



Compilation of Quantum Programs with Control Flow Primitives in Superposition

Master Thesis

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Introduction

- Idea of Quantum control flow first used by [Altenkirch and Grattage, 2005].
 - Later, it was defined by [Ying et al., 2012].
 - Most languages focus on classical control flow.
 - Only recently, Quantum Control Machine, proposed by [Yuan et al., 2024].
 - An instruction set architecture.
 - Allows for quantum control flow with an assembly-like syntax.
 - Low-level design of the architecture and syntax → complex programs.
- ⇒ Build upon the ideas, define a high-level quantum language with quantum control flow.
- ⇒ Plan to implement a compiler for it.

Quantum Control Flow

- First used by [Altenkirch and Grattage, 2005] to define functional programming language.
- For example, used to define Hadamard gate as function *had*:

$$\begin{aligned} \textit{had} &: Q \rightarrow Q \\ \textit{had} : x &\mapsto \text{if}^\circ x \\ &\quad \text{then } \{ \textit{false} \mid \neg \textit{true} \} \\ &\quad \text{else } \{ \textit{false} \mid \textit{true} \} \end{aligned}$$

- Later, formally defined by [Ying et al., 2012].
- Allows for the execution of functions based on values in superposition.
- Results in superposition of the results of individual executions.

Limitations — Reversibility

- Mainly limited by two principles: *reversibility* and *synchronization*.
 - Instructions required to be reversible, as they are unitary transformations.
- ⇒ Control flow, as implemented in classical computers, not possible.
- Landauer Embedding [Landauer, 1961] seems to offer solution.
 - Can make non-reversible function $f : D \rightarrow D'$ reversible by outputting input value.
 - Result is $g : D \rightarrow D' \times D$ with $g(x) = (f(x), x)$.
 - However, output includes program history and the result depends on the history, they become entangled.
 - Leads to disruptive entanglement, invalid results [Yuan et al., 2024].

Limitations — Synchronization

- Program counter may become entangled with data.
- ⇒ Disruptive entanglement.
- The principle of synchronization: Control flow must become independent from the data.
 - Program may include padding operations to ensure synchronization.
- ⇒ Loops cannot depend solely on value in superposition.
- ⇒ Instead, a loop must be bounded by classical value [Yuan et al., 2024].

Quantum Control Machine

- Quantum Control Machine (QCM), proposed by [Yuan et al., 2024]
- Focused on quantum control flow.
- Syntax and logic similar to classical assembly language
- Employs a branch control register *bcr* to enable reversible jumps.
- Instead of increasing IP by 1 after statement, increased by value of *bcr*.
- The *bcr* can then be reversibly modified.
 - To jump by 5, the *bcr* increased by 5
 - At its destination, decreased by 5 again.

Background

Instructions

- Here, some instructions of the QCM are listed.
- For every instruction there exists a reversed instruction, e.g., `radd` is the subtraction operation.

Operation	Syntax	Semantics ¹
No-op	<code>nop</code>	Only increases instruction pointer by the <i>bcr</i> .
Addition	<code>add ra rb</code>	Adds register <i>rb</i> to <i>ra</i> .
Multiplication	<code>mul ra rb</code>	Multiplies register <i>ra</i> by <i>rb</i> .
Jump	<code>jmp p</code>	Increases <i>bcr</i> by <i>p</i> .
Conditional Jumps	<code>jz p ra</code>	Increases <i>bcr</i> by <i>p</i> if <i>ra</i> is 0.
	<code>jne p ra rb</code>	Increases <i>bcr</i> by <i>p</i> if <i>ra</i> is not equal to <i>rb</i> .

¹ After all operations, the instruction pointer is increased by the value of the *bcr*.

An excerpt of the QCM instruction set with instructions used in later examples.

Background

(Non-) Reversible Example

- Example of a classical program and the reversible equivalent.
- Both programs calculate x^y .
- Neither are synchronized because loop is not bound by classical value.

```
1      add    res $1
2      add    r1  y
3  l1:  jz     l2  r1
4      mul    res x
5      radd   r1  $1
6      jmp    l1
7  l2:  nop
```

A non-reversible exponentiation algorithm.

```
1      add    res $1
2      add    r1  y
3  l1:  rjne   r11 r1  y
4  r12: jz     l2  r1
5      mul    res x
6      radd   r1  $1
7  r11: jmp    l1
8  l2:  rjmp   l2
```

A reversible exponentiation algorithm.

