Compilation of Quantum Programs with Control Flow Primitives in Superposition

Master Thesis

05.02.25

Sascha Thiemann

Supervisors: apl. Prof. Dr. Thomas Noll

Prof. Dr. rer. nat. Dominique Unruh

Outline

Introduction

test [Aaby, 2003]

Quantum Control Flow

- The idea of Quantum Control Flow was first used by [Altenkirch and Grattage, 2005] to define function quantum programming language.
- For example, it was used to define the Hadamard gate as the function *had*:

$$
\begin{aligned}
had &: Q \to Q \\
had &: x \mapsto \texttt{if}^{\circ} x \\
&\qquad\quad \texttt{then}\ \{false \mid -true\} \\
&\qquad\quad \texttt{else}\ \{false \mid true\}
\end{aligned}
$$

- Later, the concept was formally defined by [Ying et al., 2012].
- Quantum branching allows fot the execution of function based on values in superposition.
- The result is the superposition of the results of individual executions.

Limitations — Reversibity

- Quantum control flow is mainly limited by two principles: *reversibility* and *synchronization*.
- Any sequence of instructions on gate-based quantum computers, excluding measurements, is required to be reversible by definition, as they are all unitary transformations.
- As a result, control flow, as implemented in classical computers, is not possible.
- For example, any classical jump instruction is inherently irreversible.
- Landauer Embedding [Landauer, 1961] seems to offer solution.
- The embedding can turn any non-reversible function into a reversible one by not only returning the output but also the input of the function.
- For example, any non-reversible function $f : D \rightarrow D'$ can be given as a reversible function $g : D \rightarrow D' \times D$ with $g(x) = (f(x), x)$.
- However, because the output is the result together with the program history and the result depends on the history, they become entangled.
- This leads to disruptive entanglement [Yuan et al., 2024].

Limitations — Synchronization

- The program counter can become entangled with the data and result in disruptive entanglement leading to an invalid result.
- The principle of synchronization states that control flow must become independent from the data.
- For example, loops cannot depend solely on value in superposition.
- *Tortoise and hare* problem
- Instead, a loop must be bounded by a classical value [Yuan et al., 2024].

Quantum Control Machine

- Quantum Control Machine (QCM), proposed by [Yuan et al., 2024], is an instruction set architecture, focused on quantum control flow.
- Both its syntax and logic are similar to classical assembly language, utilizing (conditional) jump instructions.
- The architecture employs a branch control register *bcr* to enable reversible jump instructions.
- Instead of increasing the IP by 1 after each statement, it is increased by the value in the *bcr*.
- The *bcr* can then by reversibly modified.
- To jump by 5, the *bcr* is increased by 5 and, at its destination, decreased by 5 again.

Instructions

Operation	Syntax	Semantics[1]
No-op	`nop`	Only increases instruction pointer by the *bcr*.
Addition	`add` *ra rb*	Adds register *rb* to *ra*.
Multiplication	`mul` *ra rb*	Multiplies register *ra* by *rb*.
Jump	`jmp` *p*	Increases *bcr* by *p*.
Conditional Jumps	`jz` *p ra*	Increases *bcr* by *p* if *ra* is 0.
	`jne` *p ra rb*	Increases *bcr* by *p* if *ra* is not equal to *rb*.

[1] After all operations, the instruction pointer is increased by the value of the *bcr*.

An excerpt of the QCM instruction set with instructions used in later examples.

(Non-) Reversible Example

```
     add   res $1
     add   r1  y
l1:  jz    l2  r1
     mul   res x
     radd  r1  $1
     jmp   l1
l2:  nop
```

A non-reversible exponentiation algorithm.

```
      add   res $1
      add   r1  y
l1:   rjne  rl1 r1  y
rl2:  jz    l2  r1
      mul   res x
      radd  r1  $1
rl1:  jmp   l1
l2:   rjmp  l2
```

A reversible exponentiation algorithm.

Background

Reversible Synchronized Example

```
            add    res   $1
            add    r1    max
l1:         rjne   rl1   r1   max
rl2:        jz     l2    r1
rl3:        jg     l3    r1   y
            mul    res   x
rl4:        jmp    l4
l3:         rjmp   rl3
            nop                      ; padding
l4:         rjle   rl4   r1   y
            radd   r1    $1
rl1:        jmp    l1
l2:         rjmp   rl2
```

A synchronized, reversible exponentiation algorithm.

