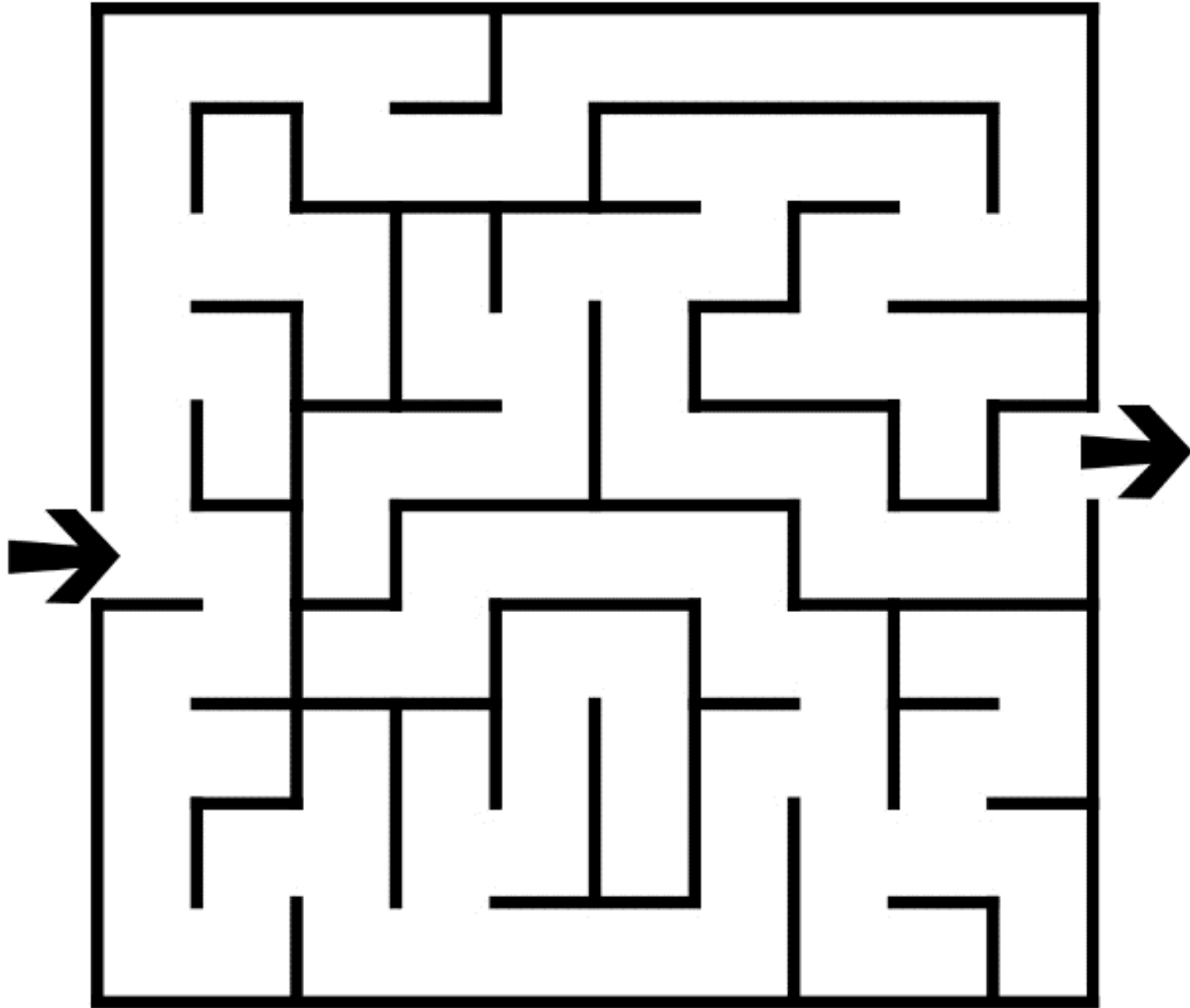


# Trajectory planning



$A^*$

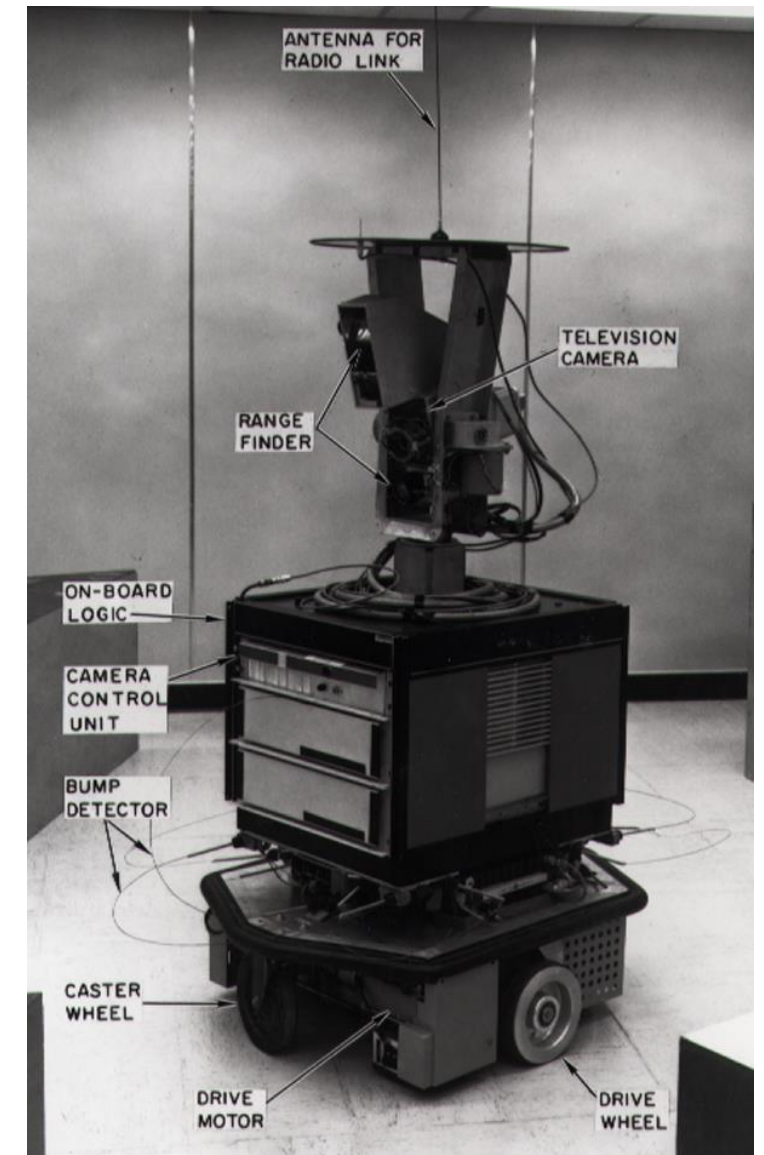
$$F = G + H$$

**F** is the total cost of the node.

**G** is the distance between the current node and the start node.

**H** is the heuristic — estimated distance from the current node to the end node.

7	6	5	6	7	8	9	10	11		19	20	21	22
6	5	4	5	6	7	8	9	10		18	19	20	21
5	4	3	4	5	6	7	8	9		17	18	19	20
4	3	2	3	4	5	6	7	8		16	17	18	19
3	2	1	2	3	4	5	6	7		15	16	17	18
2	1	0	1	2	3	4	5	6		14	15	16	17
3	2	1	2	3	4	5	6	7		13	14	15	16
4	3	2	3	4	5	6	7	8		12	13	14	15
5	4	3	4	5	6	7	8	9	10	11	12	13	14
6	5	4	5	6	7	8	9	10	11	12	13	14	15



# A\* steps

1. Add the starting square (or node) to the **open list**.
2. Repeat the following:
  - 2.1. Look for the lowest **F** cost square on the **open list**. We refer to this as the **current square**. Put in on the **closed list**.
  - 2.2. For each of the adjacent squares to the **current square**:
    - If it is not reachable or if it is on the **closed list**, ignore it. Otherwise do the following.
    - If it isn't on the open list, add it to the **open list**. Make the **current square** the parent of this square. Record the **F**, **G**, and **H** costs of the square.
    - If it is on the **open list** already, check to see if this path to that square is better, using **G** cost as the measure. A lower **G** cost means that this is a better path. If so, change the parent of the square to the **current square**, and recalculate the **G** and **F** scores of the square.

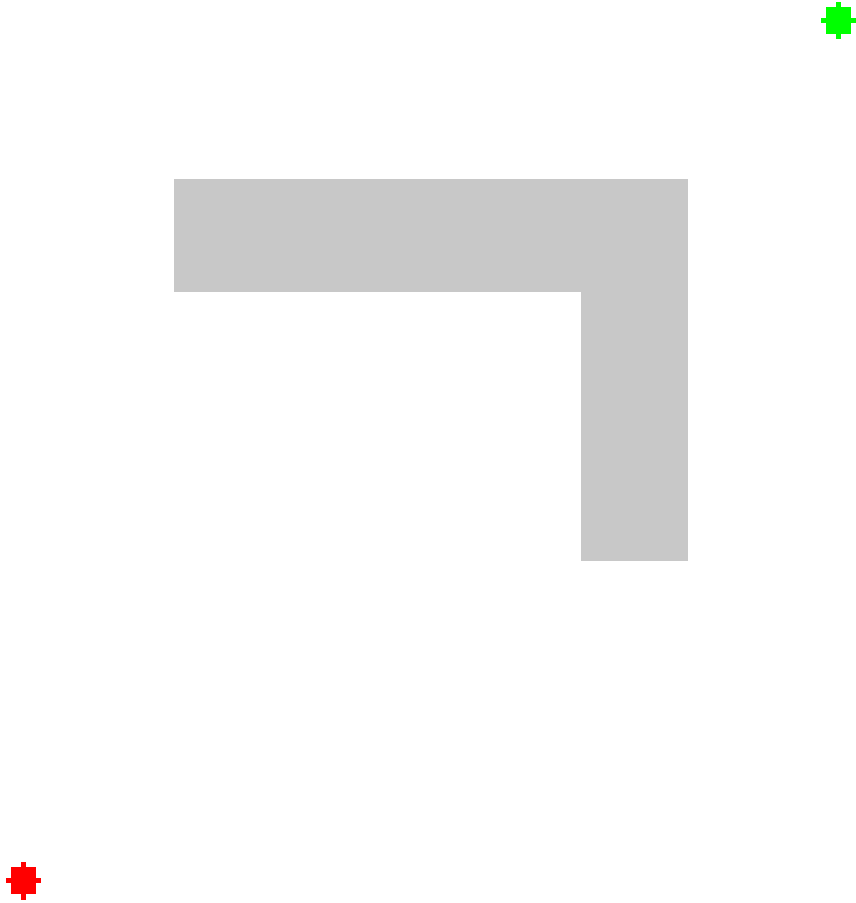
## 3. Stop when you:

- Add the target square to the **closed list**, in which case the path has been found, or
- Fail to find the target square, and the **open list** is empty. In this case, there is no path.

