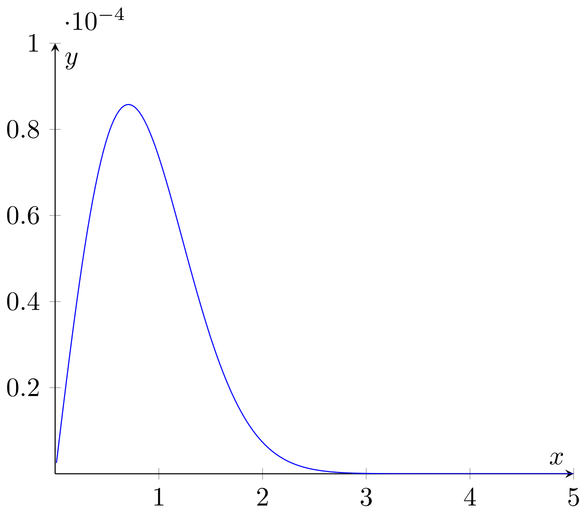
3.2 Part (2)

4 Task 4

4.1 Part (1)

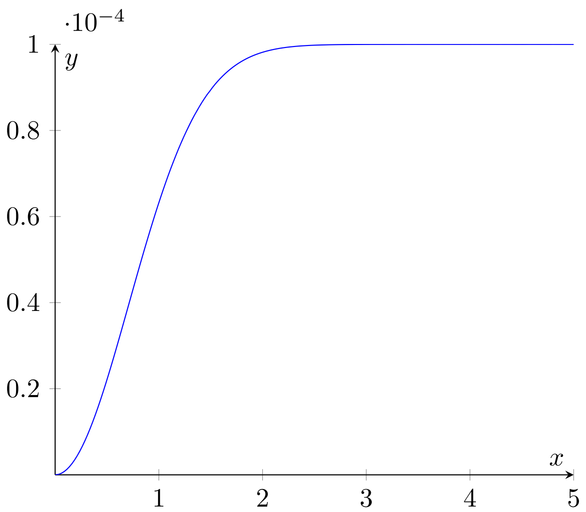
The probability density function $f(t)$ is

$$f(t) = \frac{2t \cdot \frac{\exp(-t^2)}{100}}{100} = \frac{t \cdot \exp(-t^2)}{5000}$$



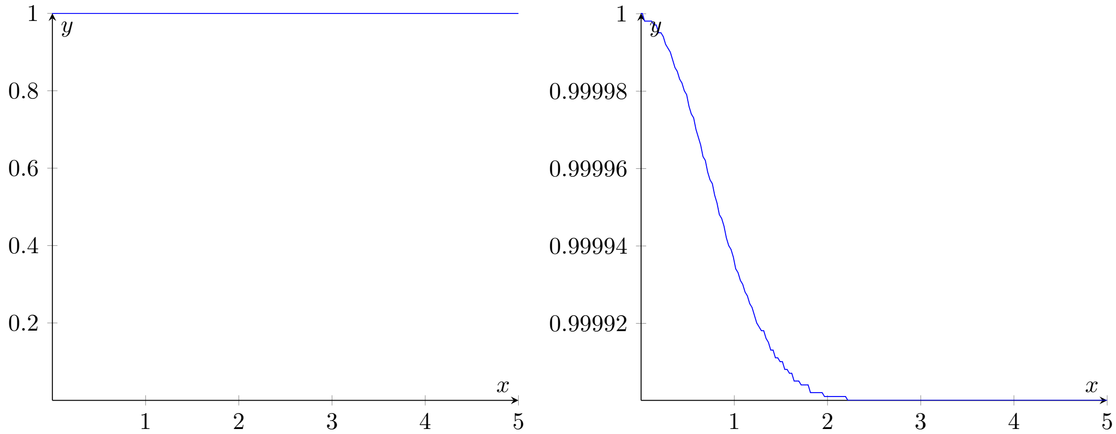
The cumulative distribution function $F(t)$ is then

$$\begin{aligned} F(t) &= \int_0^t f(\xi)\,\mathrm{d}\xi \\ &= \int_0^t \frac{\xi \cdot \exp(-\xi^2)}{5000}\,\mathrm{d}\xi \\ &= \frac{\exp(-t^2)\Big(\exp(t^2) - 1\Big)}{10000} \end{aligned}$$