Background

Reversible Synchronized Example

- Synchronized implementation, calculating $x^{\min y, \max}$.
- *max* is a classical bound for loop.
- Line 9 includes padding instruction.

```
1      add    res    $1
2      add    r1     max
3  l1:    rjne   r11   r1    max
4  r12:    jz     l2     r1
5  r13:    jg     l3     r1    y
6      mul    res    x
7  r14:    jmp    l4
8  l3:    rjmp   r13
9      nop                    ; padding
10 l4:    rjle   r14   r1    y
11      radd   r1     $1
12 r11:    jmp    l1
13 l2:    rjmp   r12
```

A synchronized, reversible exponentiation algorithm.

Language Overview

- Idea for language: Provide high-level language with the capabilities of the QCM.
- Want to remove low-level concepts, add high-level ones.
- Jump instructions in superposition are removed → need to add other control flow primitives.
- Introduce multiple high-level concepts and control flow statements:
 - blocks and scopes,
 - different data types,
 - composite gates,
 - loop statements, unrolled at compile time, and
 - quantum if- and else-statements.

Language

Syntax

- Define CFG_{Luie} for our language.
- Start symbol is the program, consisting of gate declarations and a block.
- A block is a list of translatables, either statements or declarations.

$$\begin{aligned} CFG_{Luie} &= (V_{Luie}, \Sigma_{Luie}, R_{Luie}, prg_{Luie}) \\ V_{Luie} &= \{exp, rExp, gate, prg_{Luie}, \dots\} \\ \Sigma_{Luie} &= \{.., range, (,), \dots\} \end{aligned}$$

$$\begin{aligned} Program : prg_{Luie} & ::= gDcl_1 \dots gDcl_n blk \mid \\ & blk \\ Block : blk & ::= t_1 \dots t_n \mid \\ & \epsilon \end{aligned}$$

```
1 /* Gate Declaration */
2 gate c_h_reg(control, reg) do
3     qif control do
4         for i in range(sizeof(reg)) do
5             h reg[i];
6         end
7     end
8 end
9
10 /* Block Start */
11 const regSize : int = 3; // Decl.
12 qubit c; // Decl.
13 qubit[regSize] a; // Decl.
14 c_h_reg c, a;
15 /* Block End */
```

Luie program with structure highlighted.

Language

Syntax

- Three different statements: quantum if-statement, a loop statement, gate application.
- To combine qubit or register access: qubit argument.
- Expressions for numeric values or ranges.
- Fix set of defined gates \rightarrow differentiate translations.

Statement : *stm* ::= *qif* *qArg* *do blk end* |
 for *id* *in rExp* *do blk end* |
 id *qArg*₁, ..., *qArg*_{*n*};
QubitArg : *qArg* ::= *id* | *id*[*exp*]

```
1 gate c_h_reg(control, reg) do
2   /* If-Statement */
3   qif control do
4     /* Loop Stat. |  $\nabla$  Expression */
5     for i in range(sizeof(reg)) do
6       /*  $\nabla$  Range Exp. */
7       h reg[i];
8     end
9   end
10 end
11
12 const regSize : int = 3;
13 qubit c;
14 qubit[regSize] a;
15 c_h_reg c, a; // Gate Application
16   /*  $\nabla$   $\nabla$  Qubit Argument */
```

Luie program with statements and arguments highlighted.

ConstGates = {h, x, y, z, cx, ccx}

Symbol Table

- Saves symbol information relevant for translation.
- Contains four types of symbols:
 1. named constants,
 2. quantum registers and qubits,
 3. qubit arguments, and
 4. composite gates.

$$\begin{aligned} \textit{SymbolTable} := \{st \mid st : \textit{Identifier} \dashrightarrow & (\{\textit{const}\} \times \mathbb{Q}) \\ & \cup (\{\textit{qubit}\} \times \mathbb{N} \times \textit{Identifier}) \\ & \cup (\{\textit{arg}\} \times \textit{QubitArgument}) \\ & \cup (\{\textit{gate}\} \times \textit{Block} \times \textit{Identifier}^+)\} \end{aligned}$$