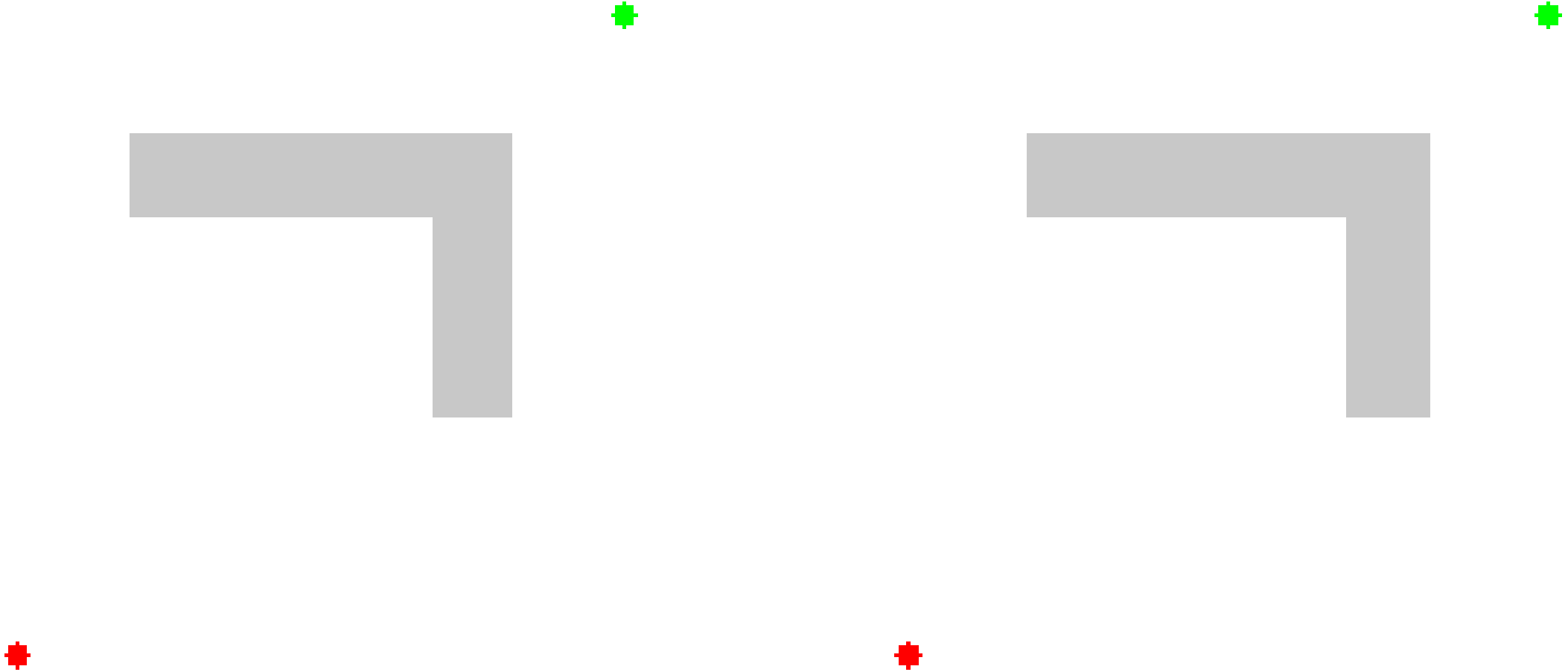
## 4. Restore the path.

4	5	6	7	8	9		17	18	19
3	4	5	6	7	8		16	17	18
2	3	4	5	6	7		15	16	17
1	2	3	4	5	6		14	15	16
2	3	4	5	6	7		13	14	15
3	4	5	6	7	8		12	13	14