Language Overview

- The idea for our language is to provide a high-level language with the capabilities of the QCM.
- We want to remove low-level concepts and add high-level ones.
- Additionally, since jump instructions in superposition are removed, we need to add other control flow statements so that the language is as expressive as the QCM.
- For this, we introduce multiple high-level concepts and two basic control flow statements:
  - Blocks and scopes
  - Different data types
  - Composite gates
  - Loop statements, unrolled at compile time
  - Quantum if- and else-statements

Syntax

- We define a CFG CFG_{Luie} for our language.
- The start symbol is the program, consisting of arbitrarily many gate declarations and a block.
- A block is a list of translatables, either statements of declarations.

$$
\begin{aligned}
CFG_{Luie} &= (V_{Luie}, \Sigma_{Luie}, R_{Luie}, prg_{Luie}) \\
V_{Luie} &= \{exp, rExp, gate, qArg, stm, prg_{Luie}, \dots\} \\
\Sigma_{Luie} &= \{\texttt{..}, \texttt{range}, \texttt{(}, \texttt{)}, \dots\} \quad \texttt{where} \quad n \in \mathbb{N}_0, id \in Identifier
\end{aligned}
$$

$$
\begin{aligned}
Program : prg_{Luie} &::= gDcl_1 \dots gDcl_n \; blk \mid blk \\
Block : blk &::= t_1 \dots t_n \mid \epsilon \\
Translatable : t &::= stm \mid dcl \\
Declaration : dcl &::= \texttt{const} \;\; id \;\; \texttt{=} \;\; exp\texttt{;} \mid \\
&\qquad \texttt{qubit} \;\; id\texttt{;} \\
GateDeclaration : gDcl &::= \texttt{gate} \;\; id \;\; \texttt{(}id_1, \dots, id_n\texttt{)} \;\; \texttt{do} \;\; blk \;\; \texttt{end}
\end{aligned}
$$

Language

Syntax

- There are three different statements: quantum if-statement, loop statement, and application of predefined or composite gate.
- The qubit argument differentiates between qubit or register access.
- For the register access or constant declarations, expressions can be used.
- Additionally, we defined a set of defined gates to differentiate the corresponding translations.

$$
\begin{aligned}
\textit{Statement} : \textit{stm} &::= \texttt{qif}\ \ \textit{qArg}\ \ \texttt{do}\ \ \textit{blk}\ \ \texttt{end}\ \mid \\
&\quad\ \texttt{for}\ \ \textit{id}\ \ \texttt{in}\ \ \textit{rExp}\ \ \texttt{do}\ \ \textit{blk}\ \ \texttt{end}\ \mid \\
&\quad\ \textit{id}\ \textit{qArg}_1, \ldots, \textit{qArg}_n\texttt{;} \\
\textit{QubitArgument} : \textit{qArg} &::= \textit{id} \mid \textit{id}[\textit{exp}] \\
\textit{Expression} : \textit{exp} &::= n \mid \textit{id} \mid \textit{exp}_1\ +\ \textit{exp}_2 \mid \textit{exp}_1\ -\ \textit{exp}_2 \mid \ldots \\
\textit{RangeExpression} : \textit{rExp} &::= n_1..n_2 \mid \texttt{range(}\textit{exp}\texttt{)} \mid \texttt{range(}\textit{exp}_1, \textit{exp}_2\texttt{)}
\end{aligned}
$$

$$\textit{ConstGates} = \{\texttt{h}, \texttt{x}, \texttt{y}, \texttt{z}, \texttt{cx}, \texttt{ccx}\}$$

Language

Example Program

```
gate c_h_reg(control, reg) do
    qif control do
        for i in range(sizeof(reg)) do
            h reg[i];
        end
    end
end

const regSize : int = 3;
qubit c;
qubit[regSize] a;
c_h_reg c, a;
```

An example Luie program.

Symbol Table

- The symbol tables saves the symbol information relevant for the translation.
- It contains four different types of symbols:
  1. Named constants
  2. Quantum registers and qubits
  3. Qubit arguments
  4. Composite gates

$$\begin{aligned} \mathit{SymbolTable} := \{ st \mid st : \mathit{Identifier} \dashrightarrow & (\{\texttt{const}\} \times \mathbb{Q}) \\ & \cup (\{\texttt{qubit}\} \times \mathbb{N} \times \mathit{Identifier}) \\ & \cup (\{\texttt{arg}\} \times \mathit{QubitArgument}) \\ & \cup (\{\texttt{gate}\} \times \mathit{Block} \times \mathit{Identifier}^{+}) \} \end{aligned}$$

Translation Function and Block Translation

- The *trans* function translate the Luie program to OpenQASM.
- The initial symbol table st_ϵ contains no mappings.
- Next, the block translation function *bt* translates all translatables, i.e., statements and declarations.

$$trans : Program \dashrightarrow QASMProgam$$

$$trans(gDcl_1 \dots gDcl_n\ blk) = bt(blk, update(update(update(st_\epsilon, gDcl_1), \dots), gDcl_n))$$

$$bt : Block \times SymbolTable \dashrightarrow QASM$$

$$\begin{aligned} bt(t_1 \dots t_n, st_1) = {} & tr_1 \quad \text{where } (tr_1, st_2) = tt(t_1, st_1) \\ & \dots \\ & tr_n \quad \text{where } (tr_n, \_) = tt(t_n, st_n) \end{aligned}$$

