國立臺北大學
National Taipei University

Causal Inference

Intro

Speaker: 吳淳硯

Dept. of Stat.
National Taipei University

Preliminary Knowledge

- Basic logical thinking.
- Basic knowledge of probability theory and statistical inference.
- Familiar to linear model (generalized linear model).
- Basic matrix algebra operation.
- Experience with any command language.
  (Ex. R, Python, VBA)

Syllabus

- Course Outline
  - Introduction to Causal Inference
  - Causal Diagram (DAGs) and d-separation (NPSEM)
  - Randomized Experiment vs Observational Studies
  - Econometric Method for Causal Inference
  - Causal Mediation Analysis
    - Single Mediator / Binary Mediator
    - Multiple Mediators
    - Time-Varying Mediator
- Evaluation Method
  - Regular grades

    Participation 20% + Assignment 20%
  - Mid-term grades 30%
  - Final grades 30%

Textbooks and Reference

- No specific textbook
  - The material is based on lecture notes and various classical papers.
- Ref (Causal Inference)
  - Causal inference What if – Hernan and Robin
  - Fundamentals of Causal Inference With R – Babette A. Brumback
  - The book of Why – Judea Pearl
- Ref (Econometric)
  - Principles of Econometrics – R. Carter Hill, William E. Griffiths and Guay C. Lim
  - Econometric Analysis – William H. Greene
  - Introductory Econometrics A Modern Approach – Jeffrey M. Wooldridge

Traditional view of Statistic

- Statistical inference is concerned with associational inference and used for finding association among variables, rather then for inferring causation relationship from observations.

- Two famous aphorisms based on this view are

Correlation does not imply causation.

You can’t prove causality with statistics.

- The mainly focus is to introduce the ″**formal language for causal inference**″ and ″**develop statistical methods to estimate causal effects**″ in randomized experiments and observational studies.

Causal inference ≈ Causal model
+ Statistical inference

Is Data access Causality?

Data Are Not Enough~ Hurray For Causality!

- **Data Are Not Enough**
  - Just collecting large amounts of data – even high-quality data – does not automatically tell us why things happen.
  - Data can show patterns and associations, but without context, it can mislead.
- **Hurray For Causality**
  - This is celebration of causal reasoning – the ability to determine what causes what.
  - With causality, we move beyond just describing the world to explaining, predicting and intervening effectively.
- So in short

Data ≠ Understanding

Causality = Power to explain, predict and change

Causal Reasoning

- Many of the inferences one wants to draw about real-world, spatiotemporal, contextual knowledge involve **Cause and Effect**.
- **Deductive Reasoning**
  - Deriving consequence (or effects, outcomes) from premises (or causes).
  - The Conclusion is **Certainly True**.
- **Inductive Reasoning**
  - Deriving relationship between causes and effects, rules that lead from one to another.
  - The Conclusion is **Probably True**.
- **Abductive Reasoning**
  - Deriving possible causes and effects.
  - The Conclusion is the **Best Explanation**.
- **Causal reasoning** is generally considered a form of induction reasoning.

Example - 1

- **Deductive Reasoning** Example
  - Premise 1: The sum of the interior angles of triangle is 180^o
  - Premise 2: ΔABC is a triangle.

  $\Rightarrow$ The sum of the interior angles of ΔABC is 180^o.
  (A guaranteed conclusion based on definitions)

