Flexibility and resilience in network design

Supply Chain Analytics 42380

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Contents

Work division

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Part 1	70%	10%	10%	10%
Part 2	10%	10%	10%	70%
Part 3	10%	10%	70%	10%
Part 4	10%	70%	10%	10%

1 Part 1

1.1 Problem

SportStuff.com is a retailer of sports equipment. We are asked to determine the optimal location and sizes of warehouses and allocation of demand over a period from 2007 to 2011. The problem can be formulated as a *Capacitated fixed-charge location problem* (CFLP), with the addition of a minimum leasing period of three years.

1.2 Data

Below is the given data for the problem.

Sets:

- Demand Zones:

  $I = \{$Northwest, Southwest, Upper Midwest, Lower Midwest, Northeast, Southeast$\}$

- Facility Locations:

  $J = \{$Seattle, Denver, St. Louis, Atlanta, Philadelphia$\}$

- Years:

  $P = \{2007, 2008, 2009, 2010, 2011\}$

- Warehouse size:

  $S = \{$Small,Large$\}$

Parameters:

- Demand growth:

  $Growth = 1.80$

- Demand for zone i in year p:

  $Demand_{i,p} = Demand_{i,p-1} \cdot Growth \quad \forall\, i \in I, p \in P - \{2007, 2011\}$

- Fixed cost for leasing a facility in location j of size s

  $Fixed_{j,s} \quad j \in J, s \in S$

- Minimum leasing length in years

  $MinLease = 3$

- Variable cost for producing one unit in facility location j of size s:
  $Var_{j,s} \quad j \in J, s \in S$
- Capacity for warehouse of size s
  $Capacity_s \quad s \in S$
- Inventory fixed cost
  $InvFixed = \$475.000$
- Inventory variable cost
  $InvVar = \$0.165$
- Shipping costs to demand zone i from facility j:
  $ShippingCost_{i,j} \quad i \in I, j \in J$

1.3 Model

Decision Variables:

- The decision to open a facility of size s in location j in year p:
  $x_{j,s,p} \in \{0,1\} \quad j \in J, s \in S, p \in P$
- Fraction of demand from zone i to allocate to facility in location j in year p:
  $y_{i,j,s,p} \quad i \in I, j \in J, s \in S, p \in P$
- Variable that tracks the year that a facility was first opened
  $z_{j,s,p} \in \{0,1\} \quad j \in J, s \in S, p \in P$

Objective function

- Minimize the total cost

$$\begin{aligned}&\text{Minimize}\\&\sum_{j\in J, s\in S, p\in P} x_{j,s,p} \cdot (Fixed_{j,s} + InvFixed) +\\&\sum_{i\in I, j\in J, s\in S, p\in P} y_{i,j,s,p} \cdot Demand_{i,p} \cdot (Var_{j,s} + InvVar + ShippingCost_{i,j})\end{aligned} \tag{1}$$

Constraints

- Cannot open a small and a large facility in the same location

$$\sum_{s\in S} x_{j,s,p} \leq 1 \quad \forall j \in J, p \in P \tag{2}$$

- Must update the tracking variable z if a facility f was not open in the previous year.

$$z_{j,s,p} \geq x_{j,s,p} - x_{j,s,p-1} \mid p > 2007 \quad \forall\, j \in J, s \in S, p \in P \tag{3}$$

- If a facility is leased, the lease must be of minimum 3 years length.

$$\sum_{p \in P} z_{j,s,p} \cdot \min(MinLease, |P| - p + 1) \leq \sum_{p \in P} x_{j,s,p} \quad \forall\, j \in J, s \in S \tag{4}$$

- Can only assign demand to open facilities

$$y_{i,j,s,p} \leq x_{j,s,p} \quad \forall\, i \in I, j \in J, s \in S, p \in P \tag{5}$$

- Must allocate a full fraction of demand from every demand zone i

$$\sum_{j \in J, s \in S} y_{i,j,s,p} = 1 \quad \forall\, i \in I, p \in P \tag{6}$$

- Cannot allocate more demand to a facility j than there is capacity for

$$\sum_{i \in I, s \in S} y_{i,j,s,p} \cdot Demand_{i,p} \leq \sum_{s \in S} x_{j,s,p} \cdot Capacity_s \quad \forall\, j \in J, p \in P \tag{7}$$

1.4 Julia program

See *Appendix 1 - SportStuff_part1.jl*

1.5 Result

Figure 1 demonstrates the solution obtained. As expected, the network is seen to heavily rely on small warehouses. Facilities in Atlanta and Seattle opened to cater to demand in the first three years. The smaller units are then upgraded to handle the increased demand. A small facility in Philadelphia is opened that appears to handle the residual volume. The total cost of the network design for the given demand is **$ 81.6 Million**