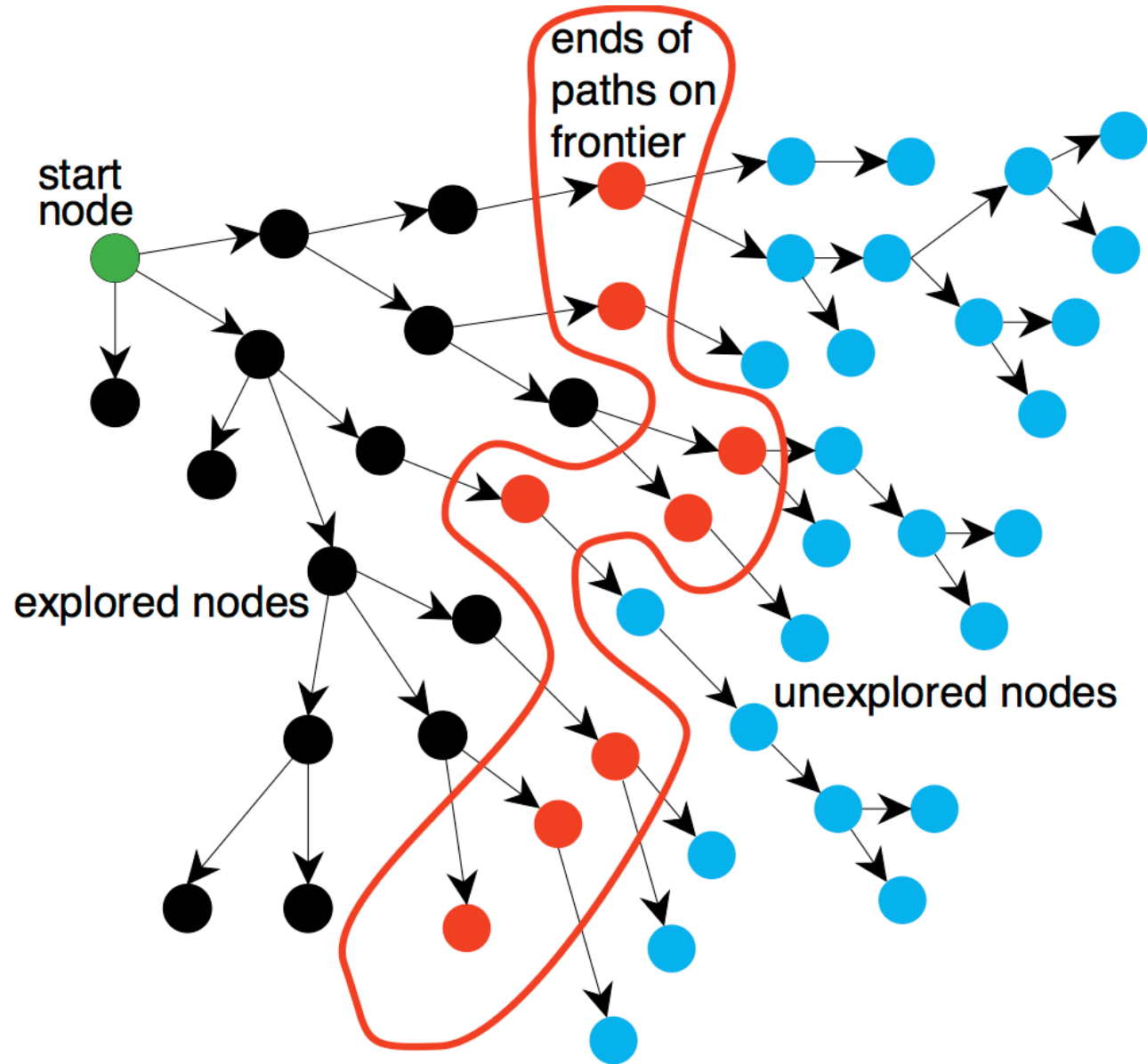
# A\* vs. Dijkstra's Algorithm



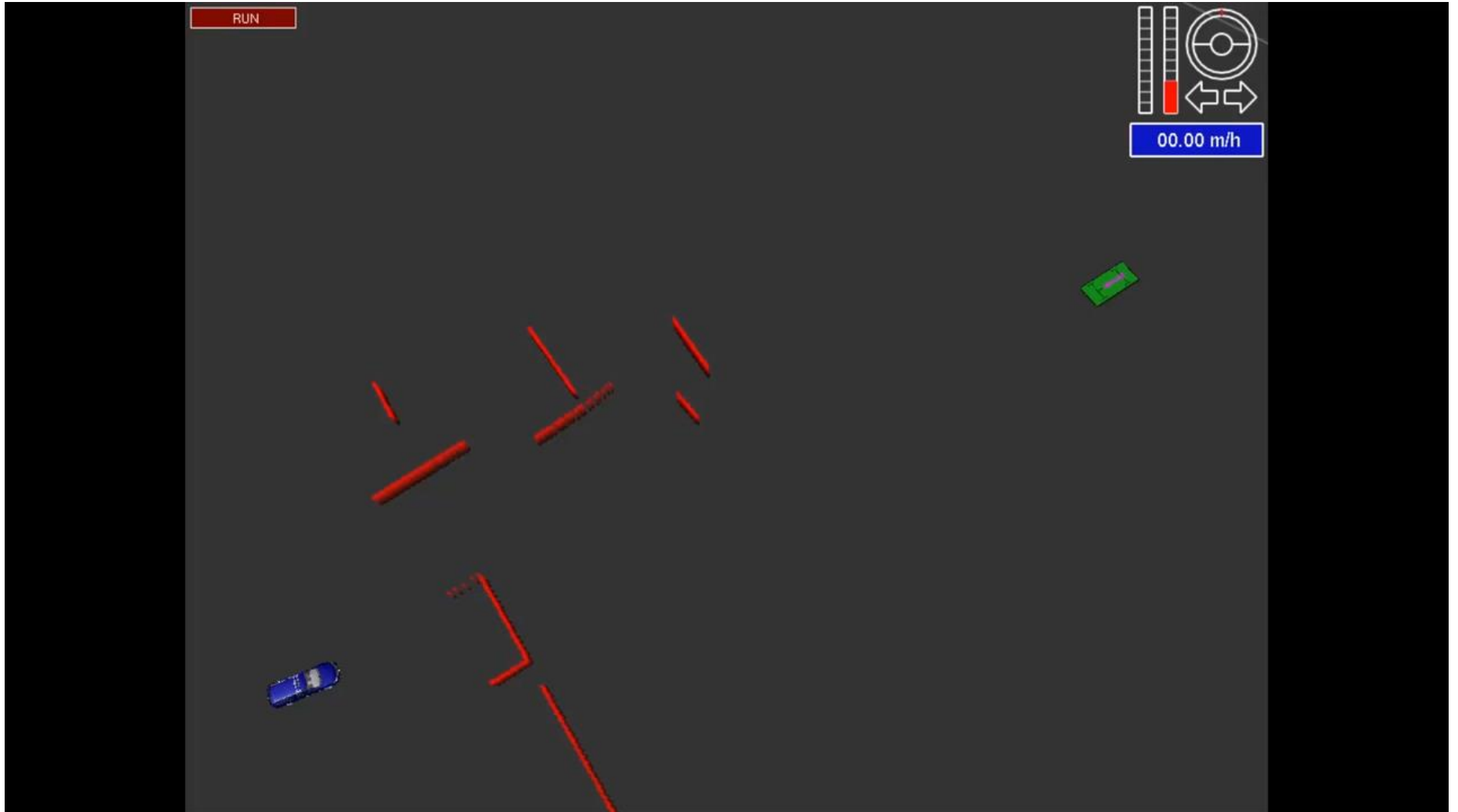
# A\* vs. Dijkstra's Algorithm



$A^*$

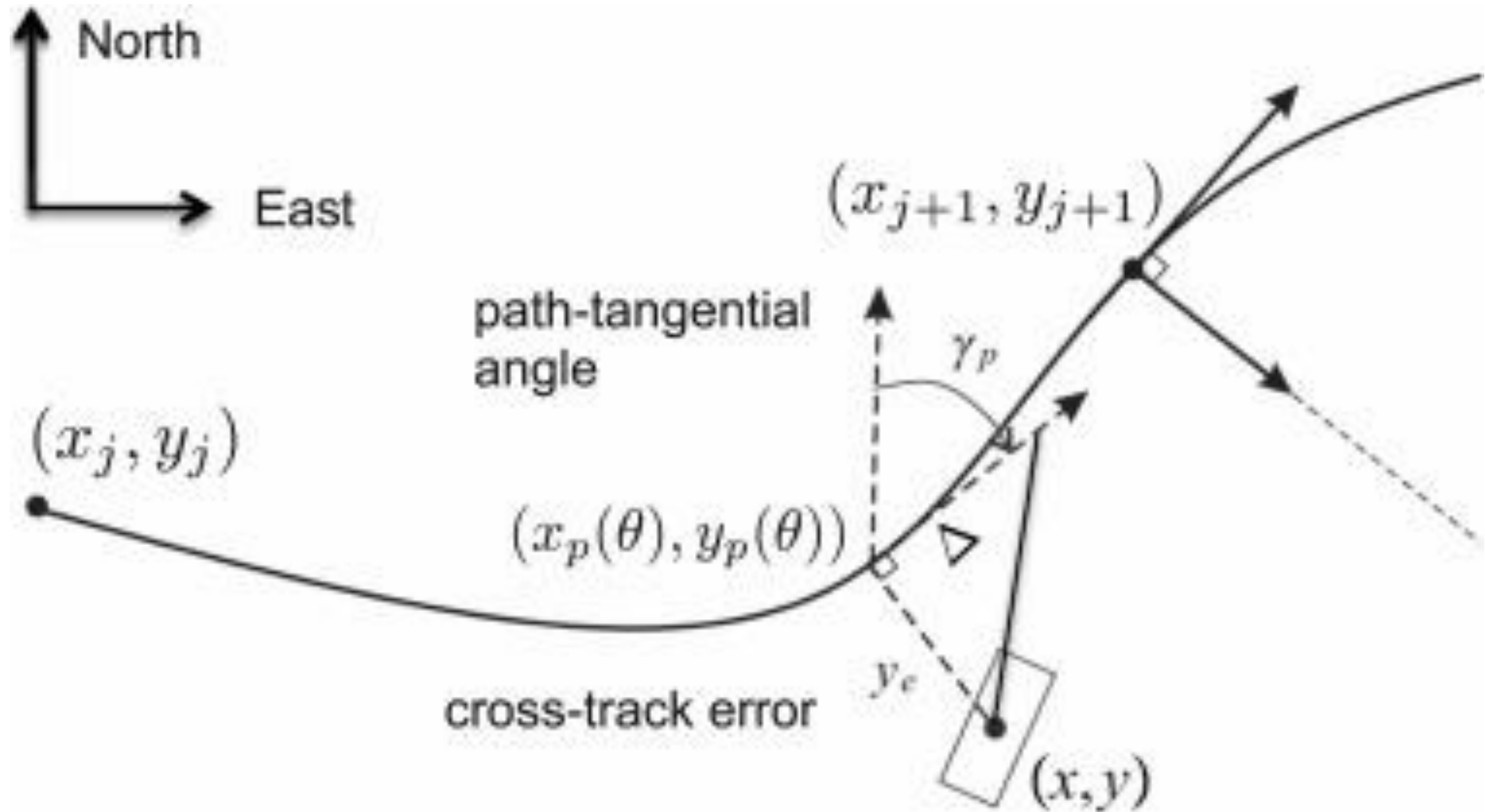


A\*



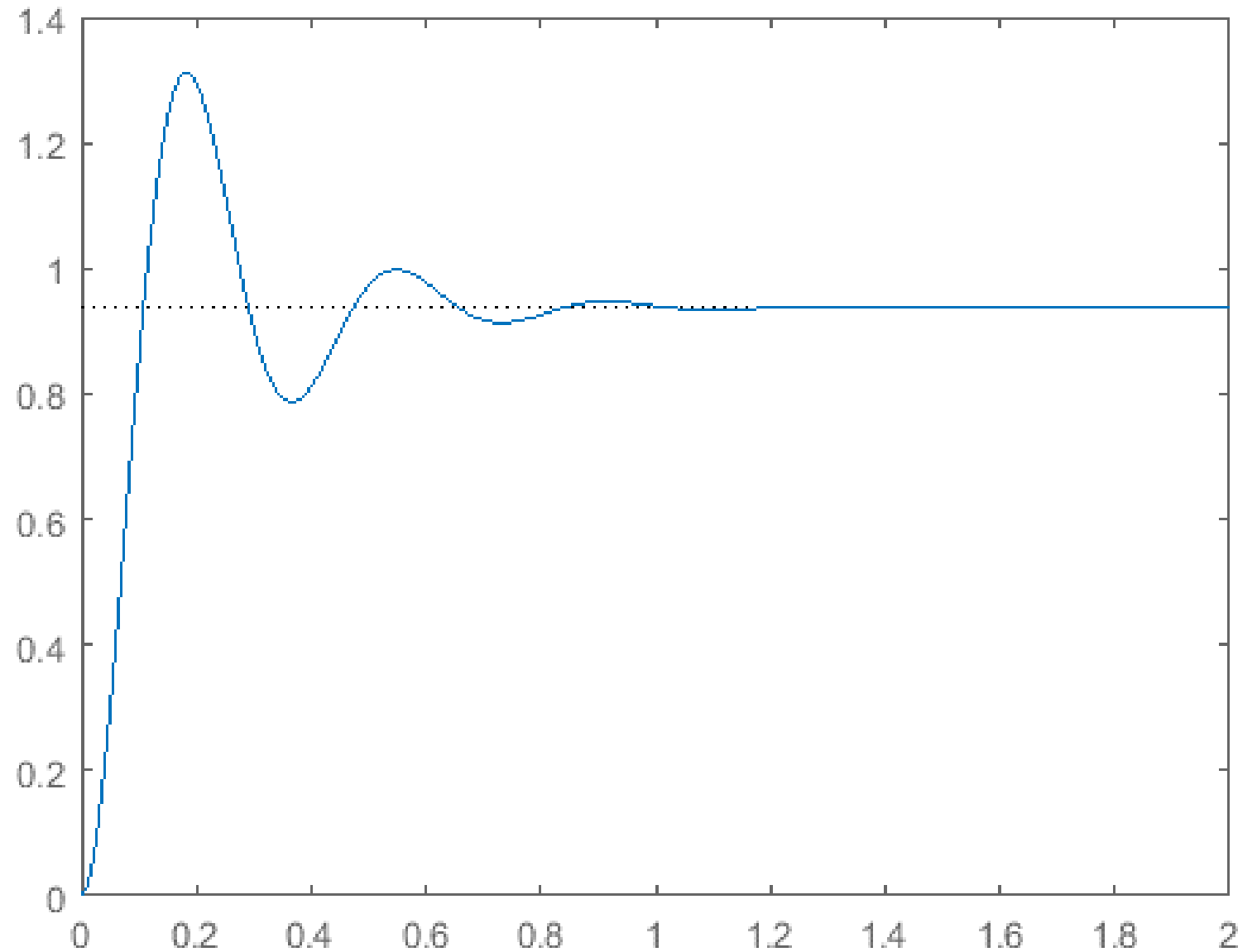


# Cross Track Error



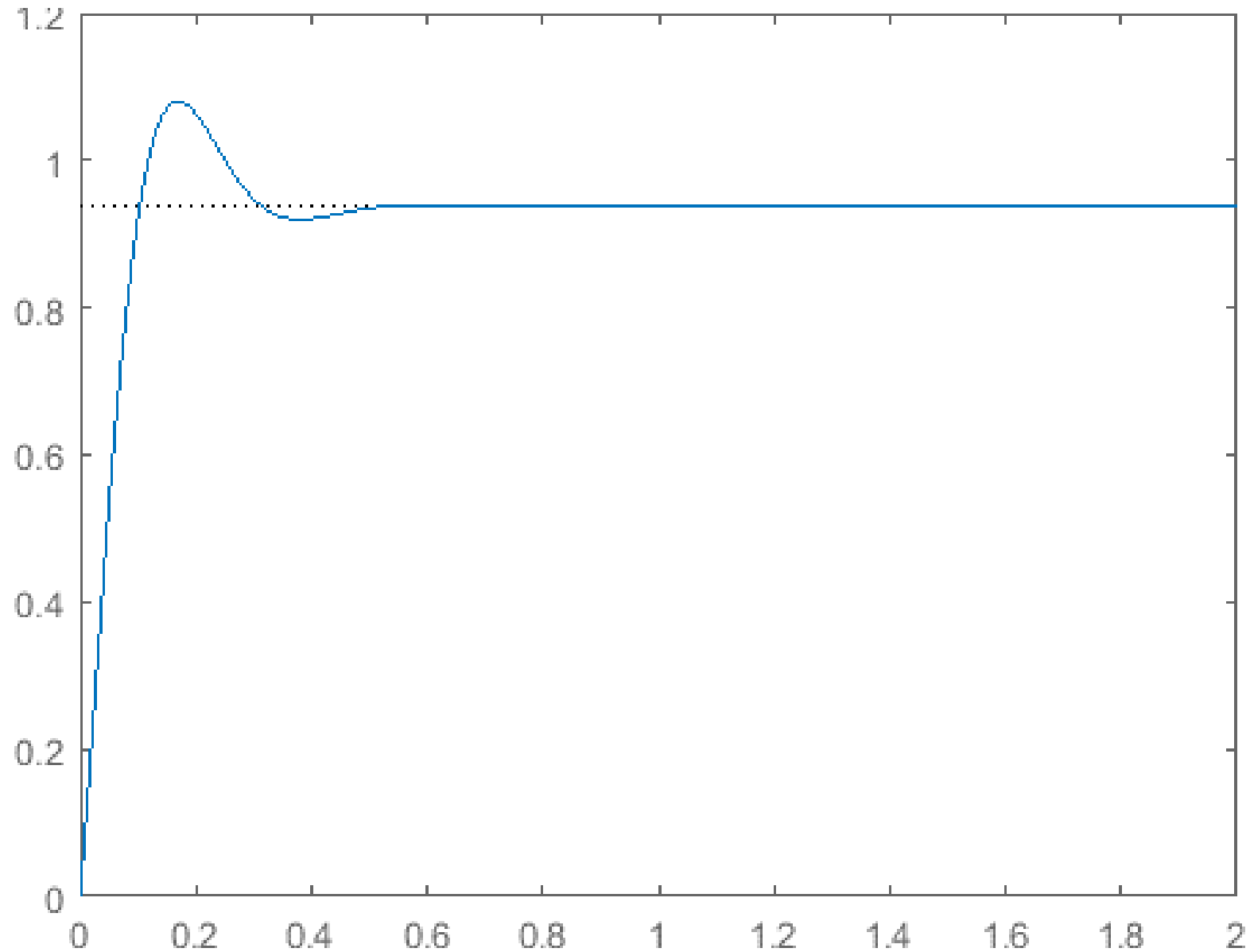
# Proportional control

$$u(t) = K_p e(t)$$



# PD control

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$



# PID control

$$u(t) = K_p e(t) + K_i \int_0^t e(t') dt' + K_d \frac{de(t)}{dt}$$

Proportional



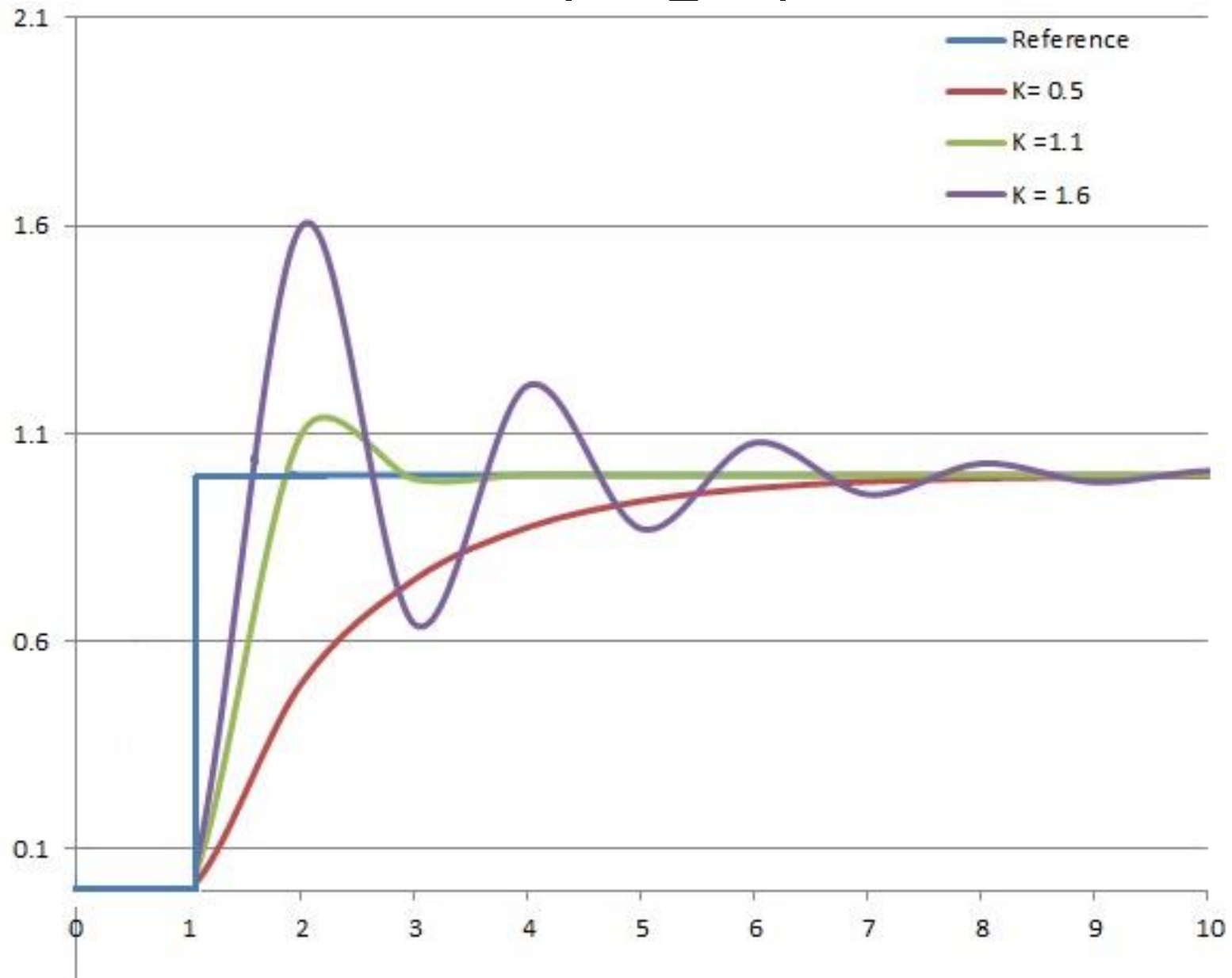
Integral