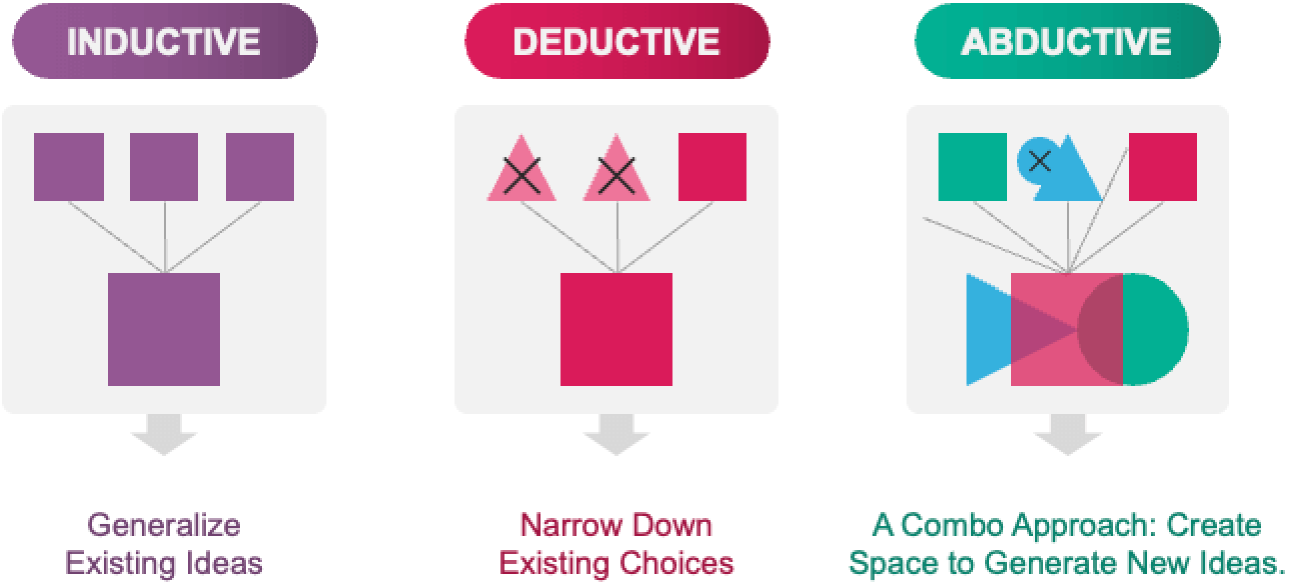
- **Inductive Reasoning** Example
  - Observations (Premises):
    - Most men have 2 testicles (Some men, have 1 or 0).
    - Women have 0 testicles.
    - Average of testicles across the entire population, the number is approximate 1.

  $\Rightarrow$ Therefore, everyone has one testicle.
  (This is technically true as a statistical average, but it's a misleading inductive conclusion if interpreted to describe a real, individual human.)

Example - 2

- **Abductive Reasoning** Example
  - Observation (Effect): The girl has a noticeably big belly.
  - Hypothesis (Possible Cause): The girl might be pregnant.

  ⇒ Therefore, the girl is probably pregnant.
  (This is not guaranteed to be true, but it’s the most plausible explanation given what we observe)
- Like **Inductive reasoning**, **Abductive reasoning** are not logically watertight. Although a hypothesis may seem to be best explanation, other explanations are still logically possible.