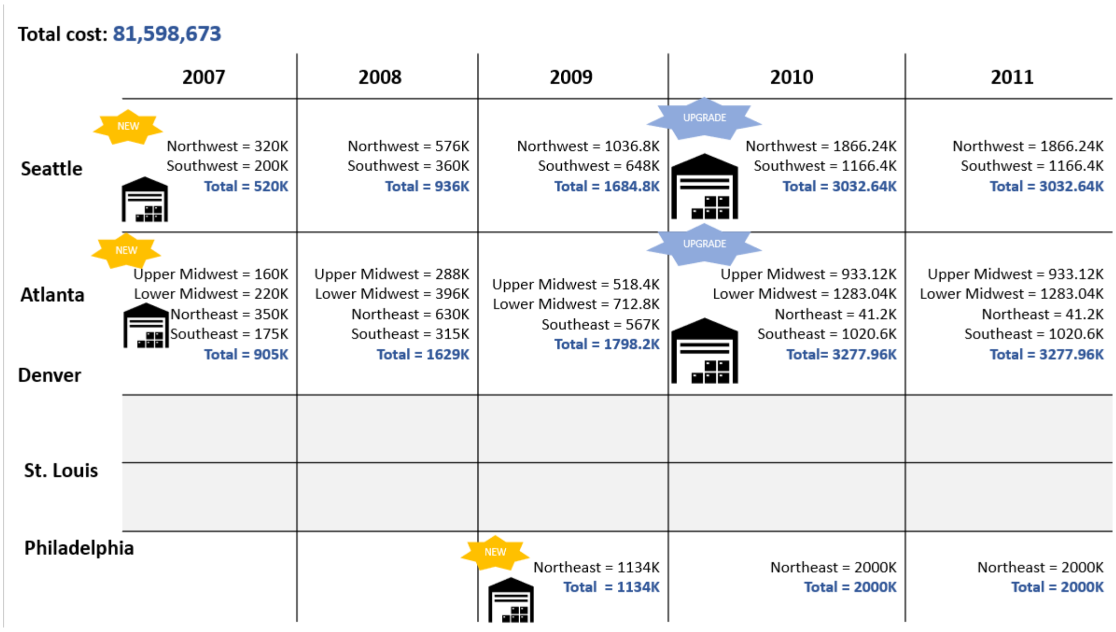

Figure 1: Part 1: Solution

2 Part 2

2.1 Problem

Consider the following two types of increased flexibility in the lease agreements:

1. The lease is fixed for one year only, instead of three.
2. It is not necessary to lease the entire warehouse, but only part of it as needed.

The model from part 1 can be adapted to include the above considerations. To allow for the first added flexibility, we may simplify the model significantly. In this case, we may remove the $z_{j,s,p}$ variable entirely, and along with it the constraints 3 and 4. For the second flexibility, we may simply change the binary $x_{j,s,p}$ variable into a continuous variable, bounded between 0 and 1. But now we also need to alter constraints 2, 5 and 7, to account for the non-binary nature of the x-variable.

2.2 Data

Same as in Part 1 with the following additions

Parameters:

- Big M

  $M \gg 1$

2.3 Model

Decision Variables:

- The decision to open a facility of size s in location j in year p:

  $0 \leq x_{j,s,p} \leq 1 \quad j \in J, s \in S, p \in P$

- Fraction of demand from zone i to allocate to facility in location j in year p:

  $y_{i,j,s,p} \quad i \in I, j \in J, s \in S, p \in P$

Objective function

- Minimize the total cost

$$\begin{aligned}&\text{Minimize}\\ &\sum_{j\in J,s\in S,p\in P} x_{j,s,p}\cdot(Fixed_{j,s}+InvFixed)+\\ &\sum_{i\in I,j\in J,s\in S,p\in P} y_{i,j,s,p}\cdot Demand_{i,p}\cdot(Var_{j,s}+InvVar+ShippingCost_{i,j})\end{aligned} \tag{8}$$

Constraints

- Cannot open a small and a large facility in the same location

$$\sum_{s\in S} x_{j,s,p}\leq 1 \quad \forall\, j\in J, p\in P \tag{9}$$

- Can only assign demand to open facilities

$$y_{i,j,s,p}\leq x_{j,s,p}\cdot M \quad \forall\, i\in I, j\in J, s\in S, p\in P \tag{10}$$

- Must allocate a full fraction of demand from every demand zone i

$$\sum_{j\in J,s\in S} y_{i,j,s,p}=1 \quad \forall\, i\in I, p\in P \tag{11}$$

- Cannot allocate more demand to a facility j than there is capacity for

$$\sum_{i\in I} y_{i,j,s,p}\cdot Demand_{i,p}\leq x_{j,s,p}\cdot Capacity_s \quad \forall\, j\in J, p\in P, s\in S \tag{12}$$

- Non-negativity constraint. Cannot allocate negative demand

$$y_{i,j,s,p}\geq 0 \quad \forall\, i\in I, j\in J, s\in S, p\in P \tag{13}$$

2.4 Julia program

See *Appendix 2 - SportStuff_part2.jl*

2.5 Result

Figure 2 demonstrates the solution obtained with the percentage representing the warehouse capacity. The solution only consists of small warehouses. Compared with part 1, the increase in demand is handled by opening more small warehouses instead of upgrading capacity, utilizing the added flexibility to hold down the cost. Furthermore, the one-year leasing period's added flexibility makes it possible to open and close warehouses as demands shift. However, it depends on how geographically widespread the demand is whether it is cheapest to increase capacity or open a new warehouse. An example can be seen in 2009 when the warehouses in Seattle and Atlanta were closed while the capacity for the last three is increased. The flexibility on both the leasing period and capacity results in a cheaper total cost for the network design on **$79.7 million** compared with $81.6 in part 1. Not surprising given the added flexibility.