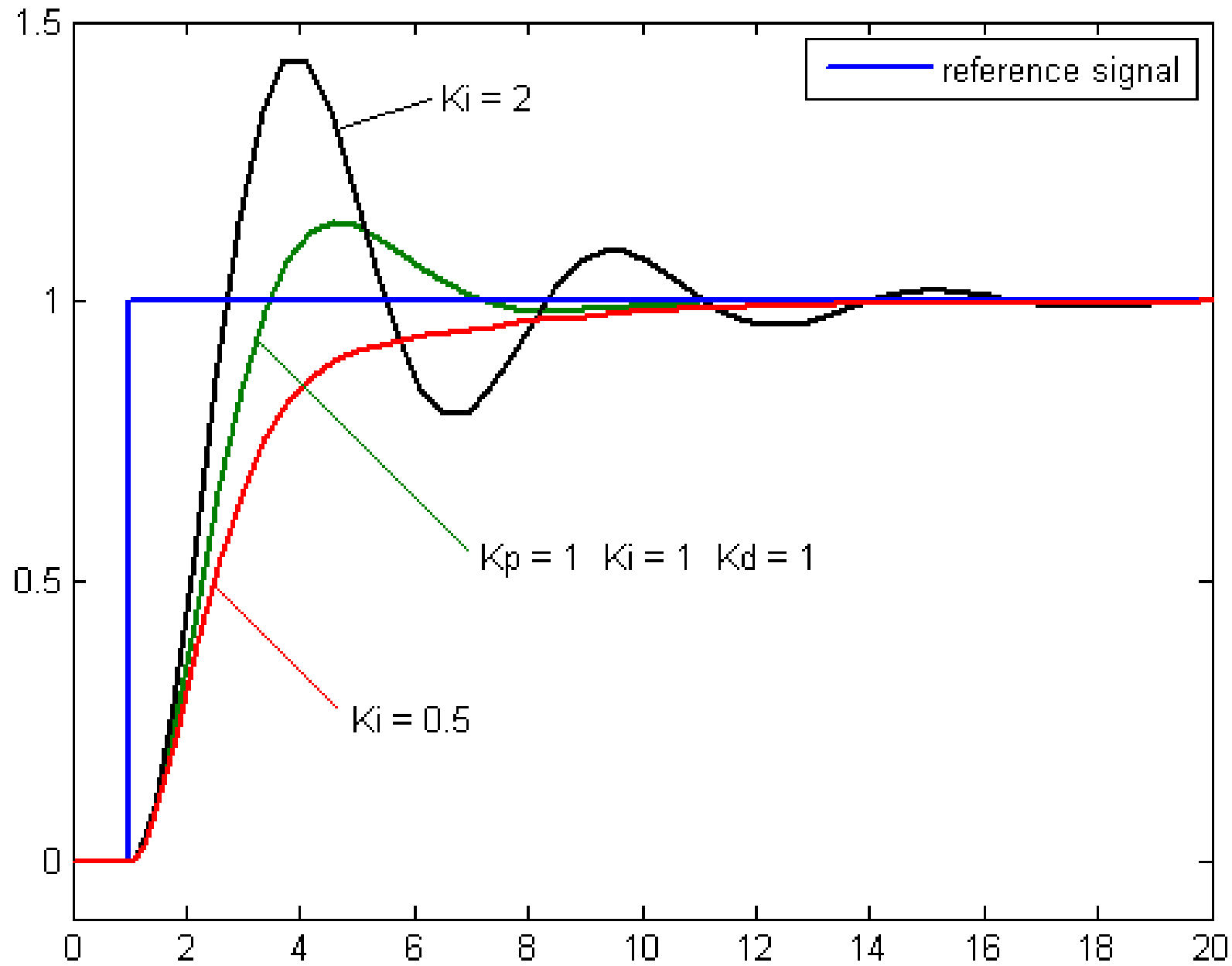
Differential



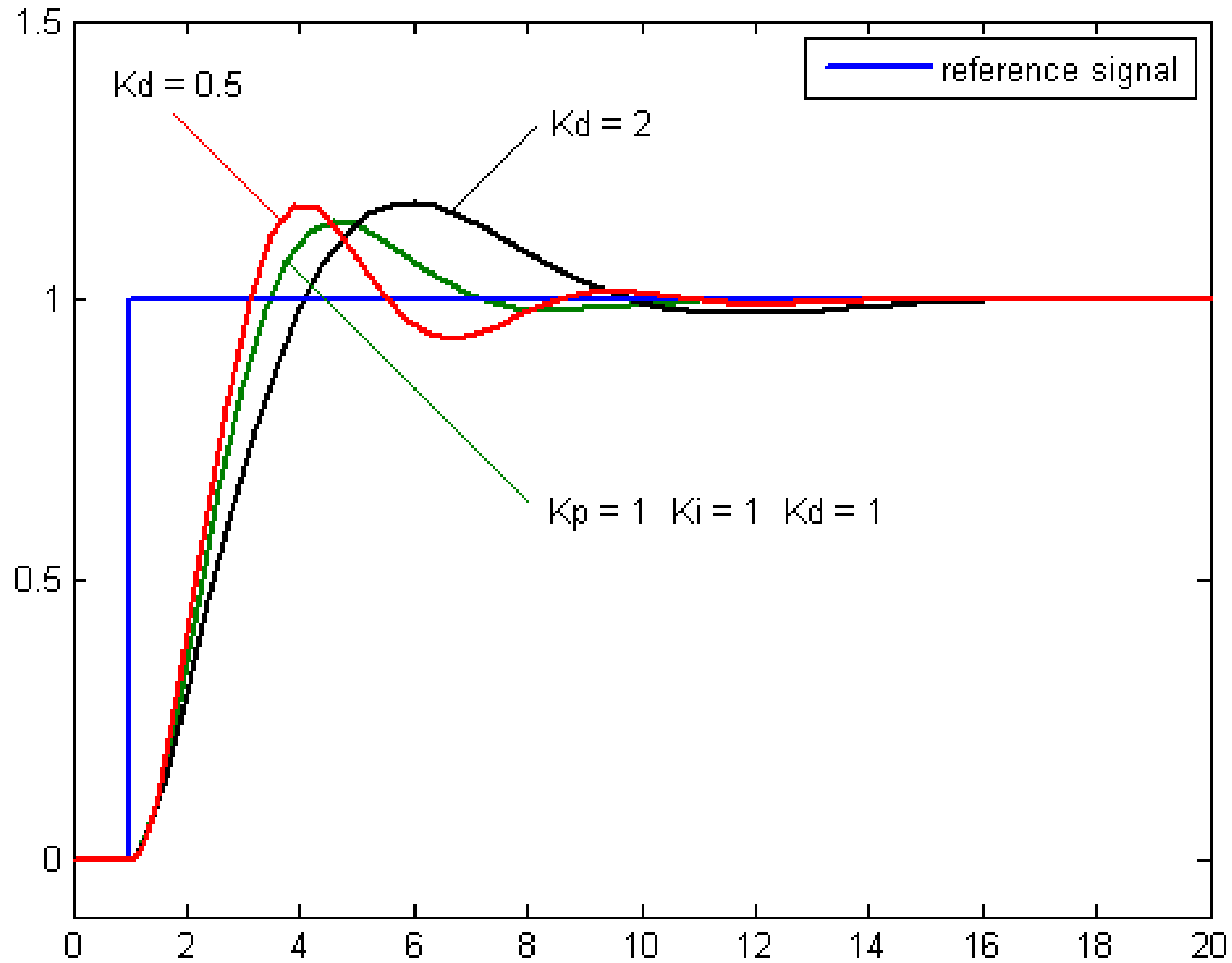
# Varying $K_p$



# Varying $K_i$



# Varying Kd

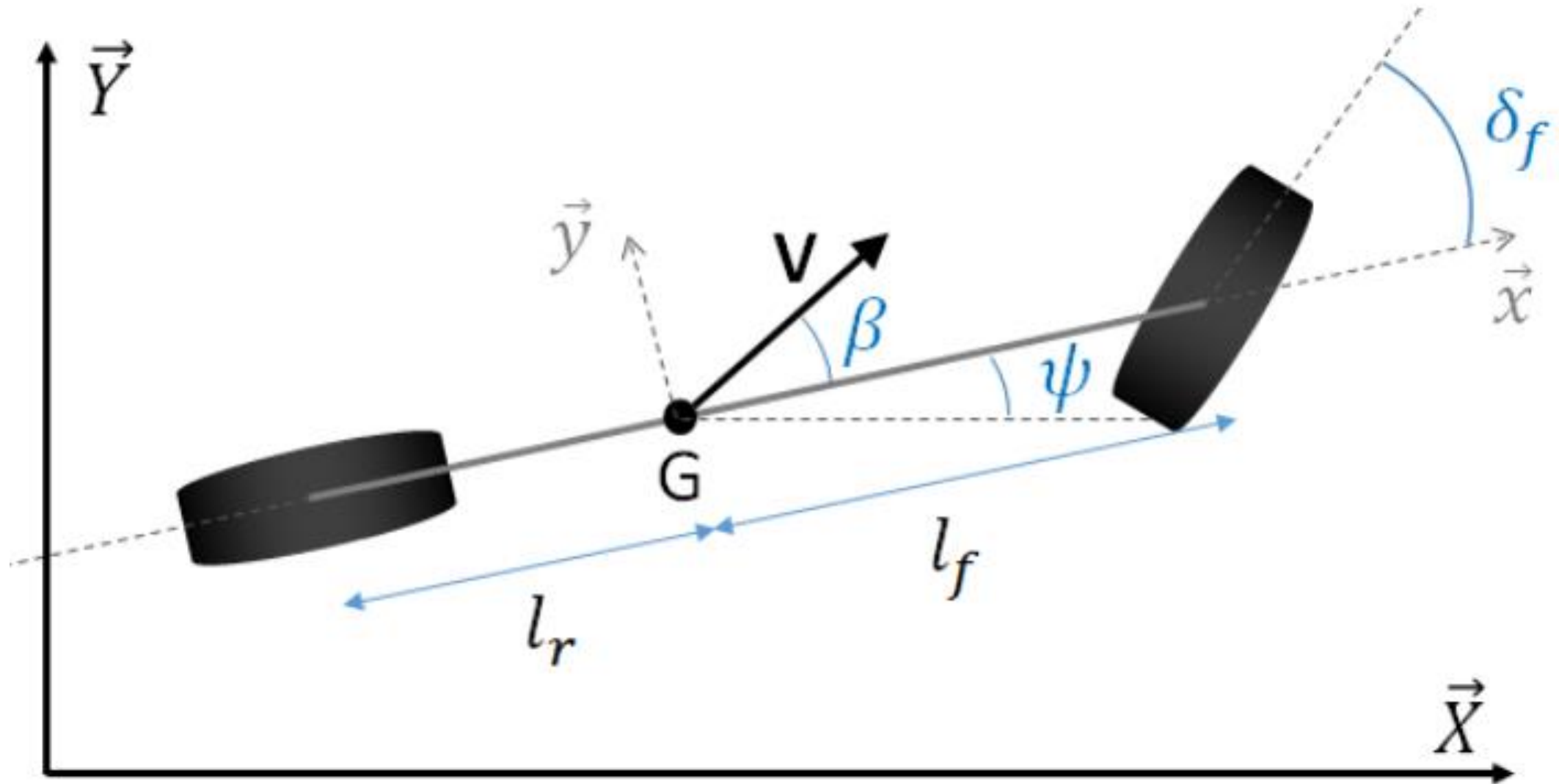


# PID control in simulated environment

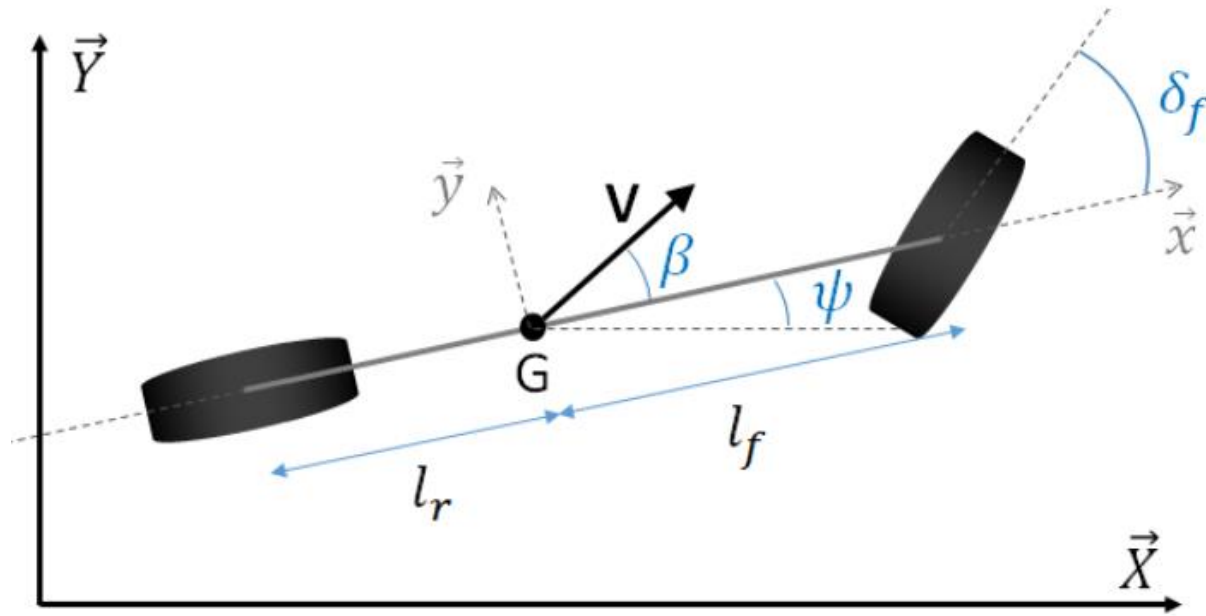




# Kinematic bicycle model



# Kinematic bicycle model



$$\dot{X} = V \cos(\psi + \beta(u_2))$$

$$\dot{Y} = V \sin(\psi + \beta(u_2))$$

$$\dot{V} = u_1$$

$$\dot{\psi} = \frac{V}{l_r} \sin(\beta(u_2))$$

$u_1$  - acceleration

$u_2$  - steering angle

$$\beta(u_2) = \arctan\left(\tan(u_2) \frac{l_r}{l_f + l_r}\right)$$