For the survival function we get

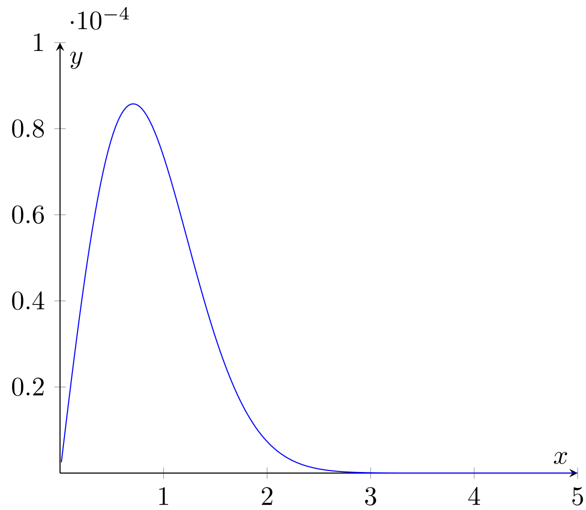
$$
\begin{aligned}
R(t) &= 1 - F(t) \\
&= \frac{\exp(-t^2) + 9999}{10000}
\end{aligned}
$$



To get the reliability of the component at $t = 7$ we simply evaluate $R(7)$ which is 0.9999.

The hazard function is defined as

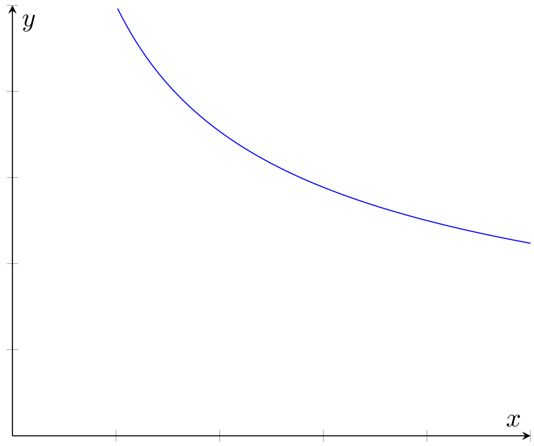
$$
\begin{aligned}
h(t) &= \frac{f(t)}{1 - F(t)} \\
&= \frac{2t}{9999 \cdot \exp(t^2) + 1}
\end{aligned}
$$



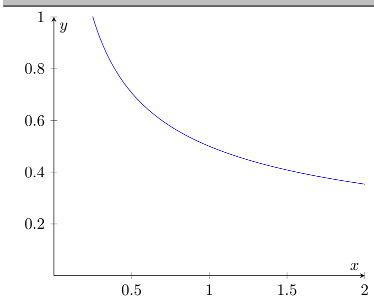
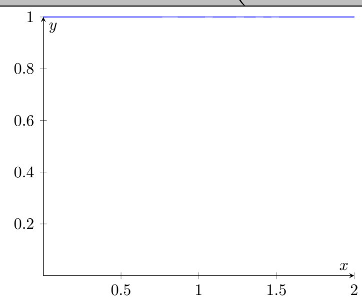
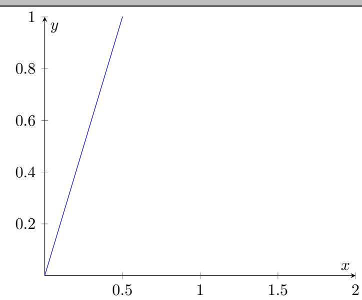
The hazard function describes how an item ages where t affects the risk of failure. It is the frequency with which the item fails, expressed in failures per unit of time.

4.2 Part (2)

Given $h(x) \sim (\sqrt{x})^{-1}$ we will try to find out the *shape*-parameter of the WEIBULL distribution first.



Comparing this graph to graphs of the hazard function with different *shape*-parameters we see that $shape = 0.5$ fits best.

$shape = 0.5$	$shape = 1$	$shape = 2$
Hazard function $\left(h = \frac{\text{PDF}}{1-\text{CDF}}\right)$		
		

To get the *scale*-parameter of the distribution we use the other provided information:

$$
\begin{aligned}
5 &= \mu \\
&= scale \cdot \Gamma\left(1 + \frac{1}{shape}\right) \\
&= scale \cdot \Gamma(3) \\
\Rightarrow scale &= \frac{5}{2}
\end{aligned}
$$

Let's build the survival function:

$$
\begin{aligned}
R(t) &= 1 - \left(1 - \exp\left(-\sqrt{\frac{x}{5/2}}\right)\right) \\
&= \exp(-\sqrt{x} \cdot \sqrt{2.5})
\end{aligned}
$$

That mean that the probability of surviving 6 years (30 years) is $R(6) = 0.0208$ ($R(30) = 0.0002$).