Translatable and Declaration Translation

- The translatable function *tt* translates each translatable.
- Since declarations update the symbol table, the function returns the updated symbol table.
- The language allows for different variable scopes and, in turn, an identifier can be used multiple times.
- Therefore, a unique identifier *uid* is generated for the translation of identifiers.

$$tt : \mathit{Translatable} \times \mathit{SymbolTable} \dashrightarrow \mathit{QASM} \times \mathit{SymbolTable}$$

$$tt(t, st) = \begin{cases} dt(t, st) & \text{if } t \in \mathit{Declarations} \\ (ct(t, st), st) & \text{otherwise} \end{cases}$$

$$dt : \mathit{Declaration} \times \mathit{SymbolTable} \dashrightarrow \mathit{QASM} \times \mathit{SymbolTable}$$

$$dt(\underbrace{\texttt{qubit}\ \ id;}_{decl}, st) = (\texttt{qubit}\ \ uid\,\boldsymbol{;}\,, st')$$

$$\text{where } st' = update(decl, st) \text{ and } st'[id] = (\texttt{qubit}, 1, uid)$$

Command Translation

- The commands are translated with the *ct* function.
- We take a look at an example translation of a quantum if-statement.
- The qubit argument translation *qt* is used to differentiate between qubits and register accesses and looks up the *uid*.
- The *control* function adds the translated *qArg* as a guard to all gate applications in the block translation.

$$ct : \mathit{Statement} \times \mathit{SymbolTable} \dashrightarrow \mathit{QASM}$$

$$ct(\texttt{qif}\ \ \mathit{qArg}\ \ \texttt{do}\ \ \mathit{blk}\ \ \texttt{end}, \mathit{st}) = \mathit{control}(\mathit{qt}(\mathit{qArg}, \mathit{st}), \mathit{bt}(\mathit{blk}, \mathit{st}))$$

$$\begin{aligned}
\mathit{control}(\mathit{qArg},\ & \mathit{id}\ \mathit{qArg}_1, \ldots, \mathit{qArg}_{n'}\,\texttt{;}\,) = \\
\texttt{ctrl(1) @}\ \ & \mathit{id}\ \mathit{qArg}, \mathit{qArg}_1, \ldots, \mathit{qArg}_{n'}\,\texttt{;} \\
\mathit{control}(\mathit{qArg}, \texttt{ctrl(}n\texttt{) @}\ \ & \mathit{id}\ \mathit{qArg}_1, \ldots, \mathit{qArg}_{n'}\,\texttt{;}\,) = \\
\texttt{ctrl(}n+1\texttt{) @}\ \ & \mathit{id}\ \mathit{qArg}, \mathit{qArg}_1, \ldots, \mathit{qArg}_{n'}\,\texttt{;}
\end{aligned}$$

Overview

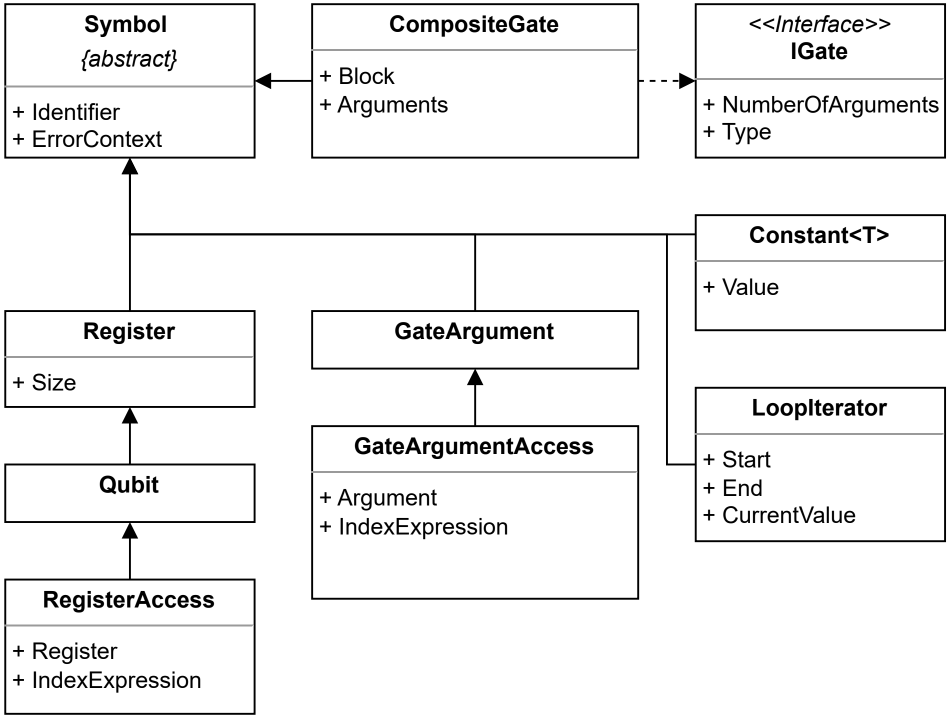
- The implementation of the compiler is differentiate into four different stages:
  1. the lexical and syntactic analysis,
  2. semantic analysis,
  3. code generation, and
  4. optimizations
- The process is managed by a static compiler class.
- It parses the command line parameters, handles the input and output of files, and calls the different stages.

```
./LUIECompiler --input "./program.luie"               \
               --output "./build/program.qasm"        \
               --optimization nullgate+peepingcontrol
```

A command line interface example.