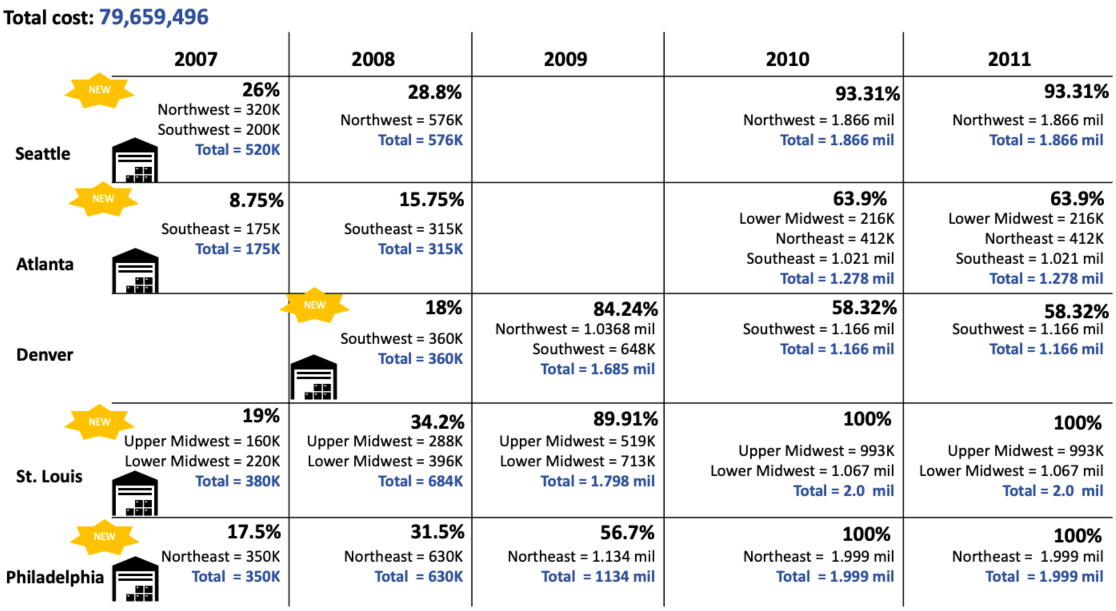

Figure 2: Part 2: Solution

3 Part 3

3.1 Problem

Investigate the resilience of the network design against significant deviations from what is expected. In particular, investigate situations where (a) demand is much larger than initially forecasted and where (b) warehouse capacity is much less than initially agree upon.

3.2 Theory

Thus far we have assumed all variables to be deterministic. This assumption is most often unrealistic, but for many facility location problems small fluctuations in parameters will not change the decision-variables of the given solutions. But how about situations where the parameters may vary greatly? In a situation like this, we can construct and include scenarios into our model, and consider these scenarios in the calculation of our solution. Starting from this part, scenarios are set up to demonstrate the uncertainty elements in the demand and the warehouse capacity.

- Demand scenarios We set up four demand scenarios representing the different directions of the changes in the demand. The scenarios are implemented to the Julia code using a demand factor which represent the number of times that the demand increases or decreases. The scenarios are as listed:
  1. Demand scenario 1: Normal demand as given in the assignment
  2. Demand scenario 2: Demand for each region decreases slightly from year 2007 to year 2009 and stabilizes. Demands decrease equally for every region.
  3. Demand scenario 3: Demand for each region increases slightly from year 2007 to year 2009 and stabilizes. Demands increase equally for every region.
  4. Demand scenario 4: Demand spikes in Southwest region whereas other demands are normal demand throughout the time period. This scenario illustrates randomness in having demand extremes.
- Capacity scenarios We set up four capacity scenarios representing different capacity level that the warehouse may have. This reflects the reality by which warehouses sometime are used to temporary store other products and so they have less capacity to storage this specific product.
  1. Capacity scenario 1: Normal warehouse capacity as given in the assignment
  2. Capacity scenario 2: Capacity for warehouses of any sizes is lowered to 0.7 times of the normal capacity level, from year 2008 to 2011.

3. Capacity scenario 3: Capacity for small warehouses is lowered to 0.7 times of the normal capacity level, from year 2008 to 2011.
4. Capacity scenario 4: Capacity for large warehouses is lowered to 0.7 times of the normal capacity level, from year 2008 to 2011.

First-stage and second-stage decisions

The first thing that we need to make a distinction between is the dimensionality of the decisions being taken in a stochastic model. Given a set of likely scenarios S, we will need to make decisions at certain points in time. Some of these decisions will have to be made independently of which scenario will occur in reality, and should thus be the same under every scenario. These are our first-stage decisions. On the other hand, some decisions may be delayed until a later time, when some events have already occured. This means that these delayed decisions may take additional information into consideration. And this information will be dependent on which scenario is happening in reality. These are your second-stage, or sometimes multi-stage, decisions, and they may be distinct under each scenario $s \in S$.

Stochastic optimization and Robust optimization

Two popular ways of solving models with parameters varying under a set of scenarios S is the stochastic optimization method, where you optimize for the expected value of your objective function. This method requires an estimation of probabilities of all scenarios $s \in S$ occuring. On average, given that the estimation of probabilities are accurate, this method will perform the best. Another way of approaching a model under a set of scenarios is the robust optimization method, where you construct the model such that it optimizes for the worst possible outcome for all scenarios $s \in S$. This is valid for many cases where the cost of failure is unacceptable or if you do not have the resources available to scale your operations for a regression to the mean. For robust optimization you do not need to estimate probabilities of the scenarios occurring.