Translation Function and Block Translation

- *trans* function translates program.
- Symbol table updated with gate declaration by *up* function.
- Block translation *bt* translates all translatable.
- Updates symbol table used in next translation.

$$st_1 = up(st_e, \text{gate } c_h_reg \dots) = [c_h_reg \mapsto (\text{gate}, \text{qif control do } \dots \text{ end, control, reg})]$$
$$\begin{aligned} bt(\text{const } regSize = 3; \dots, st_1) &= tr_1 && \text{where } (tr_1, st_2) = tt(\text{const } regSize = 3; , st_1) \\ &tr_2 && \text{where } (tr_2, st_3) = tt(\text{qubit } c; , st_2) \\ &tr_3 && \text{where } (tr_3, st_4) = tt(\text{qubit}[regSize] a; , st_3) \\ &tr_4 && \text{where } (tr_4, -) = tt(c_h_reg c, a; , st_4) \end{aligned}$$

Translatable and Declaration Translation

- *tt* translates each translatable, differentiates between declaration and command.
 - Declarations translated by *dt*.
 - Update symbol table \rightarrow function returns (updated) symbol table.
 - Language allows for different variable scopes \rightarrow same identifier can be used multiple times.
- \Rightarrow Unique identifier *uid* is generated for translation.

$$tt(c_h_reg\ c,\ a;\ ,st) = (ct(c_h_reg\ c,\ a;\ ,st), st)$$

$$tt(qubit\ c;\ ,st) = dt(qubit\ c;\ ,st)$$

$$dt(qubit\ c;\ ,st) = (qubit\ uid_c;\ ,up(st, qubit\ c;\))$$

$$up(st, qubit\ c;\) = st[c \mapsto (qubit, 1, uid_c)]$$

Command Translation

- Commands are translated with *ct*.
- Argument translation *qt* used to differentiate between qubits and register accesses, get *uid*.
- *control* adds the translated *qArg* as guard to all gate applications in translation.

$$st = [c \mapsto (\text{qubit}, 1, uid_c), \\ a \mapsto (\text{qubit}, 3, uid_a)]$$

$$ct(\text{if } c \text{ do } x \ a[0]; \text{ end}, st) = control(qt(c, st), bt(x \ a[0];, st)) \\ = control(uid_c, x \ uid_a[0];)$$

$$qt(c, st) = st[c]_3 = uid_c \\ bt(x \ a[0];, st) = x \ qt(a[0]); = x \ uid_a[0];$$

$$control(uid_c, x \ uid_a[0];) = ctrl(1) \ @ \ x \ uid_c, uid_a;$$

Implementation

Overview

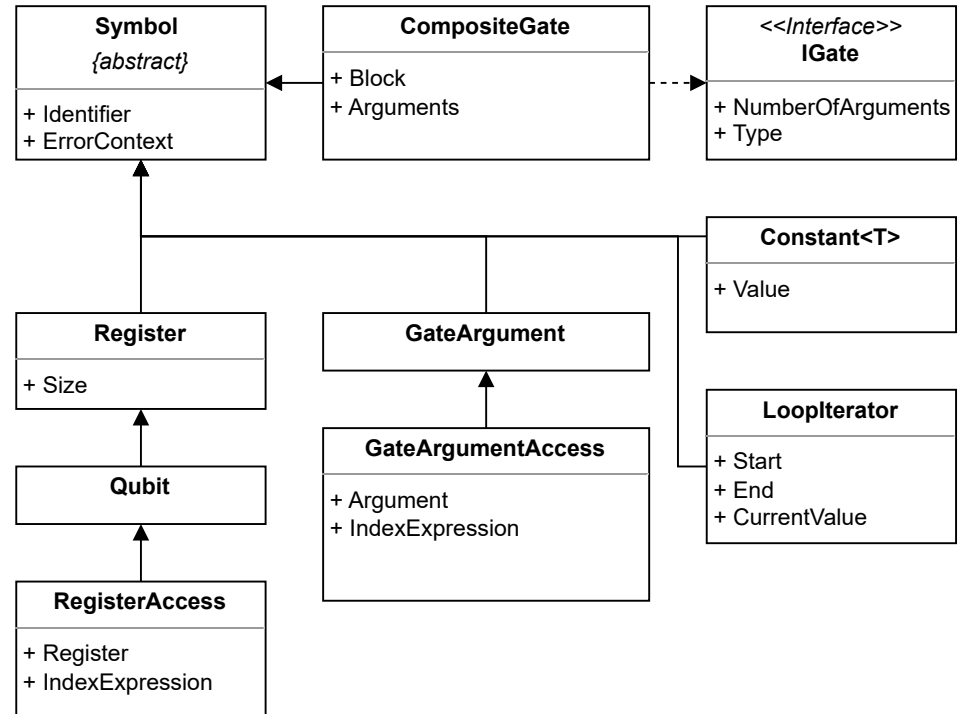
- Four compilation stages:
 1. the lexical and syntactic analysis,
 2. semantic analysis,
 3. code generation, and
 4. optimizations
- Process managed by a static compiler class.
 - Parses command line parameters.
 - Handles input and output of files.
 - Calls the different stages.
 - Handles logging and error messages.

```
1 ./LUIECompiler --input "./program.luie" \
2                  --output "./build/program.qasm" \
3                  --optimization nullgate+peepingcontrol
```