Symbols and Symbol Table

- Through out the whole compilation process, the symbol table is used to save and propagate symbol information.
- Additionally, it handles higher level concepts such as variable contexts.
- It contains a dictionary that maps identifier to the symbol objects.
- All symbol objects are derived from an abstract symbol class.



A diagram showing the hierarchy of symbol classes.

Implementation

Lexical and Syntactic Analysis

- The first compilation stage is the lexical and syntactic analysis.
- Both the lexer and parser are created with the ANTLR4 tool.
- It generates the source code based on a given grammar.
- The implementation of the grammar is a more elaborate version of the syntax given previously.

```
parse     : mainblock EOF;

mainblock : gateDeclaration* (declaration | statement)*;

block     : (declaration | statement)*;
```

The basic structure of parsing rules for Luie.

Semantic Analysis

- The next step is the semantic analysis.
- An analysis without any context is not sufficient for non-syntactic constraints of the program.
- For example, the syntactic analysis may ensure gate declarations are always at the beginning of a program, but it cannot ensure that all identifiers in a gate application were previously defined.
- Our semantic analysis is divided into two parts:
  1. the declaration analysis and
  2. the type checking.
- Both parts are implemented as ANTLR listener classes.
- These traverse the parse tree and call both an enter and exit function for each grammar rule, e.g., `EnterBlock` and `ExitBlock`.

Implementation

Semantic Analysis

- The declaration analysis ensure that all identifiers used were previously declared and all identifiers used in declarations are not already declared.
- This includes throwing warnings for declared but unused identifiers.
- Additionally, it prevents the use of a qubit in a code block that is guarded by the same qubit because this would lead to irreversible gates.
- The type checking ensures that symbols are used in the correct context.
- For example, while a qubit symbol can be used as the argument for a gate application, it does not represent a classical numerical value and, therefore, cannot be used in the context of a factor.
- Since we do not give the type of composite gate argument, its body cannot be type checked and any invalid types are thrown at generation time.

Implementation

Code Generation

- First, the parse tree is traversed and the source code is translated to an in-memory representation.
- Next, this source code representation (SCR) is translated to the target code representation (TCR).
- Then, the TCR can be translated directly to the textual OpenQASM code.
- We want to go through the process with the example program from before.
- For simplicity, the named constant was replaced with a constant value.

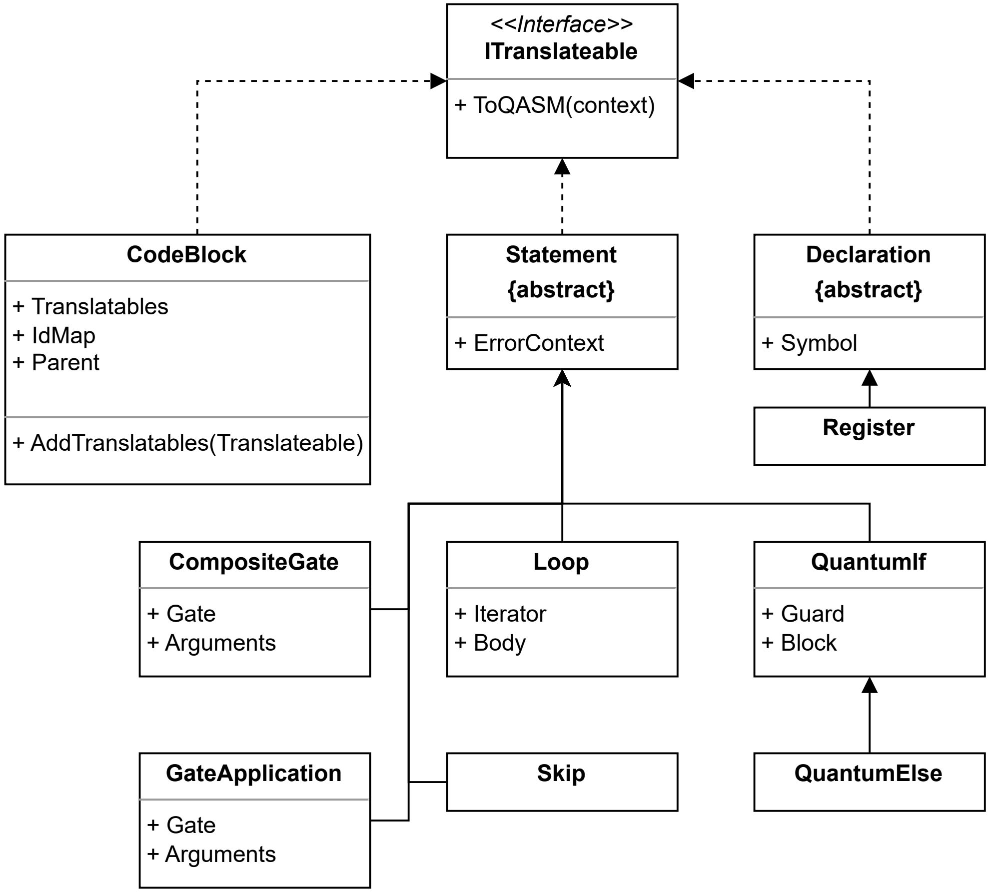
```
gate c_h_reg(control, reg) do
    qif control do
        for i in range(sizeof(reg)) do
            h reg[i];
        end
    end
end

qubit c;
qubit[3] a;
c_h_reg c, a;
```