3.3 Data

The data builds on the data from part 1, with the following additions

Sets

- 4 possible demand scenarios

  $SD \in \{1, 2, 3, 4\}$

- 4 possible capacity scenarios

  $SC \in \{1, 2, 3, 4\}$

Parameters

- Demand-factors for zone i in period p under demand scenario sd
  $Dfactor_{i,p,sd} \quad i \in I, p \in P, sd \in SD$
- Capacity-factors for warehoues of size s in period p under demand scenario sd
  $Cfactor_{s,p,sc} \quad s \in S, p \in P, sc \in SC$
- Probability of demand scenario sd occuring
  π_{sd}
- Probability of capacity scenario sc occuring
  π_{sc}

3.4 Stochastic Optimization Model

Decision Variables:

- The decision to open a facility of size s in location j in year p:
  $x_{j,s,p} \in \{0,1\} \quad j \in J, s \in S, p \in P$
- Fraction of demand from zone i to allocate to facility in location j in year p under demand scenario sd:
  $y_{i,j,s,p,sd,sc} \quad i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC$
- Variable that tracks the year that a facility was first opened
  $z_{j,s,p} \in \{0,1\} \quad j \in J, s \in S, p \in P$

Objective function

- Minimize the expected cost

$$\text{Minimize} \tag{14}$$
$$\sum_{j \in J, s \in S, p \in P} x_{j,s,p} \cdot (Fixed_{j,s} + InvFixed) +$$
$$\pi_{sd}\pi_{sc} \sum_{i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC} y_{i,j,s,p,sd,sc} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \cdot (Var_{j,s} + InvVar + ShippingCost_{i,j})$$

Constraints

- Cannot open a small and a large facility in the same location

$$\sum_{s \in S} x_{j,s,p} \leq 1 \quad \forall\, j \in J, p \in P \tag{15}$$

- Must update the tracking variable z if a facility f was not open in the previous year.

$$z_{j,s,p} \geq x_{j,s,p} - x_{j,s,p-1} \mid p > 2007 \quad \forall\, j \in J, s \in S, p \in P \tag{16}$$

- If a facility is leased, the lease must be of minimum 3 years length.

$$\sum_{p \in P} z_{j,s,p} \cdot \min(MinLease, |P| - p + 1) \leq \sum_{p \in P} x_{j,s,p} \quad \forall\, j \in J, s \in S \tag{17}$$

- Can only assign demand to open facilities

$$y_{i,j,s,p,sd,sc} \leq x_{j,s,p} \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC \tag{18}$$

- Must allocate a full fraction of demand from every demand zone i

$$\sum_{j \in J, s \in S} y_{i,j,s,p,sd,sc} = 1 \quad \forall\, i \in I, p \in P, sd \in SD, sc \in SC \tag{19}$$

- Cannot allocate more demand to a facility j than there is capacity for

$$\sum_{i \in I, s \in S} y_{i,j,s,p,sd,sc} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \leq \sum_{s \in S} x_{j,s,p} \cdot Capacity_s \cdot Cfactor_{s,p,sc} \tag{20}$$
$$\forall\, j \in J, p \in P, sd \in SD, sc \in SC$$

- Non-negativity constraint. Cannot allocate negative demand

$$y_{i,j,s,p,sd,sc} \geq 0 \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC \tag{21}$$

Julia program

See *Appendix 3 - SportStuff_part3_one-stage.jl*

Results

Figure 4 demonstrates the solution for the stochastic optimization model. Compared to part 1, large capacity, continuous and widespread warehouses characterize the solution. The difference in the network design is partly due to the scenarios with a 10% probability of demand increase making it more profitable to increase the number of large warehouses in the network. Furthermore, the larger warehouses cater to the lower capacity scenarios with a 15% and 5% probability. Of course, accounting for different scenarios comes at a higher cost, which is why the total cost for this network design is higher than in part 1 with **\$84.2 million** compared to \$81.6 million.