A command line interface example.

Symbols and Symbol Table

- Symbol table used to save and propagate symbol information.
- Handles higher-level concepts such as variable contexts.
- Contains dictionary that maps identifiers to symbol objects.
- All symbol objects derived from abstract symbol class.



A diagram showing the hierarchy of symbol classes.

Lexical and Syntactic Analysis

- First compilation stage is lexical and syntactic analysis.
- Both lexer and parser created with the ANTLR4 tool.
- Generates the source code based on a given grammar.
- Implementation of the grammar is more elaborate version of previously disused one.

```
1 parse      : mainblock EOF;  
2  
3 mainblock  : gateDeclaration* (declaration | statement)*;  
4  
5 block      : (declaration | statement)*;
```

The basic structure of parsing rules for Luie.

Semantic Analysis

- Analyses non-syntactic constraints of program, mainly:
 1. declaration analysis and
 2. type checking.
- Declaration analysis ensures all identifiers used were previously declared and all identifiers used in declarations are not already declared.
- Type checking ensures that symbols are used in the correct context.

```
1 qubit c;                /* ↙ Type error (cannot use qubits in arith. expression) */
2 const regSize : int = c + 2;
3                        /* ↙ Already declared */
4 qubit[regSize] c;
5                        /* ↙ Type error (expects a register or qubit) */
6 c_h_reg c, a, regSize;
7                        /* ↗ Undeclared identifier */
```

Luise program with semantic errors highlighted.

Implementation

Code Generation

- Parse tree is traversed and source code is translated to in-memory representation.
- Source code representation (SCR) is translated to target code representation (TCR).
- TCR can be translated directly to the textual OpenQASM code.
- Example code generation with program:

```
1 gate c_h_reg(control, reg) do
2     qif control do
3         for i in range(sizeof(reg)) do
4             h reg[i];
5         end
6     end
7 end
8
9 qubit c;
10 qubit[3] a;
11 c_h_reg c, a;
```

An example Luie program to show the code generation process.