# Predictive model

$$x_{t+1} = x_t + v_t \cos(\psi_t) dt$$

$$y_{t+1} = y_t + v_t \sin(\psi_t) dt$$

$$\psi_{t+1} = \psi_t + \frac{v_t}{l_f} \delta_t dt$$

$$v_{t+1} = v_t + a_t dt$$

$$cte_{t+1} = f(x_t) - y_t + v_t \sin(e\psi_t) dt$$

$$e\psi_{t+1} = \psi_t + \psi_{des_t} \frac{v_t}{l_f} \delta_t dt$$

# MPC cost function and constraints

- Cross-track error.
- Heading error.
- Speed cost.
- Steering cost.
- Acceleration cost.
- Steering rate change.
- Acceleration rate change (jerk).

$$\begin{aligned} J = & \sum_{t=1}^N w_{cte} \|cte_t\|^2 + w_{e\psi} \|e\psi_t\|^2 + w_v \|v_t - v_{target}\|^2 \\ & + \sum_{t=1}^{N-1} w_{\delta} \|\delta_t\|^2 + w_a \|a_t\|^2 \\ & + \sum_{t=2}^N w_{rate_{\delta}} \|\delta_t - \delta_{t-1}\|^2 + w_{rate_a} \|a_t - a_{t-1}\|^2 \end{aligned}$$

Constraints:

$$\delta \in [-25^{\circ}, 25^{\circ}]$$

$$a \in [-1, 1]$$

**Solve for next N points with QP solver.**

# MPC in simulated environment

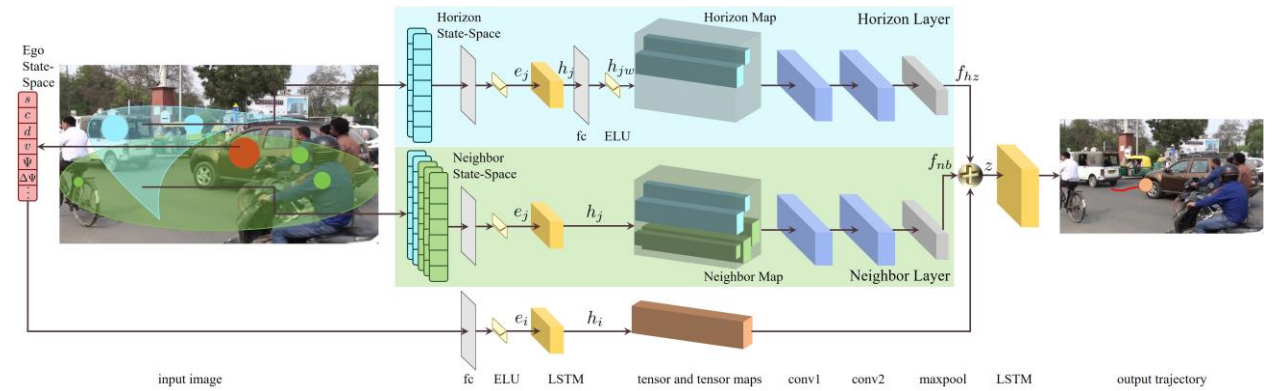


# Prediction

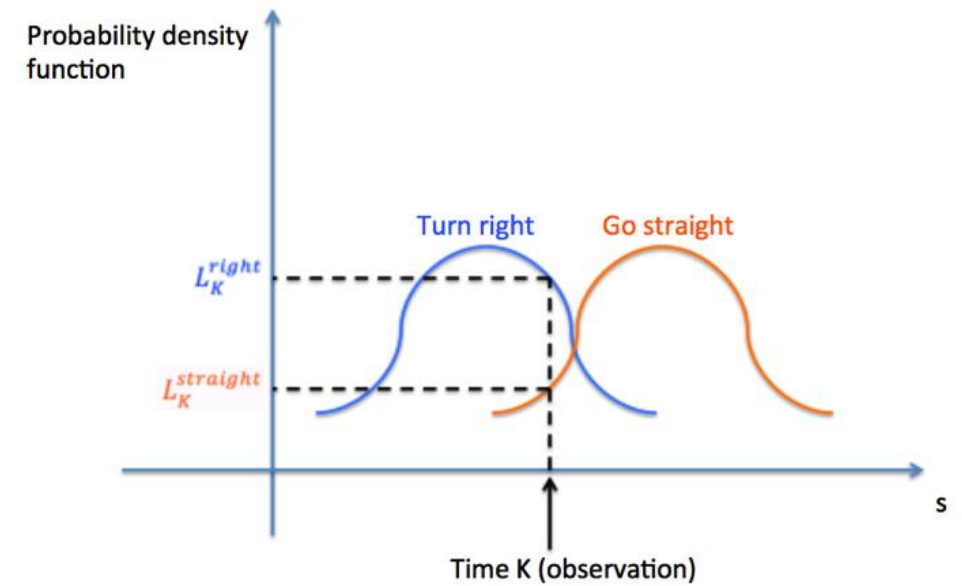
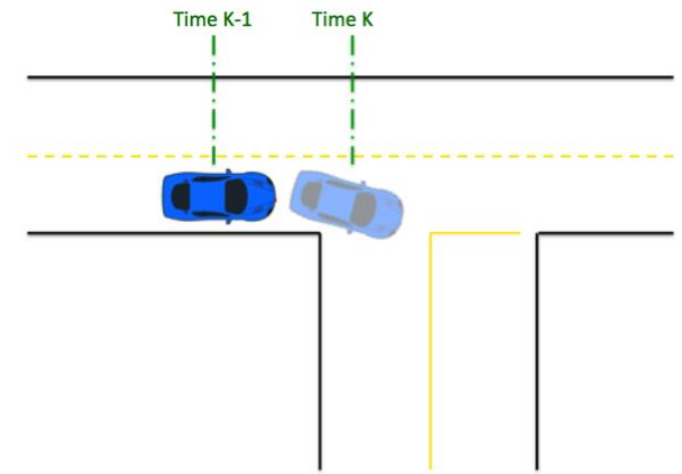
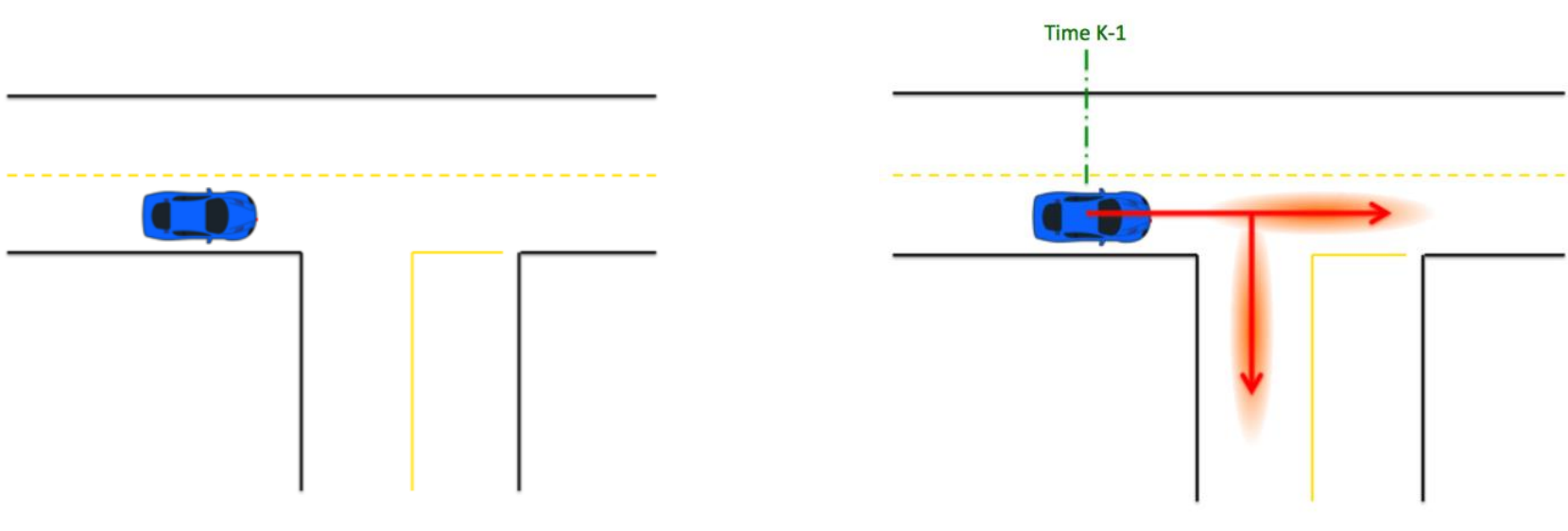
Model-based

$$\mu_k^{(i)} = \frac{\mu_{k-1}^{(i)} L_k^{(i)}}{\sum_{j=1}^M \mu_{k-1}^{(j)} L_k^{(j)}}$$