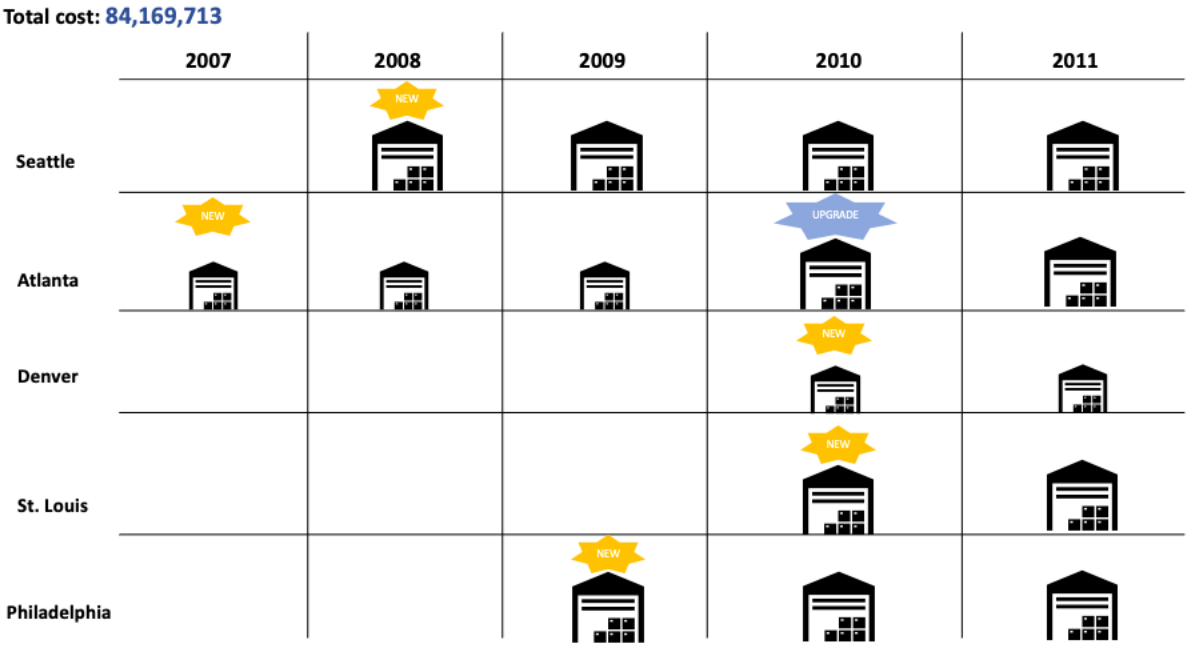

Figure 3: Part 3 Stochastic Optimization Model: Solution

3.5 Robust Optimization Model

The robust optimization problem can be constructed as the following minimax problem

$$\text{Minimize} \tag{22}$$

$$\max \Big[\sum_{j \in J, s \in S, p \in P} x_{j,s,p} \cdot (Fixed_{j,s} + InvFixed) +$$

$$\sum_{i \in I, j \in J, s \in S, p \in P, sd \in SD} y_{i,j,s,p,sd} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \cdot (Var_{j,s} + InvVar + ShippingCost_{i,j}) \Big]$$

In order to implement it in julia/JuMP, we need to create an additional variable w

Decision Variables:

- The decision to open a facility of size s in location j in year p:

  $x_{j,s,p} \in \{0, 1\} \quad j \in J, s \in S, p \in P$

- Fraction of demand from zone i to allocate to facility in location j in year p under demand scenario sd:

  $y_{i,j,s,p,sd} \quad i \in I, j \in J, s \in S, p \in P, sd \in SD$

- Variable that tracks the year that a facility was first opened

  $z_{j,s,p} \in \{0, 1\} \quad j \in J, s \in S, p \in P$

- Variable that tracks the worst possible outcome
  $0 \leq w$

Objective Function:

$$\text{Minimize} \quad w \tag{23}$$

Constraints

- Cannot open a small and a large facility in the same location

$$\sum_{s \in S} x_{j,s,p} \leq 1 \quad \forall\, j \in J, p \in P \tag{24}$$

- Must update the tracking variable z if a facility f was not open in the previous year.

$$z_{j,s,p} \geq x_{j,s,p} - x_{j,s,p-1} \mid p > 2007 \quad \forall\, j \in J, s \in S, p \in P \tag{25}$$

- If a facility is leased, the lease must be of minimum 3 years length.

$$\sum_{p \in P} z_{j,s,p} \cdot \min(MinLease, |P| - p + 1) \leq \sum_{p \in P} x_{j,s,p} \quad \forall\, j \in J, s \in S \tag{26}$$

- Can only assign demand to open facilities

$$y_{i,j,s,p,sd} \leq x_{j,s,p} \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD \tag{27}$$

- Must allocate a full fraction of demand from every demand zone i

$$\sum_{j \in J, s \in S} y_{i,j,s,p,sd} = 1 \quad \forall\, i \in I, p \in P, sd \in SD \tag{28}$$

- Cannot allocate more demand to a facility j than there is capacity for

$$\sum_{i \in I, s \in S} y_{i,j,s,p,sd} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \leq \sum_{s \in S} x_{j,s,p} \cdot Capacity_s \cdot Cfactor_{s,p,sc} \tag{29}$$
$$\forall\, j \in J, p \in P, sd \in SD, sc \in SC$$

- Non-negativity constraint. Cannot allocate negative demand

$$y_{i,j,s,p} \geq 0 \quad \forall\, i \in I, j \in J, s \in S, p \in P \tag{30}$$

Julia program

See *Appendix 4 - SportStuff_part3_robust.jl*

Results

Figure 4 demonstrates the solution for the robust optimization model. Many large warehouses characterize the solution as demand increases to safeguard against the worst-case scenario. Compared to part 1, more new warehouses with higher capacity are opened. This also results in a higher total cost for the network design compared to part 1. The total cost for this solution is **$85.3 million**. If compared to the stochastic optimization model the robust model is more costly, as the solution is based on minimizing the cost of the worst case scenario.