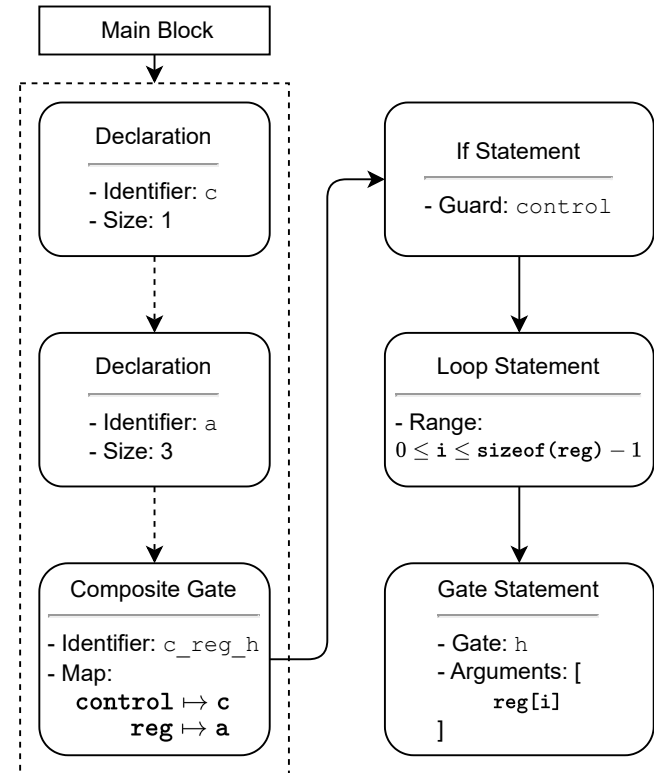
Implementation

Source Code Representation

- Three main classes:
 1. Code block: contains list of translatables.
 2. Declaration: only register declaration, constants compile-time only.
 3. Statements: variety of different classes.
- Example contains three translatables.
 - First two declarations, last is gate statement.
 - Gate's body \supset if-statement \supset loop statement \supset gate application.

```
1 gate c_h_reg(control, reg) do
2   qif control do
3     for i in range(sizeof(reg)) do
4       h reg[i];
5   end end end
6
7 qubit c; qubit[3] a;
8 c_h_reg c, a;
```

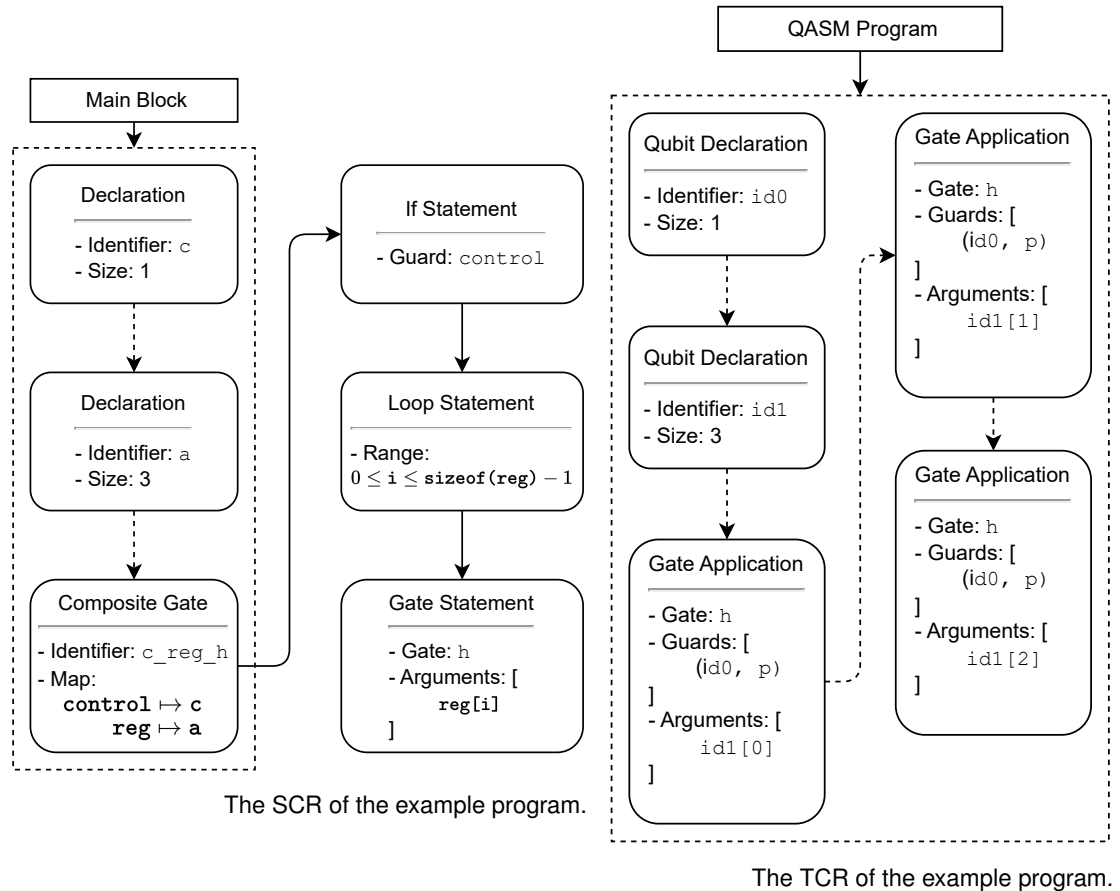
An example Luie program to show the code generation process.