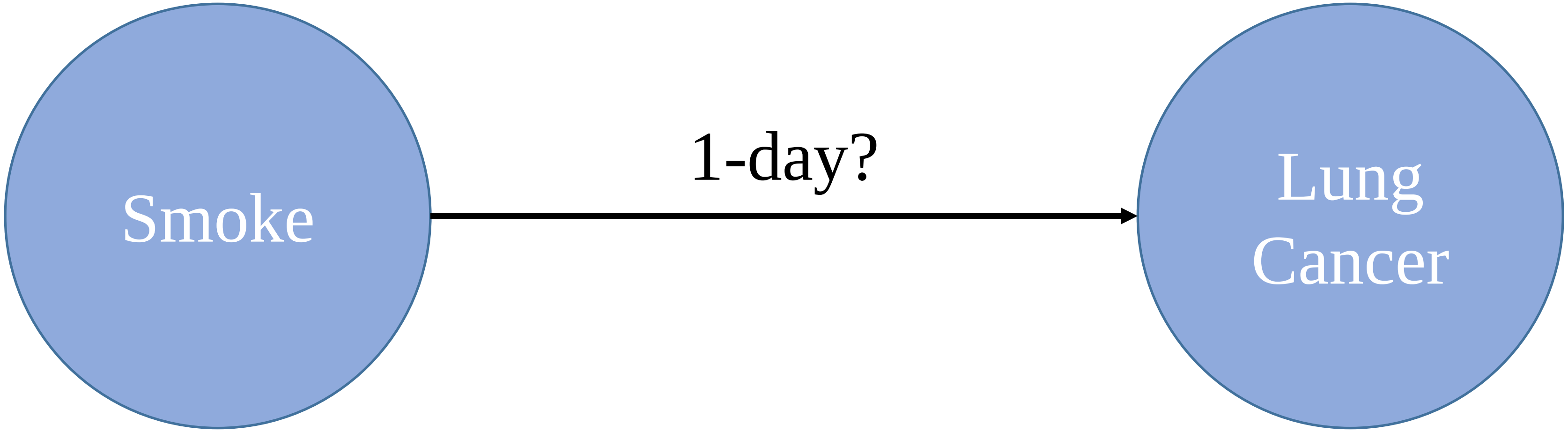
Hill's Criteria

- Some viewpoints we should study association before we cry causation
  - Strength if the association
  - Consistency
  - Specificity of the association
  - Temporality
  - Biological gradient (dose-response)
  - Plausibility (credibility)
  - Coherence
  - Experiment
  - Analogy
- None of them can bring indisputable evidence for or against the cause-and-effect hypothesis and none can be required as a *sine qua non*.
- What they can do, is to help us make up our minds on the question
  - Is there any other way of explaining the set of facts before us?
  - Is there any other answer equally, or more, likely than cause and effect?

Temporality

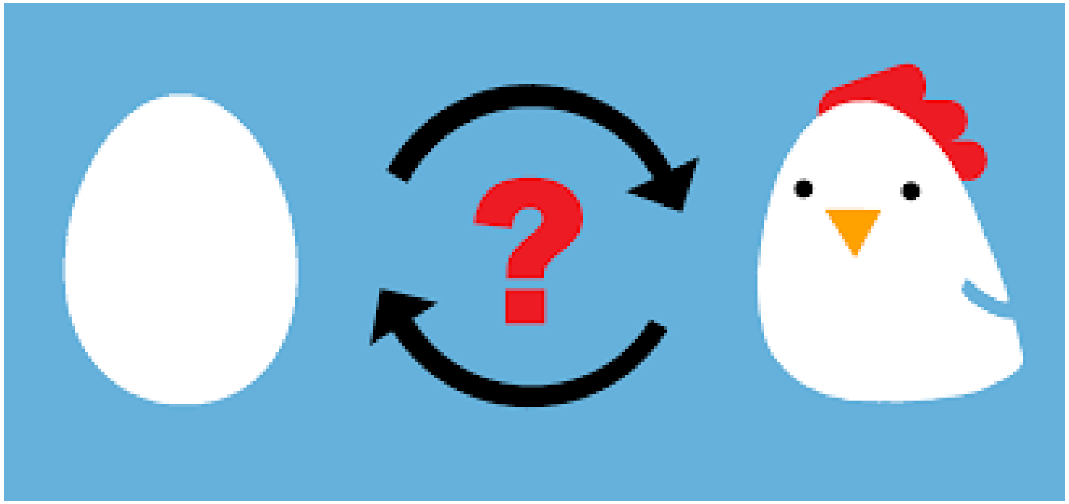
Exposure must precede outcome

- This criterion has been identified as being the most likely to be the *sine qua non* for causality.
- For an agent to be causal, its presence must precede the development of the outcome. Lack of temporality rules out causality.
- The effect must occur after the cause, with an **expected delay** between them.



Circular reasoning

- It is a logical fallacy in which the reasoner begins with what they are trying to end with.
- Circular reasoning is not a formal logical fallacy, but a pragmatic defect in an argument whereby the premises are just as much in need of proof or evidence as the conclusion.
- As a consequence, the argument becomes a matter of faith and fails to persuade those who do not already accept it.



Which happened first?

Why care about Causality?

- Allows us to:
  - Predict the future.
  - Explain the past.
  - Intervene in the present.
  - Learning and Adaptation.
  - Building and Fixing System

- Pervades daily life.
  - Naïve physics

    Building things, fixing things
  - Naïve biology

    Growing thing, cooking things
  - Naïve psychology

    Influencing, crediting, blaming.
- Great interactive features stimulate motor skill development and increase cause-effect understanding.

The Little Scientist

- How Children Learn **Cause and Effect**?
  - Natural Explorers

    From infancy, children act like little scientists - testing, observing, and predicting outcomes from their actions.
  - Trial and Error

    They repeat actions to see what happens, building an understanding of cause and effect.
  - Imitating Adults

    Kids learn by watching others, even when those actions include mistakes or imperfect outcomes.
  - Everyday Experiments

    Daily life is full of opportunities for exploration - no special tools needed. Ordinary routines become learning labs.
- As children grow, they begin to understand that their actions can cause thing to happen, and this awareness is crucial for cognitive development.