An example Luie program to show the code generation process.

Source Code Representation

- All SCR objects implement the translatable interface, which requires a translation function.
- The are three main translatables: the code block, statement, and declaration classes.
- The block contains a list of translatables and is used for both the main block and the body of some statements.
- The declaration consists only of register declarations; the compile-time-only declarations are only saved in the symbol table.



A diagram showing the hierarchy of translatable classes.

Implementation

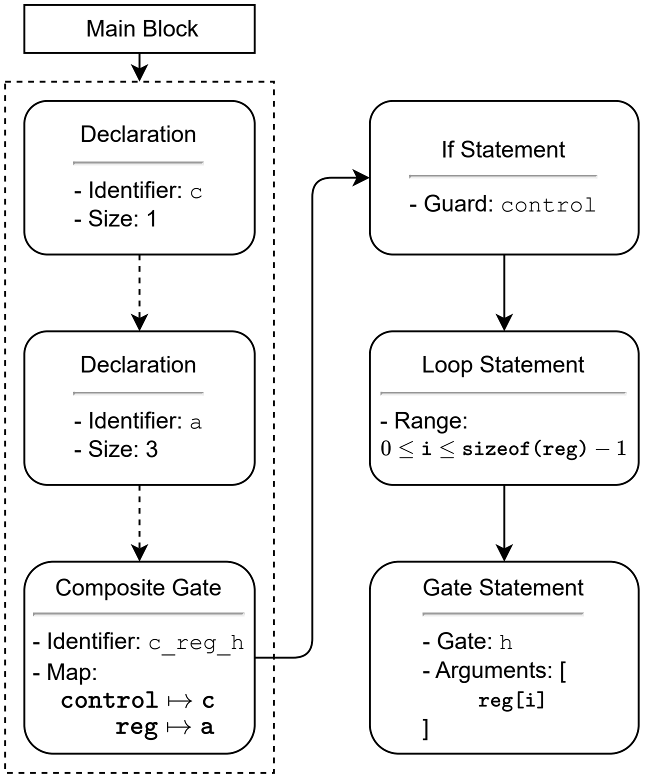
Source Code Representation — Example

- Any translation always consists of the main block.
- It contains three translatables.
- The first two are the declarations and the last is the composite gate statement.
- The gate's body contains only an if-statement, which, in turn, contains a loop statement.
- The loop statement consists of a gate application.

```
gate c_h_reg(control, reg) do
    qif control do
        for i in range(sizeof(reg)) do
            h reg[i];
end end end

qubit c; qubit[3] a;
c_h_reg c, a;
```

An example Luie program to show the code generation process.



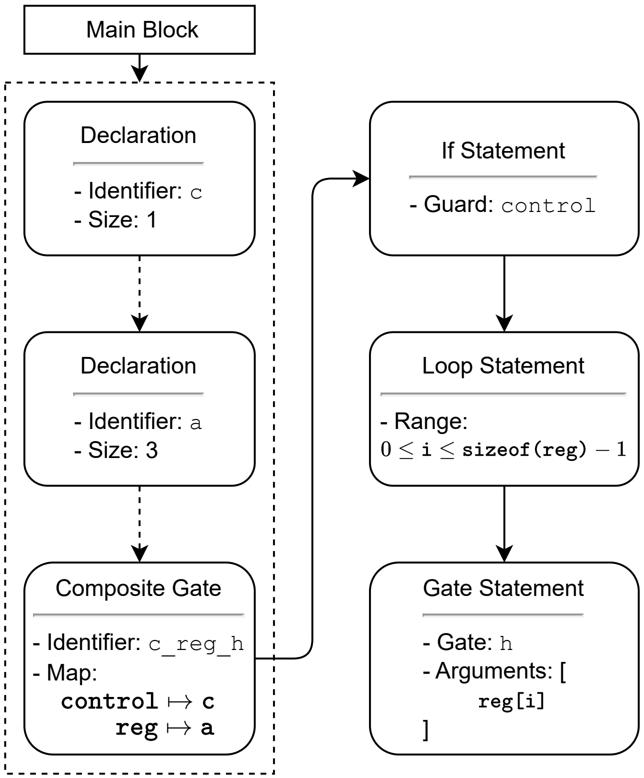
The SCR of the example program.