The SCR of the example program.

Target Code Representation

- TCR based on `QASMP` program object.
- Contains list of `Code` objects, either gate or declaration.
- All SCR objects translated to list of code objects and appended to program object.
- Translate program object: code iterated, converted to text, written file.



Translated Example Program

- TCR converted to OpenQASM program.
- Version string and include header prepended to the code.
- For each quantum register, classical one is declared, the registers are measured and saved to registers.
- Additions are performed right before result is written to output and after optimization.

```
1 OPENQASM 3.0;  
2 include "stdgates.inc";  
3 qubit id0;  
4 qubit[3] id1;  
5 ctrl(1) @ h id0, id1[0];  
6 ctrl(1) @ h id0, id1[1];  
7 ctrl(1) @ h id0, id1[2];  
8 bit id0_measurement;  
9 measure id0 -> id0_measurement;  
10 bit[3] id1_measurement;  
11 measure id1 -> id1_measurement;
```

The OpenQASM translation of the example Luie program.

Optimization

- Compiler can perform optimizations to reduce number of gates and qubits.
- Optimizations are performed to TCR → allow for more optimizations at cost of performance.
- Compiler performs general peephole optimizations based on rules presented by [Garcia-Escartin and Chamorro-Posada, 2011].
- Can be divided into four rules:
 1. null gate,
 2. peeping control,
 3. Hadamard reduction, and
 4. control reversal.

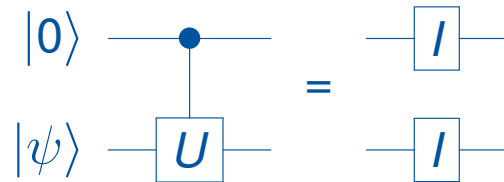
Implementation

Null Gate and Peeping Control Rules

- Null gates are combinations of gates under specific conditions equivalent to I .
 - Simplest null gate version is twofold application of self-inverse gate.
 - Can be removed entirely from circuit.
- Our peeping control rules are special case of null gates.
 - Control is $|1\rangle \rightarrow$ remove control.
 - Control is $|0\rangle \rightarrow$ remove gate.



Null gates of self-inverse gates.



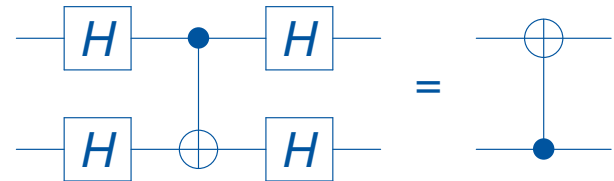
Null gates for gates in specific conditions.

Hadamard Reduction and Control Reversal Rules

- Hadamard reduction can reduce either X or Z surrounded by H to the other without surrounding H .
- Rule is basis for control reversal.
- Emerges when Hadamard reduction is combined with control reversal of CZ .
- A CX surrounded by H is equivalent to target and control qubits exchanged without H .



A Hadamard reduction rule.



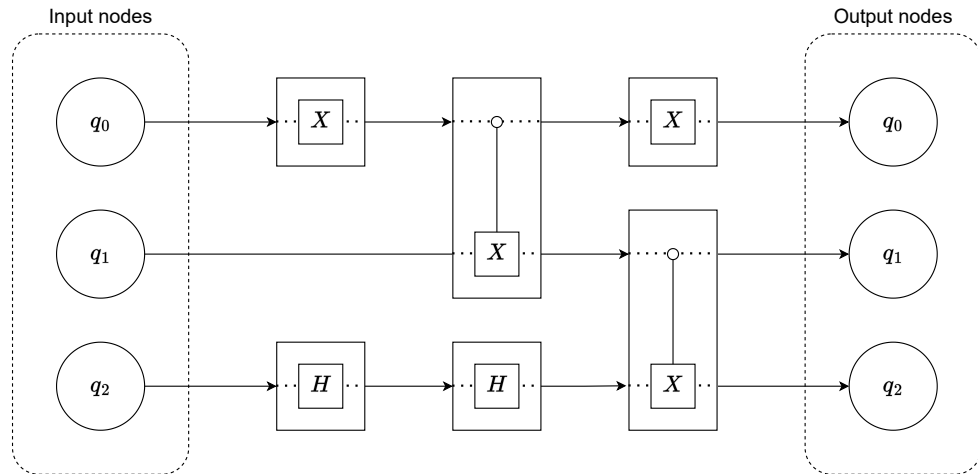
Null gates for gates in specific conditions.