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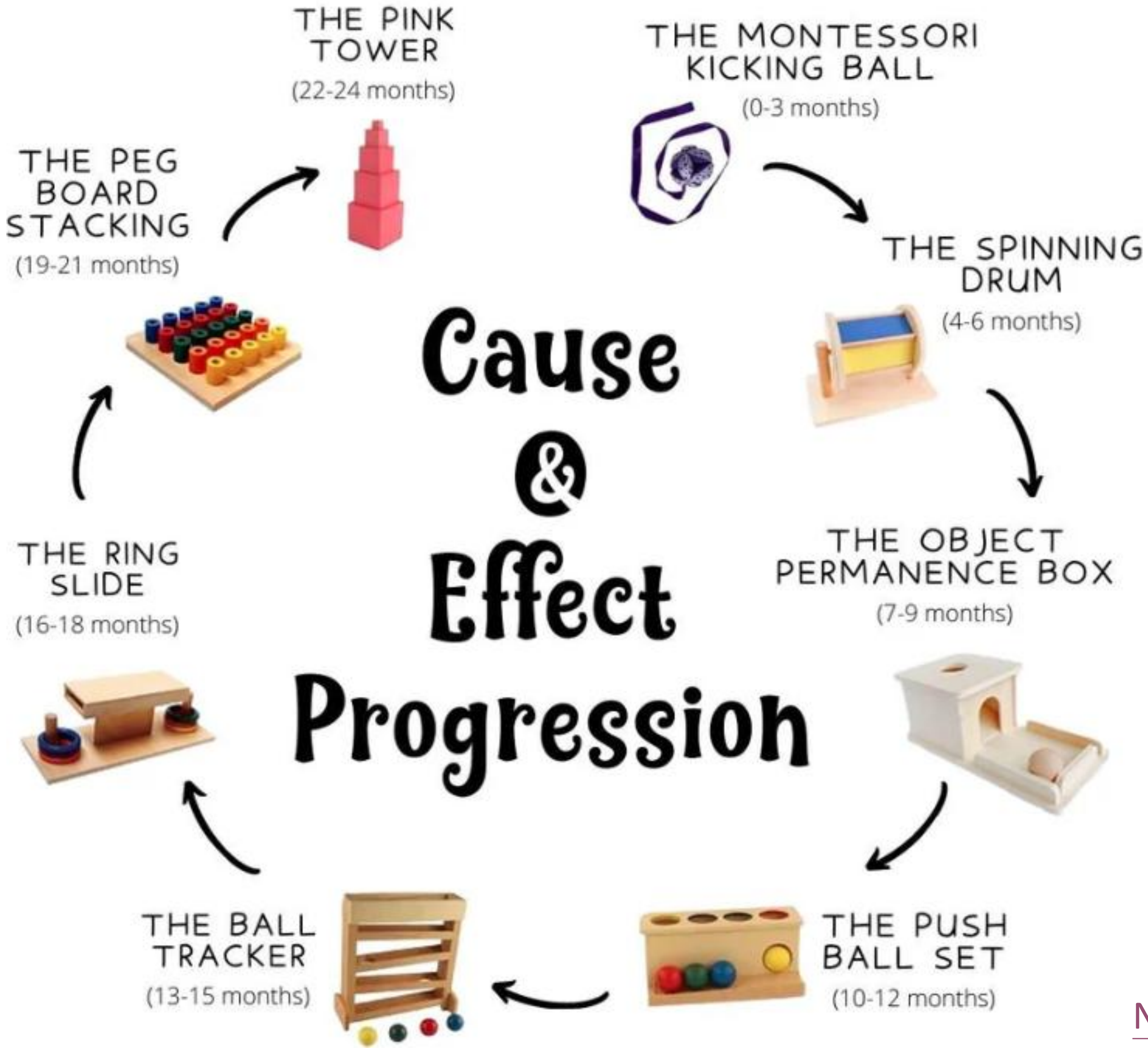
Cause and Effect Toy



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Prediction vs Causation

- Prediction and Causation are very different.
  - Prediction:

    Predict Y after **observing** $A = a \Leftrightarrow P(Y = y|A = a)$

    Ex. Predict health given that a person takes vitamin C
  - Causation:

    Predict Y after **setting** $A = a \Leftrightarrow P(Y(a) = y)$

    Ex. Predict health if I give a person vitamin C.