Target Code Representation

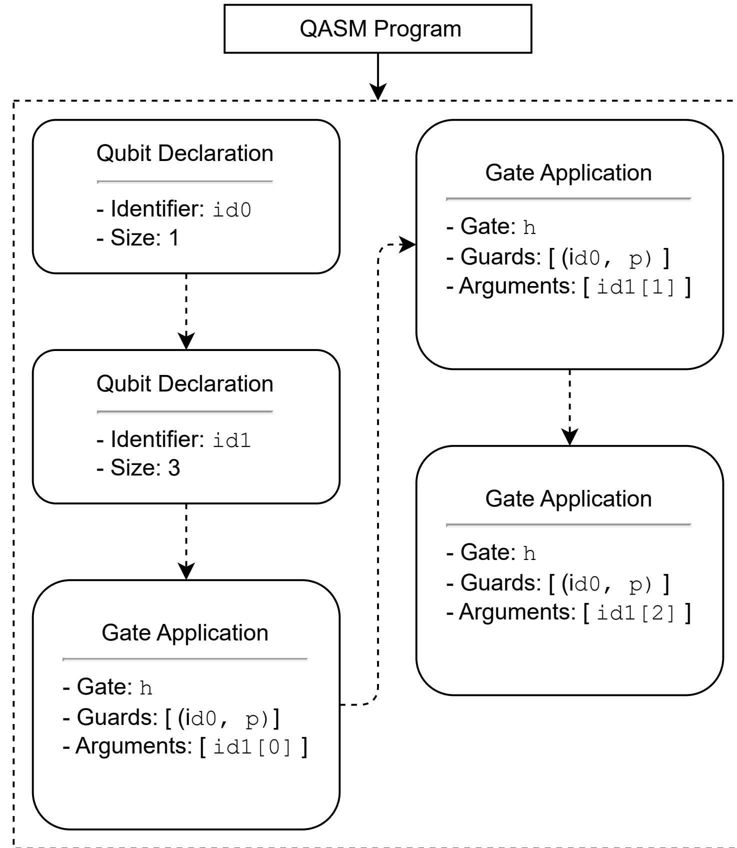
- The basis of the TCR is the `QASMProgram` object.
- It contains a list of `Code` objects, which are either declarations of gate applications.
- All SCR objects can be translated to a list of code objects and appended to the program object.
- The translations are as described in translation section.
- For example, a if-statement adds a control qubit to all gate applications in the block translation.
- All code objects implement a `ToCode` function that returns the textual representation of the statement.
- To translate the program object, the code objects are simply iterated, converted to text, and written to a file.

$$ct(\texttt{qif}\ \ qArg\ \ \texttt{do}\ \ blk\ \ \texttt{end}, st) = control(qt(qArg, st), bt(blk, st))$$

Target Code Representation — Example



The SCR of the example program.



The TCR of the example program.

Implementation

Translated Example Program

- In the end, the TCR is converted to the OpenQASM program.
- For this, the version string and include header is appended to the code.
- Additionally, for each quantum register, a classical one is declared and the registers are measured and saved to the corresponding classical registers.
- These additions are performed right before the result is written to the output and after the optimization.

```
OPENQASM 3.0;
include "stdgates.inc";
qubit id0;
qubit[3] id1;
ctrl(1) @ h id0, id1[0];
ctrl(1) @ h id0, id1[1];
ctrl(1) @ h id0, id1[2];
bit id0_measurement;
measure id0 -> id0_measurement;
bit[3] id1_measurement;
measure id1 -> id1_measurement;
```

The OpenQASM translation of the example Luie program.

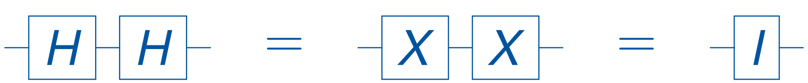
Optimization

- The compiler can perform optimizations to reduce both the number of gates and qubits.
- The optimizations are performed to the TCR to allow for more optimizations at the cost of performance.
- The compiler performs general peephole optimizations based on rules presented by [Garcia-Escartin and Chamorro-Posada, 2011].
- They can be divided into four rules:
  1. *null gate*,
  2. *peeping control*,
  3. *Hadamard reduction*, and
  4. *control reversal*.

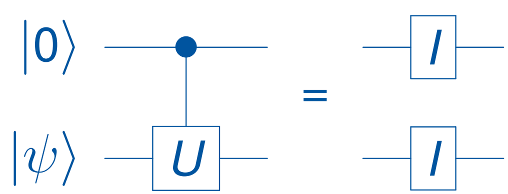
Implementation

Null Gate and Peeping Control Rules

- Null gates are combinations or gates under specific conditions that are equivalent to the identity gate.
- The simplest null gate version is the twofold application of a self-inverse gate.
- They can be removed entirely from the circuit.
- Our peeping control rules are a special case of null gates.
- In this case, the value of a control is know and the gate can be removed if it is not applied
- Since the implementation of the rules differs greatly from the other null gate rules, its separated as its own rule.