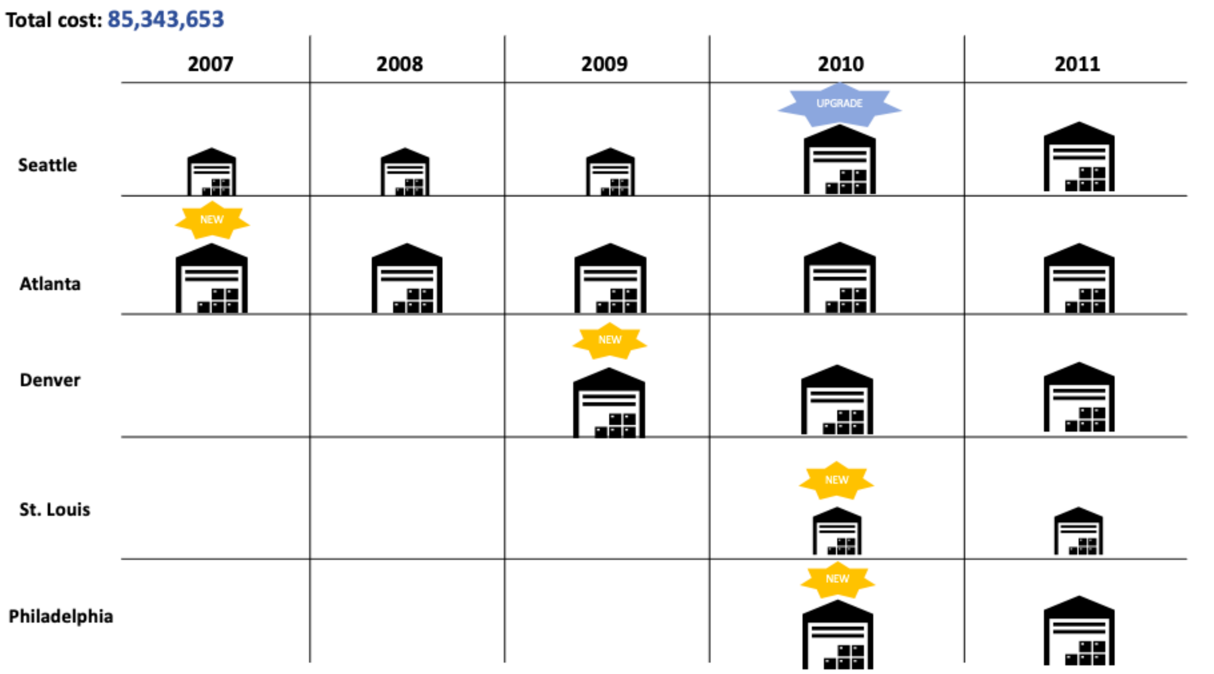

Figure 4: Part 3 Robust Optimization Model: Solution

3.6 Remarks

It is worth considering that while we have modelled leasing decisions for the full time-period as first-stage decisions, we might want to model later leasing decisions as multi-stage decisions. We could extend the model with this functionality, if we index the x-variable by the scenarios $sd \in SD$ and $sc \in SC$ and add non-anticipatory constraints accordingly. The logic behind non-anticipatory constraints is that while you may wish to be able to alter your decisions according to which scenario unfolds, such as we have done with the y variable, but it is not given that you know which scenario you are in at all given points. So you may allow your variable to change under given scenarios, but you should take care when doing this, and constrain your variables at points in the program where there is no diverging information available. For example, if you wanted to construct our leasing decisions as multi-stage variables, but at year 2007 we have the same demand for all forecasted scenarios, we would have to constrain the x variable across all scenarios like such:

$$x_{j,s,1,sc,sd} = x_{j,s,1,sd,sc} \quad \forall\, j \in J, s \in S, sd \in SD, sc \in SC \tag{31}$$

And this would ensure that the leasing decisions in year 2007 would be the same across all scenarios. We have a julia/JuMP implementation that extends the model with this functionality in the file *Appendix 5 - SportStuff_part3_two-stage.jl*, but since it is out of the scope of the assignment we shall not go into further detail. The solution to a multi-stage problem like this is a set of optimal decisions, one for each scenario, which in our case is 4x4 total combinations of scenarios of demand and capacity.

4 Part 4

4.1 Problem

Investigate the flexibility and the resilience of the network design against significant deviations in the demands and warehouse capacity as mentioned in part 3. The flexibility of the network design is based on the conditions mentioned in part 2 with a fixed one-year leasing and the ability to lease warehouse partially.

4.2 Data

The data builds on the data from part 2 and 3, with the basis construction from part 1. This section provides data for part 4 that are beyond the baseline data in part 1. Essentially, this part takes over data sets from part 3, and parameters from part 2 and 3.

Sets

- 4 possible demand scenarios

  $SD \in \{1, 2, 3, 4\}$

- 4 possible capacity scenarios

  $SC \in \{1, 2, 3, 4\}$

Parameters

- Big M

  $M \gg 1$

- Demand-factors for zone i in period p under demand scenario sd

  $Dfactor_{i,p,sd} \quad i \in I, p \in P, sd \in SD$

- Capacity-factors for warehoues of size s in period p under demand scenario sd

  $Cfactor_{s,p,sc} \quad s \in S, p \in P, sc \in SC$

- Probability of demand scenario sd occuring

  π_{sd}

- Probability of capacity scenario sc occuring

  π_{sc}

4.3 Stochastic Optimization Model

Decision Variables

- The decision to open a facility of size s in location j in year p:

  $0 \leq \mathrm{x}_{j,s,p} \leq 1 \quad j \in J, s \in S, p \in P$

- Fraction of demand from zone i to allocate to facility in location j in year p under demand scenario sd, given the probability of demand scenario sc:

  $y_{i,j,s,p,sd,sc} \quad i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC$

Objective function

- Minimize the expected cost

$$\begin{aligned}&\text{Minimize} \\ &\sum_{j \in J, s \in S, p \in P} x_{j,s,p} \cdot (Fixed_{j,s} + InvFixed) + \\ &\pi_{sd}\pi_{sc} \sum_{i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC} y_{i,j,s,p,sd,sc} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \cdot (Var_{j,s} + InvVar + ShippingCost_{i,j})\end{aligned} \tag{32}$$

Constraints

- Cannot open a small and a large facility in the same location

$$\sum_{s \in S} x_{j,s,p} \leq 1 \quad \forall\, j \in J, p \in P \tag{33}$$

- Can only assign demand to open facilities

$$y_{i,j,s,p,sd,sc} \leq x_{j,s,p} \cdot M \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC \tag{34}$$