  In general

$$P(Y = y|A = a) \neq P(Y(a) = y)$$

- Causation (Counterfactual prediction)
  - Involves prediction the effect of an intervention.
- The difference between passively observing $A = a$ and actively intervening and setting $A = a$ is significant and requires different techniques and, typically, much stronger assumptions.

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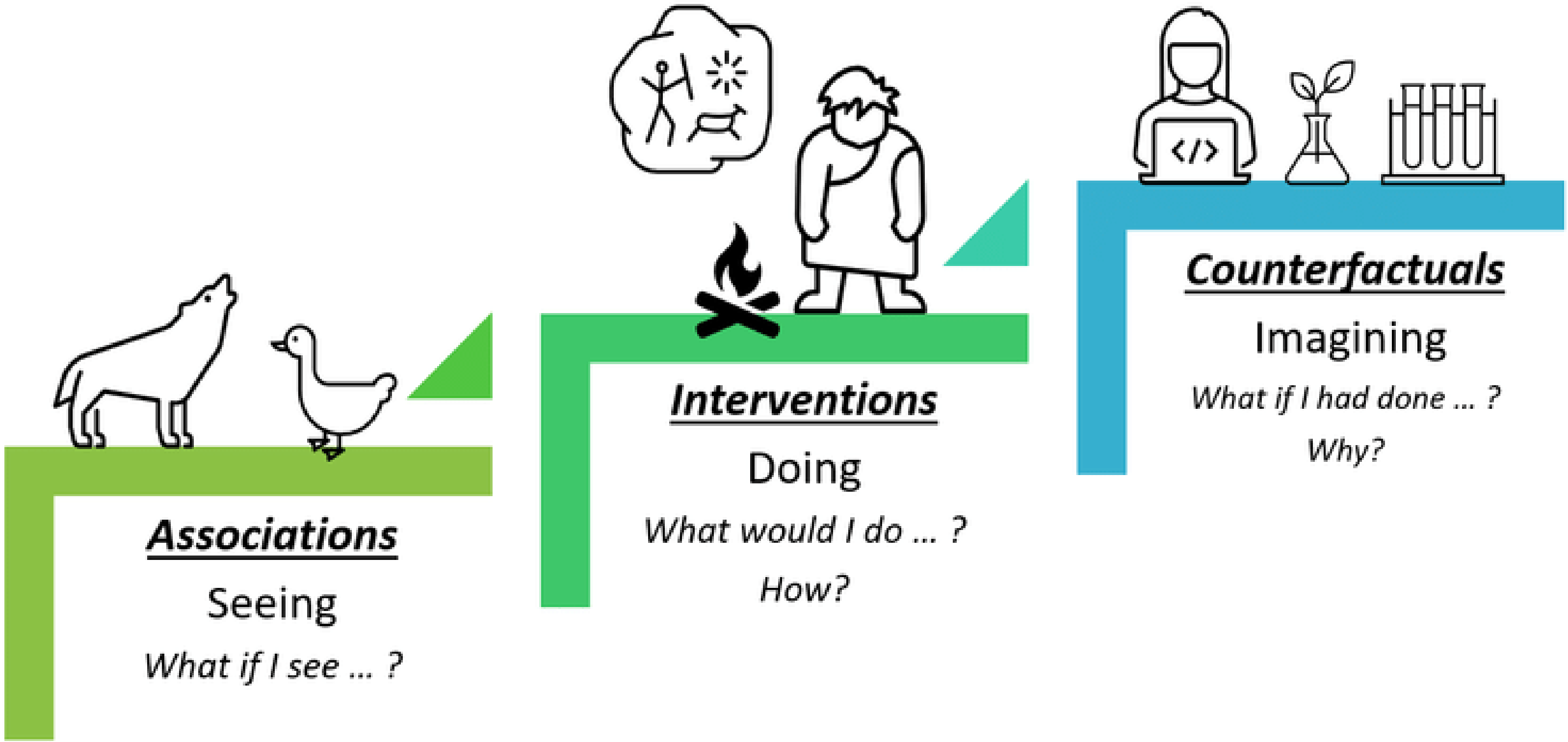
The Ladder of Causations