Null gates of self-inverse gates.

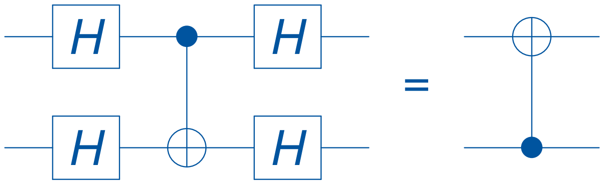


Null gates for gates in specific conditions.

Hadamard Reduction and Control Reversal Rules

- The Hadamard reduction can reduce either an X or a Z gate surrounded by H gates to the other without the surrounding gates.
- The rule is also the basis for the control reversal.
- It emerges then the Hadamard reduction is combined with the control reversal of the controlled-Z gate.
- A controlled-not gate surrounded by Hadamard gates is equivalent to the target and control qubits exchanged without Hadamard gates.

A Hadamard reduction rule.



Null gates for gates in specific conditions.

Cricuit Graph

- Instead of applying the rules directly to the program, it is translated to a circuit graph.
- Then, the graph is optimized and the result is translated back to a OpenQASM program.
- The circuit graph C is acyclic and directed.
- It consists of a set of nodes V — split into input, output, and gate nodes — and edges E.
- Additionally, it contains a set of qubits Q and two functions that map nodes Q_V and edges Q_E to qubits.

$$C = \left(V, E, Q, Q_E, Q_V\right)$$

$$V = \underbrace{I}_{\text{Input Nodes}} \cup \underbrace{O}_{\text{Output Nodes}} \cup \underbrace{G}_{\text{Gate Nodes}}$$

$$E \subseteq \{(x, y) \mid x, y \in V \wedge x \neq y\}$$

$$Q_V : I \cup O \rightarrow Q$$

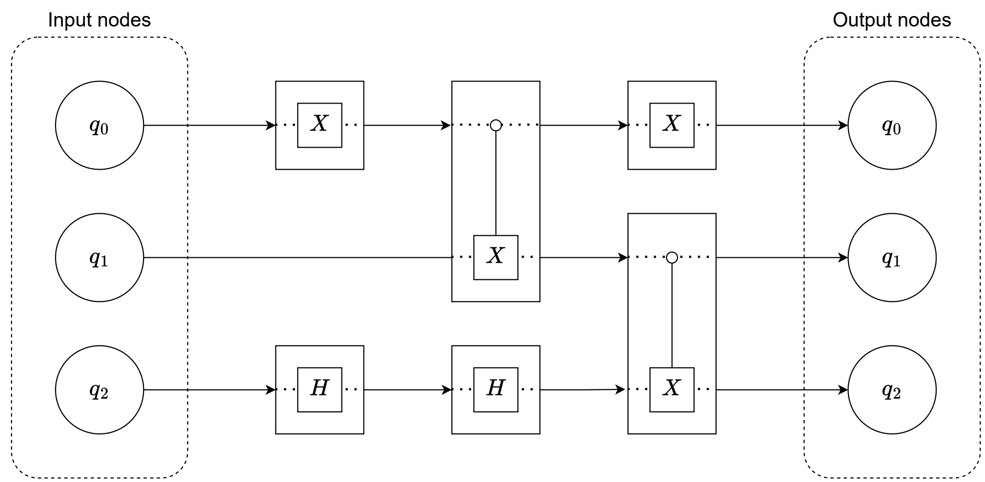
$$Q_E : E \rightarrow Q$$

Example Circuit Graph

- Input and output nodes have only one outgoing or incoming edge, respectively.
- For each qubit, C contains an input-output node pair.
- Each edge coming into a gate node has a corresponding outgoing one with the same qubit.
- These represent an argument of the gate application.
- The wire for qubit q is represented by the path from the input to the output node where all edges map to q.

```
qubit[3] q;
x q[0];
cx q[0], q[1];
x q[0];
h q[2];
h q[2];
cx q[1], q[2];
```

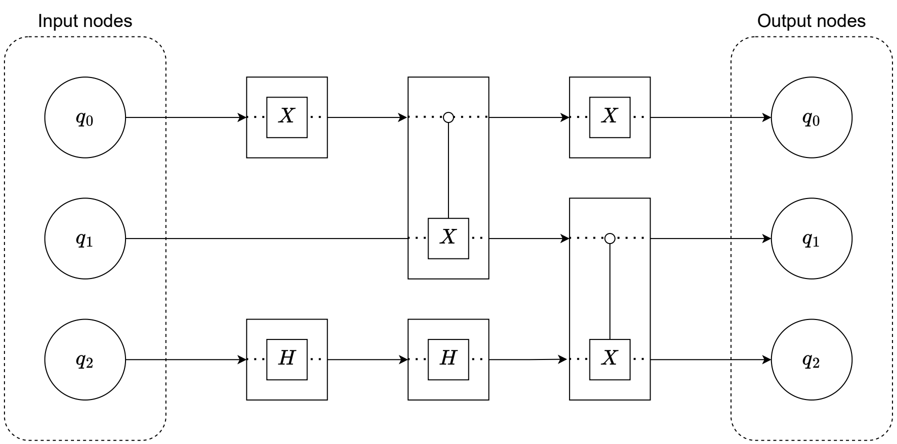
A simple, unoptimized program.