Circuit Graph

- Rules not directly applied to program, but to circuit graph.
- Program $\xrightarrow{\text{Translation}}$ Graph $\xrightarrow{\text{Optimization}}$ Graph $\xrightarrow{\text{Translation}}$ Program
- Circuit graph is acyclic and directed.
 - Nodes split into input, output, gate nodes.
 - Input, output nodes have only one outgoing, incoming edge.
 - Each input, output, edge corresponds belongs to qubit in program.
 - Qubit wire represented by path from input to output node over corresponding edges.

```
1 qubit[3] q;  
2 x q[0];  
3 cx q[0], q[1];  
4 x q[0];  
5 h q[2];  
6 h q[2];  
7 cx q[1], q[2];
```

A simple, unoptimized program.

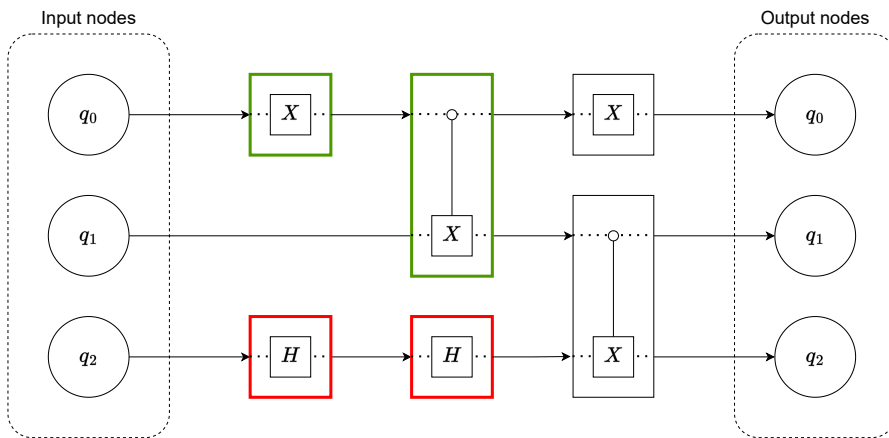


An example of a simple, unoptimized circuit graph.

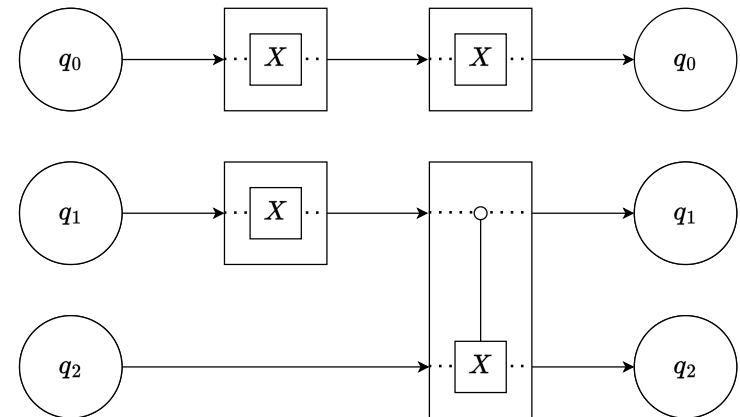
Implementation

Example Optimization Process I

- To optimize the graph, each qubit wire is iterated.
- All subpaths (up to max. length) checked for alternatives.
- Example:
 - **First wire:** Peeping control rule can be applied to first **CX** gate.
 - **Third wire:** **HH** null gate can be removed.



An example of a simple, unoptimized circuit graph.