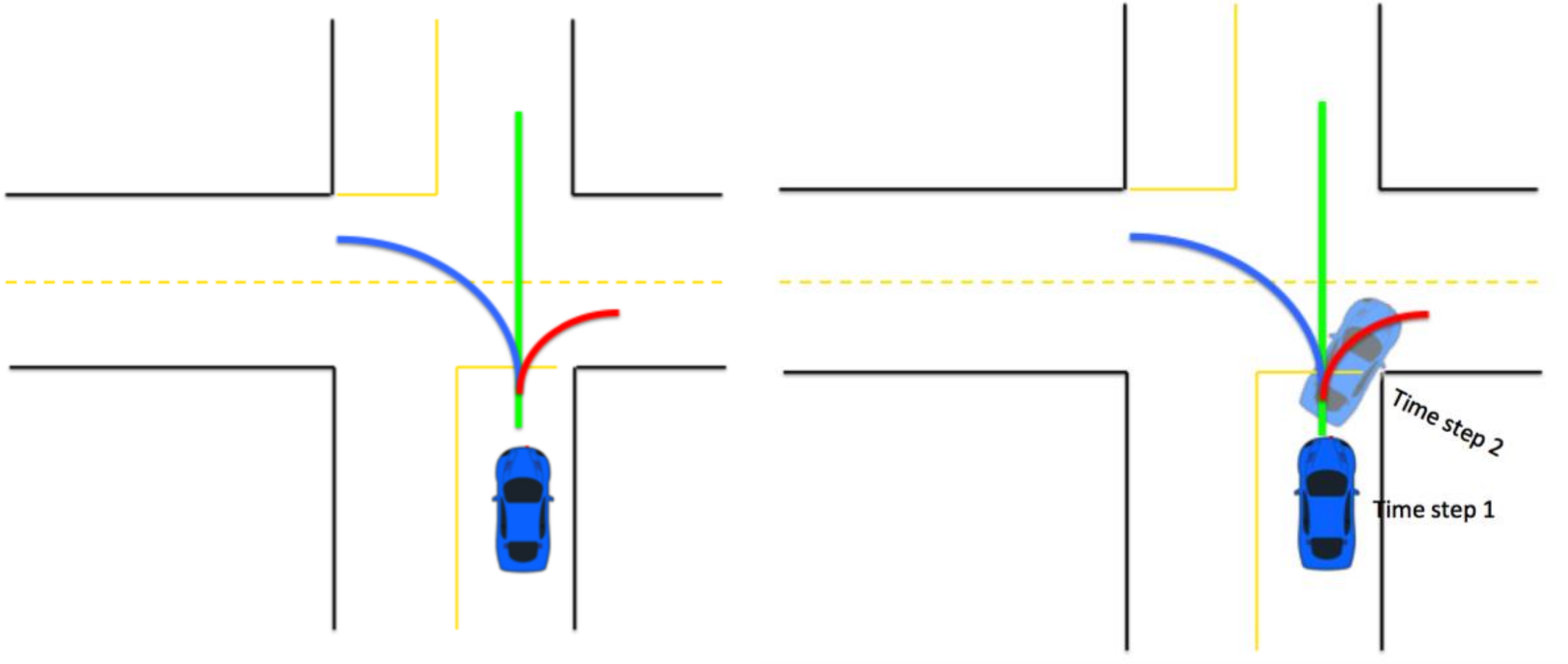
Data-driven



# Model based prediction

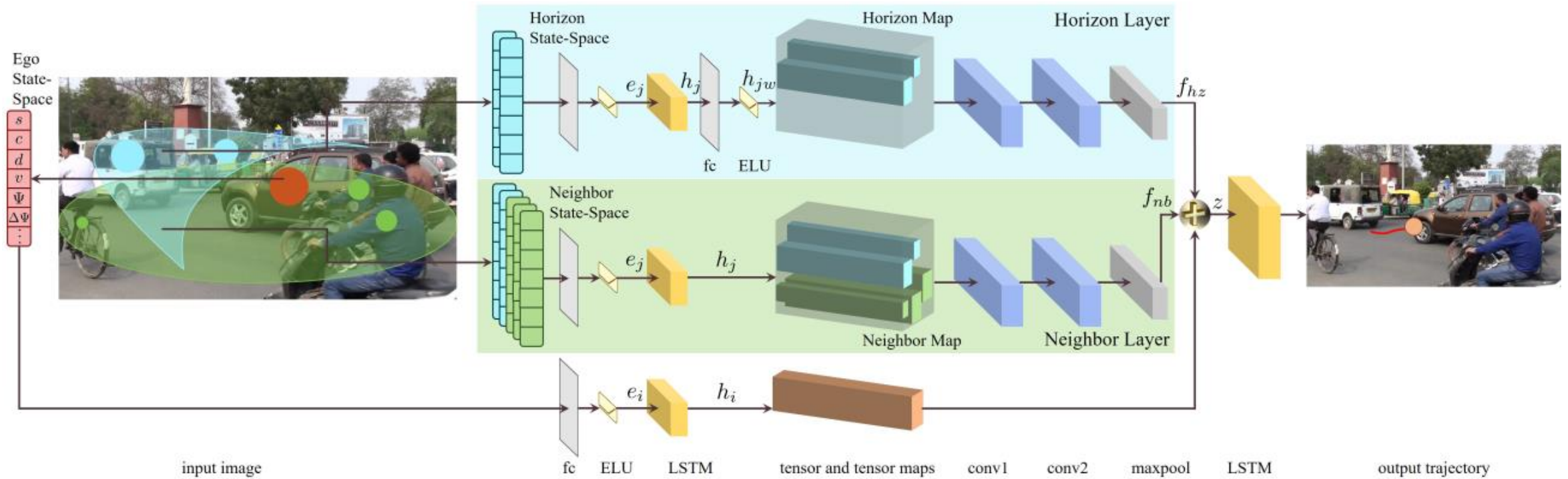


# Data-driven prediction, cluster trajectories





# TraPHic



# TraPHic

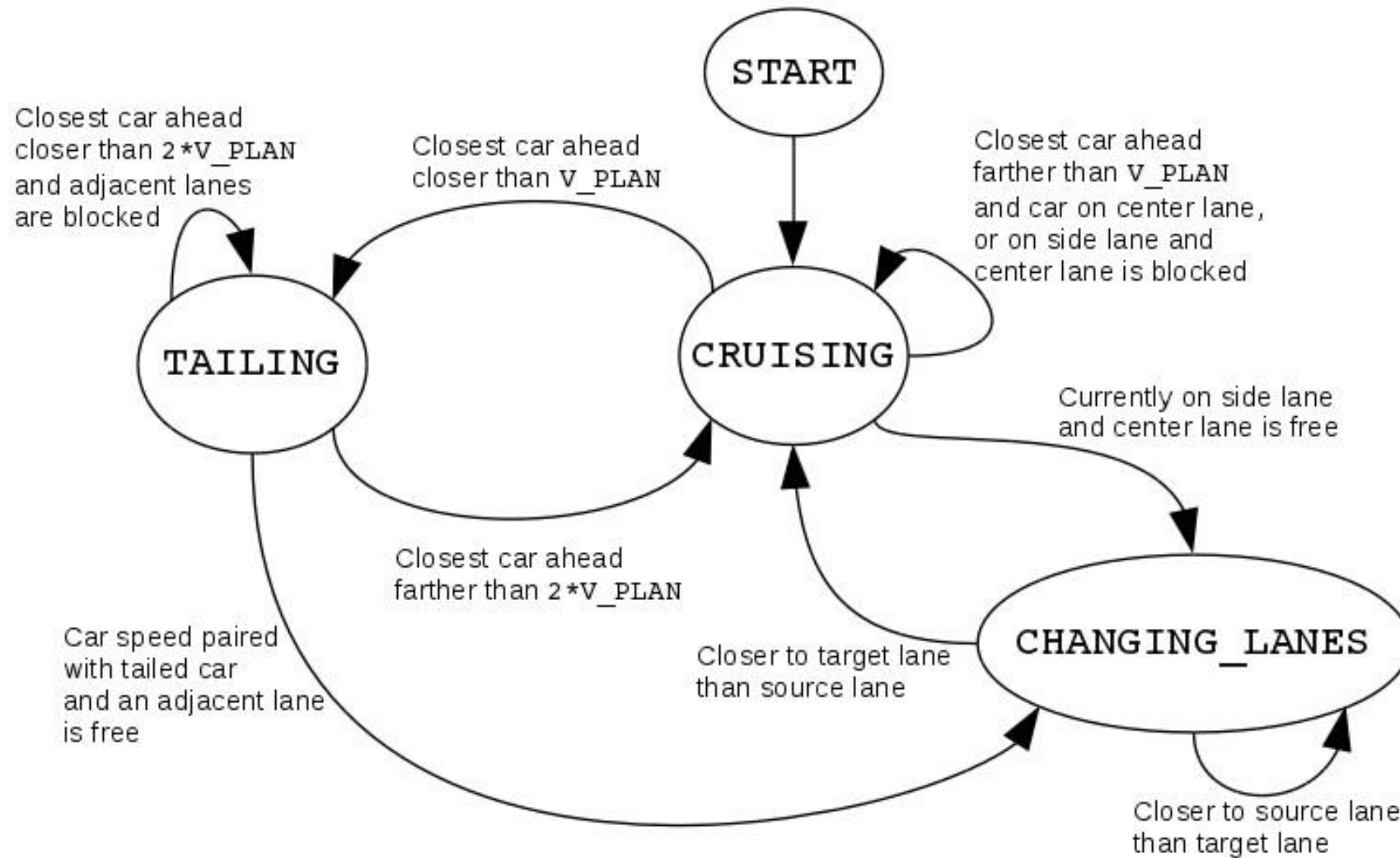




# Behavior

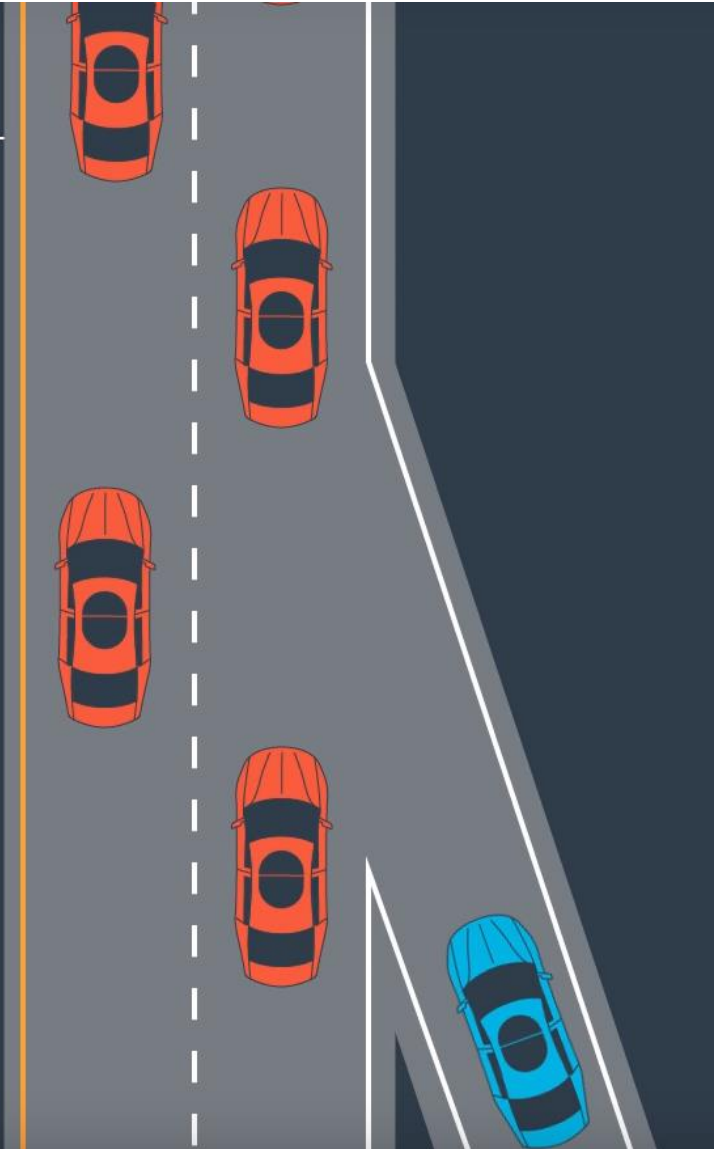


# Behavior with Finite State Machines

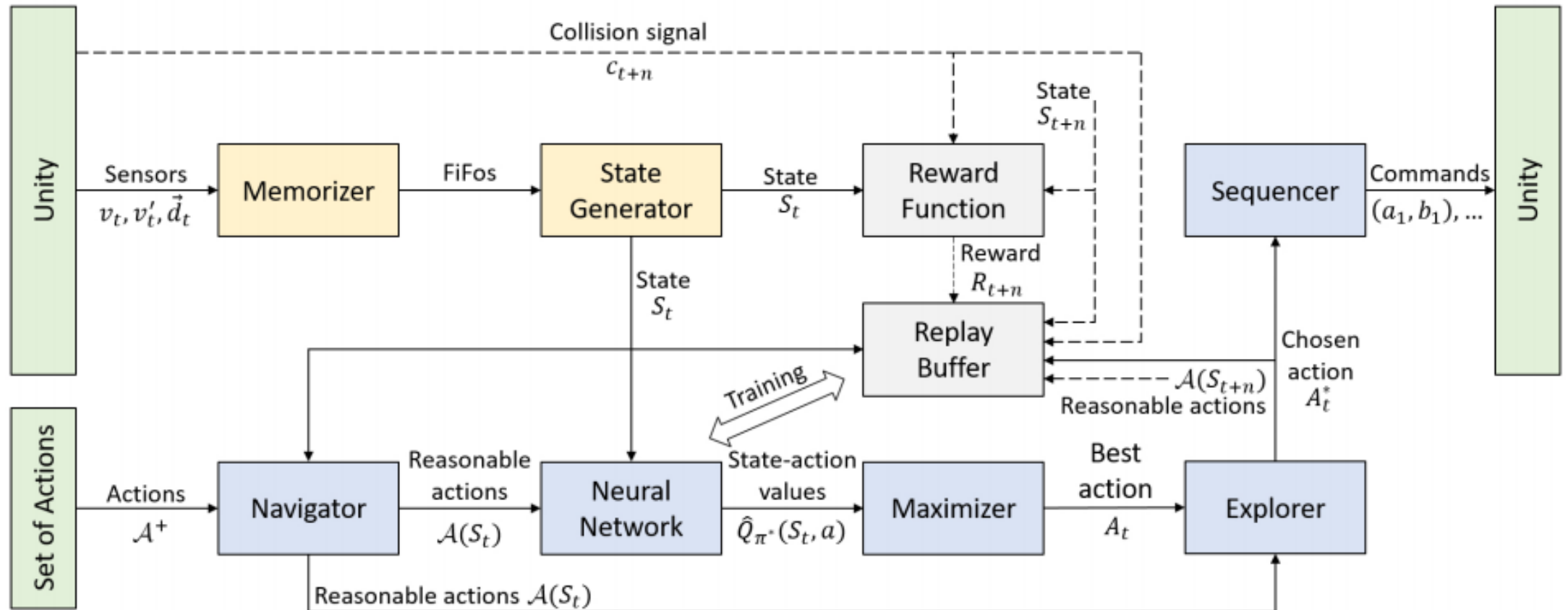


# Cost functions for FSM

Class	Position	Velocity	Acceleration
Feasibility	Avoids Collision?		Acceleration is feasible for car?
Safety	Buffer Distance	Speed $\approx$ traffic speed	
Legality	Stays on Road?	Speed < speed limit?	
Comfort	Near center of current lane		Low change in acceleration (jerk)
Efficiency	Desired Lane	Speed $\approx$ speed limit	

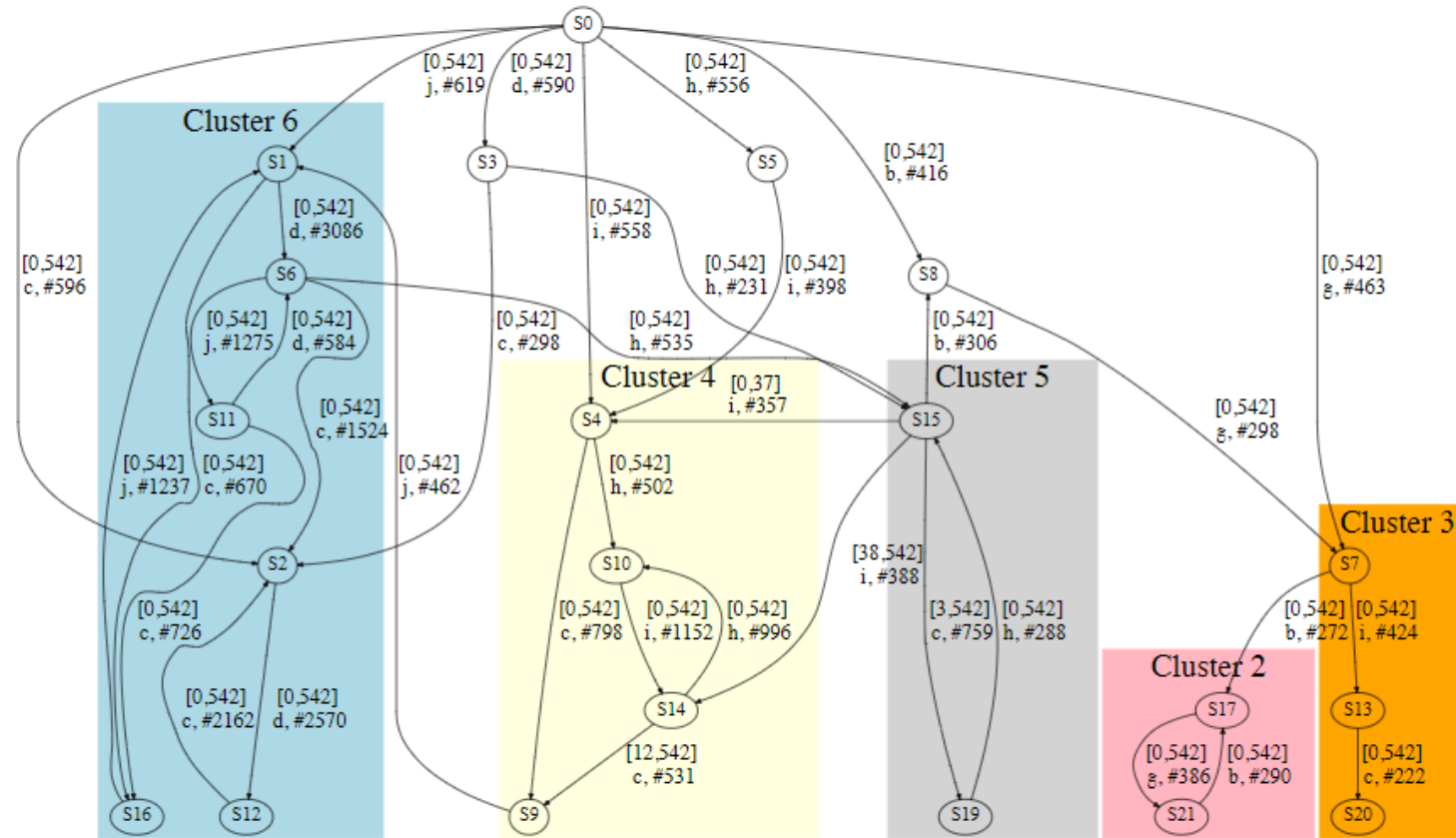
An illustration of a road with a dashed white center line and solid white edge lines. There are four red cars in the left lane and one blue car in the right lane. The blue car is positioned further to the right, near the edge of the road. The background is dark grey.

# Deep Reinforcement Learning for FSM





# Learned states



Cluster ID	Dominating states	Description
1	0,2,3,8,13	without significant meaning
2	17, 21	steady long distance car-following
3	7,13,20	intermediate process
4	4, 9, 10, 14	steady medium distance car-following
5	12, 15, 19	intermediate process
6	1, 2, 6, 11, 12, 16	steady short distance car-following

# Homework

- Create a PID controller for a simple robot.
- Write cost functions for FSM for lane changing behavior.