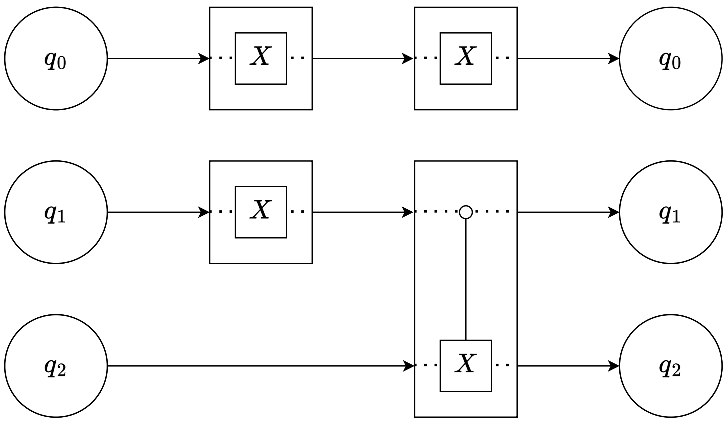
An example of a simple, unoptimized circuit graph.

Example Optimization Process I

- To optimize the graph, each qubit wire is iterated.
- All subpaths up to a maximum length are checked for optimized alternatives.
- In the example, the peeping control rule can be applied to the first *CX* gate.
- Next, the *HH* null gate can be removed.



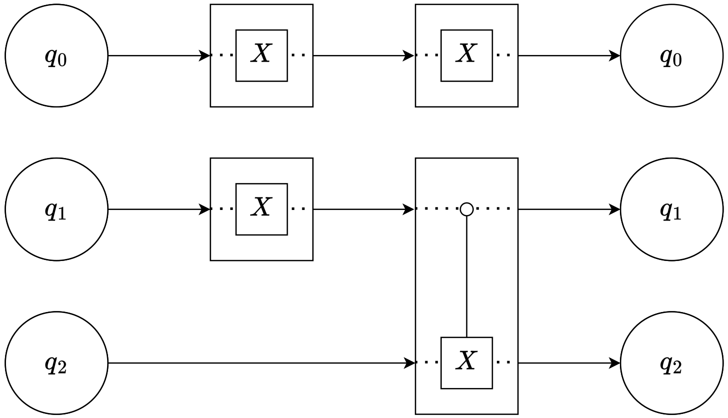
An example of a simple, unoptimized circuit graph.



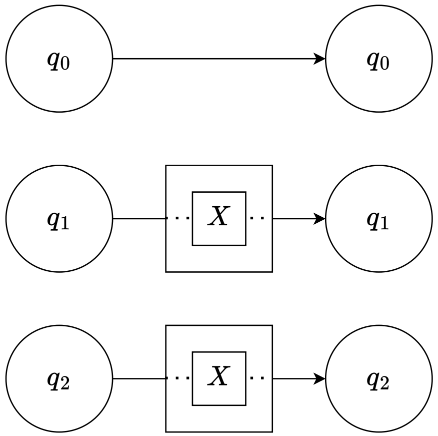
The circuit graph after the first optimization step.

Example Optimization Process II

- While all qubit wire were iterated, there are still possible optimizations.
- This is because applying optimizations, may enable others.
- Therefore, the optimization is repeated as long as the previous iteration applied optimizations.
- In the next iteration, the *XX* null gate can be removed.
- Additionally, a peeping control rule can be applied to the remaining *CX* gate.



The circuit graph after the first optimization step.



The completely optimized graph.

Evaluation

. . .

Conclusion

References

- Aaby, A. A. (2003).
  Compiler construction using flex and bison.
- Altenkirch, T. and Grattage, J. (2005).
  A functional quantum programming language.
  In *20th Annual IEEE Symposium on Logic in Computer Science (LICS' 05)*, pages 249–258. IEEE.
- Garcia-Escartin, J. C. and Chamorro-Posada, P. (2011).
  Equivalent quantum circuits.
- Landauer, R. (1961).
  Irreversibility and heat generation in the computing process.
  IBM Journal of Research and Development, 5(3):183–191.
- Ying, M., Yu, N., and Feng, Y. (2012).
  Defining quantum control flow.

References

Yuan, C., Villanyi, A., and Carbin, M. (2024).
Quantum control machine: The limits of control flow in quantum programming.
Proceedings of the ACM on Programming Languages, 8(OOPSLA1):1–28.