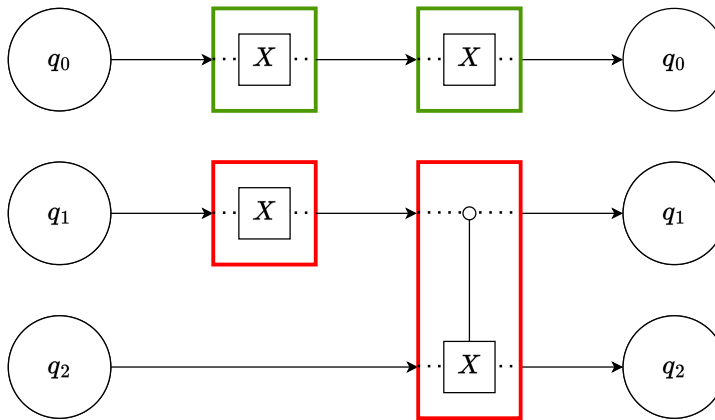


The circuit graph after the first optimization step.

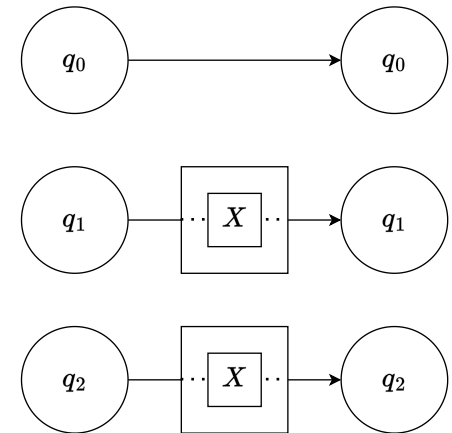
Implementation

Example Optimization Process II

- While all qubit wires iterated, still possible optimizations.
 - Applying optimizations may enable others!
- ⇒ Optimization repeated as long as previous iteration applied optimizations.
- Example:
 - **First wire:** XX null gate can be removed.
 - **Second wire:** Peeping control rule can be applied to CX gate.



The circuit graph after the first optimization step.



The completely optimized graph.

Evaluation

- Evaluation consists of two aspects:
 1. optimizations performed by compiler and
 2. execution time of compilation stages.
- Example program: quantum ripple-carry adder proposed by [Cuccaro et al., 2004].
 - Takes two registers *a* and *b*, two qubits *cin* and *cout*.
 - Adder adds *a* register to *b* register.
 - *cin* and *cout* qubits are used as input, output carry bits.
- Our implementation consists only of *CX* and *CCX*.

Optimization Evaluation

- For optimization evaluation, classical inputs and in superposition.
- First inputs: $a = |1\rangle$ and $b = |15\rangle$.
 - Classical inputs \rightarrow peeping control rules, null gate rules can be applied.
 - Optimized circuit: only gates that initialize the result.
 - Only two X gates remain.
 - First X flips $a[0]$ qubit $\rightarrow a = |1\rangle$, second flips $cout$, indicating result of $|16\rangle$.
- Second inputs: $a = \frac{1}{\sqrt{2}}(|0\rangle + |3\rangle)$ and $b = |4\rangle$.
 - Inputs in superposition \rightarrow peeping control rules, null gate rules only applied *partially*.
 - Only $\frac{12}{25}$ gates can be optimized.
 - Other inputs in superposition: even fewer gates optimized.

Evaluation

Performance Evaluation

- Compiled adder with input of $a = \frac{1}{\sqrt{2}}(|0\rangle + |3\rangle)$ and $b = 15$ for different register sizes n .
- Program size does not change \rightarrow execution times of the semantic analysis remain constant.
- Code generation stage: linear increase.
 - Unrolled loop \rightarrow compiled program increases linearly.
- Optimization worst performance, approximate quadratic increase.






Register Size n	Execution Time of Stages in ms		
	Semantic Analysis	Code Generation	Optimization
64	27.3	47.8	711.6
128	26.3	50.4	2292.4
256	26.2	59.7	10755.7
512	25.8	74.9	60204.7
1024	26.1	109.1	405376.6

The execution times compiling a quantum ripple-carry adder with different register sizes.


Conclusion

- Multiple aspects that can be improved.
- Optimizations:
 - Focused on high-level optimizations → work best in tandem with other tools.
 - Addition of hardware-focused features and transpilation to concrete devices.
 - Improve performance of optimization stage by parallelization of wire traversals.
 - Increases complexity of program, e.g., race conditions need to be considered.
- General improvements:
 - Addition of type casting for the named constants.
 - Expansion of predefined function; addition of constants, e.g., π or e .
 - Addition of explicit measurement.

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