- Must allocate a full fraction of demand from every demand zone i

$$\sum_{j \in J, s \in S} y_{i,j,s,p,sd,sc} = 1 \quad \forall\, i \in I, p \in P, sd \in SD, sc \in SC \tag{35}$$

- Cannot allocate more demand to a facility j than there is capacity for

$$\sum_{i \in I, s \in S} y_{i,j,s,p,sd,sc} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \leq \sum_{s \in S} x_{j,s,p} \cdot Capacity_s \cdot Cfactor_{s,p,sc} \quad \forall\, j \in J, p \in P, sd \in SD, sc \in SC \tag{36}$$

- Non-negativity constraint. Cannot allocate negative demand

$$y_{i,j,s,p,sd,sc} \geq 0 \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC \tag{37}$$

Julia program

See *Appendix 6 - SportsStuff_part4_one-stage.jl*

Result

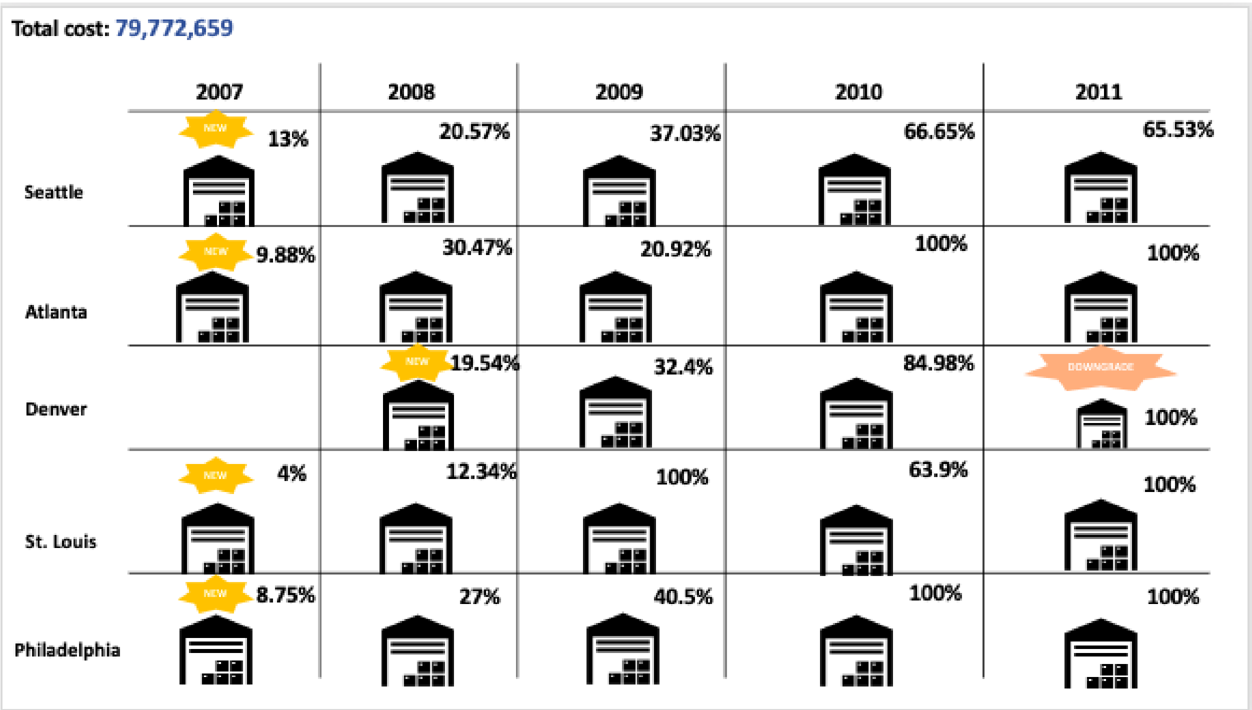

Figure 5: Part 4 Stochastic Optimization Model: Solution

Figure 5 shows the solution of the stochastic optimization model with flexibility and resilience characteristics. The solution proposes that large warehouses are opened throughout the period. This is mainly due to the ability to rent the warehouse partially. The network design opened multiple large facilities with a low usage of their capacities, instead of opening multiple small warehouses and using full capacities. This is because the capacity of the large warehouse is twice as larger than the small warehouse, whereas the fixed cost of operating the large warehouse is not as twice as larger as the cost of operating the small one. Therefore, by operating multiple large warehouses with a very low capacity usage gives lower cost than operating small warehouses with full capacity. For example, in year 2007, the cost of operating 4% a large facility in St. Louis will incur the cost of 0.04*(300,000+475,000) = \$39,000 for 0.04*4,000,000 = 160,000 units of capacity while if the model decided to open a small facility of the same capacity, it needs to use 160,000/2,000,000 = 0.08 or 8% of the small warehouse which would incur the cost of 0.08*(300,000+475,000) = $62,000.

This network design resulted in the cost of network design of **$79.7 million**. It is worth mentioning that it is lower than the network design in part 1 as the model take advantage of the flexibility condition. This indicates that a small additional investment can ensure the network's resilience to uncertainties and higher probability to cater to the demands.

4.4 Robust Optimization Model

The robust optimization model follows the same approach as the robust optimization model in part 3. The problem is constructed as minimax problem which minimized the objective function of the worst possible scenario.

$$\begin{aligned} &\text{Minimize} \\ &\max \Bigg[\sum_{j \in J, s \in S, p \in P} x_{j,s,p} \cdot (Fixed_{j,s} + InvFixed) + \\ &\sum_{i \in I, j \in J, s \in S, p \in P, sd \in SD} y_{i,j,s,p,sd} \cdot Demand_{i,p} \cdot (Var_{j,s} + InvVar + ShippingCost_{i,j}) \Bigg] \end{aligned} \tag{38}$$

Decision Variables

- The decision to open a facility of size s in location j in year p:

  $0 \leq \mathrm{x}_{j,s,p} \leq 1 \quad j \in J, s \in S, p \in P$

- Fraction of demand from zone i to allocate to facility in location j in year p under demand scenario sd:

  $y_{i,j,s,p,sd} \quad i \in I, j \in J, s \in S, p \in P, sd \in SD$

- Variable that tracks the worst possible outcome

  $0 \leq w$

Objective function

$$\begin{aligned} &\text{Minimize} \\ &w \end{aligned} \tag{39}$$

Constraints

- Cannot open a small and a large facility in the same location

$$\sum_{s \in S} x_{j,s,p} \leq 1 \quad \forall j \in J, p \in P \tag{40}$$

- Can only assign demand to open facilities

$$y_{i,j,s,p,sd} \leq x_{j,s,p} \cdot M \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD \tag{41}$$

- Must allocate a full fraction of demand from every demand zone i

$$\sum_{j \in J, s \in S} y_{i,j,s,p,sd} = 1 \quad \forall\, i \in I, p \in P, sd \in SD \tag{42}$$

- Cannot allocate more demand to a facility j than there is capacity for

$$\sum_{i \in I, s \in S} y_{i,j,s,p,sd} \cdot Demand_{i,p} \cdot Dfactor_{i,p,sd} \leq \sum_{s \in S} x_{j,s,p} \cdot Capacity_s \cdot Cfactor_{s,p,sc} \tag{43}$$
$$\forall\, j \in J, p \in P, sd \in SD, sc \in SC$$

- Non-negativity constraint. Cannot allocate negative demand

$$y_{i,j,s,p,sd,sc} \geq 0 \quad \forall\, i \in I, j \in J, s \in S, p \in P, sd \in SD, sc \in SC \tag{44}$$

Julia program

See *Appendix 7 - SportsStuff_part4_robust.jl*

Result

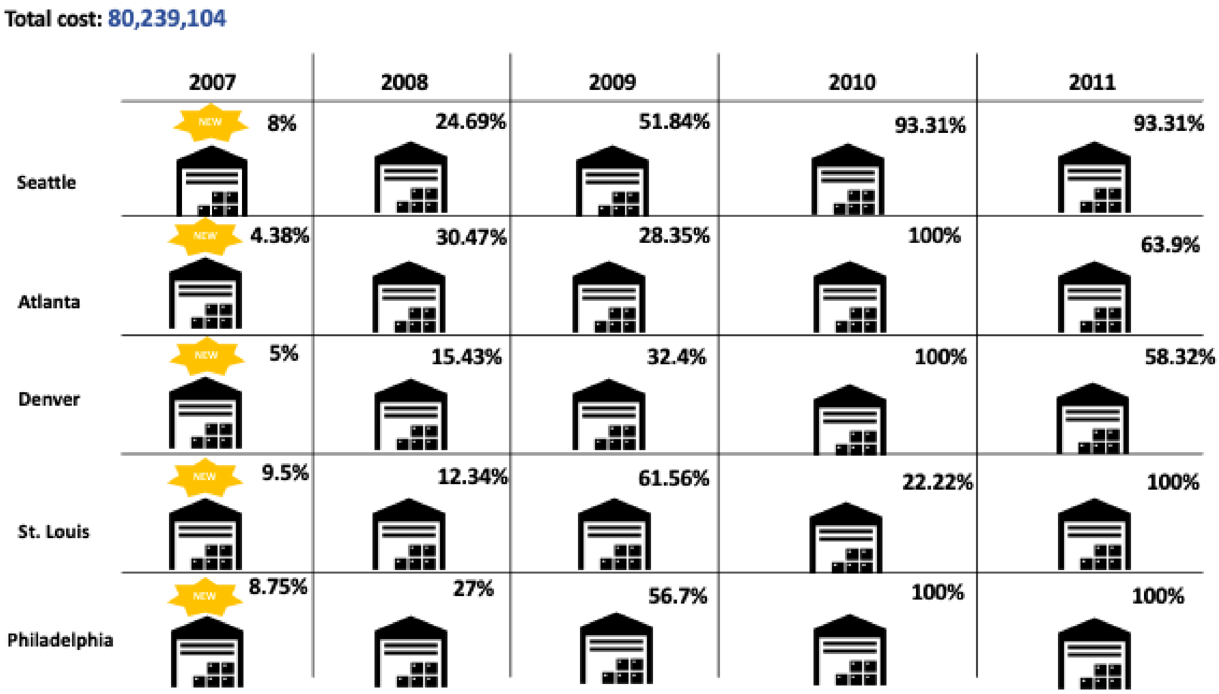

Figure 6: Part 4 Robust Optimization Model: Solution

Figure 6 shows the solution of the robust optimization model with flexibility and resilience characteristics. The solution follows the same approach as the robust optimization model in part 3 where large warehouses are opened for the whole period. However, in this model, we open the large warehouses from the beginning and utilize more and more capacity of the warehouses as demand grows throughout the years. This ensures the network resilience to the uncertainties of having higher demands and less capacity. The cost of network design of **$80.24 million**.