- A causal learner must master at least three distinct levels of cognitive ability:
  - Seeing, Observing $\Leftrightarrow P(Y = y|A = a)$
  - Doing, Intervening $\Leftrightarrow P(Y(a^*) = y)$
  - Imagining, Retrospection, Understanding $\Leftrightarrow P(Y(a^*) = y|A = a)$



Paradox Galore - 1

- Monty Hall problem (Game Show - Let's Make a Deal)

  There are 3 doors. Behind one door is a car, behind the others, goats. Pick a door, say No.1, and the host, who knows what's behind the doors, opens door No.3, which has a goat. He says to you,

Do you want to pick door No.2?

  Is it to your advantage to switch your choice of doors? **YES**

- Three possible arrangements of doors and goats

Door 1	Door 2	Door 3	Outcome If You Switch	Outcome If You Stay
Auto	Goat	Goat	Lose	Win
Goat	Auto	Goat	Win	Lose
Goat	Goat	Auto	Win	Lose

Always account for variable change.

Bayes' Theorem

$C_i \triangleq$ Car at Door No. i, $D_i \triangleq$ Open Door No. i

- **Prior**: Probability of car behind doors

$$P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}$$

- **Event**: Probability of host to open door No.3

$$P(D_3|C_1) = \frac{1}{2}, \qquad P(D_3|C_2) = 1, \qquad P(D_3|C_3) = 0$$

- **Posterior**: Chance of the car behind the doors after the event.

$$P(C_1|D_3) = \frac{P(D_3|C_1)P(C_1)}{P(D_3|C_1)P(C_1) + P(D_3|C_2)P(C_2) + P(D_3|C_3)P(C_3)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + 1 \times \frac{1}{3} + 0 \times \frac{1}{3}} = \frac{1}{3}$$

Similar way to obtained

$$P(C_2|D_3) = \frac{2}{3}, \qquad P(C_3|D_3) = 0$$

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Paradox Galore - 2

- Simpson's Paradox (**BBG** ≜ Bad/Bad/Good, Drug)

	Control Group (No Drug)		Treatment Group (Took Drug)	
	Heart attack	*No heart attack*	*Heart attack*	*No heart attack*
Female	1	19	3	37
Male	12	28	8	12
Total	13	47	11	49

  - Bad for Female

$$\frac{1}{1+19} = \frac{1}{20} < \frac{3}{40} = \frac{3}{3+37}$$

  - Bad for Male

$$\frac{12}{12+28} = \frac{6}{20} < \frac{8}{20} = \frac{8}{8+12}$$

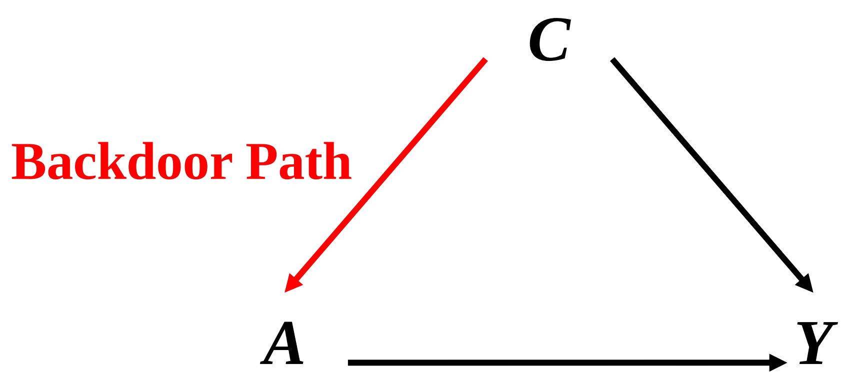
  - Good for People

$$\frac{1+12}{13+47} = \frac{13}{60} > \frac{11}{60} = \frac{3+8}{11+49}$$

- **BBG** drug indeed does not exist and will never be invented.

Blocking the Spurious Paths

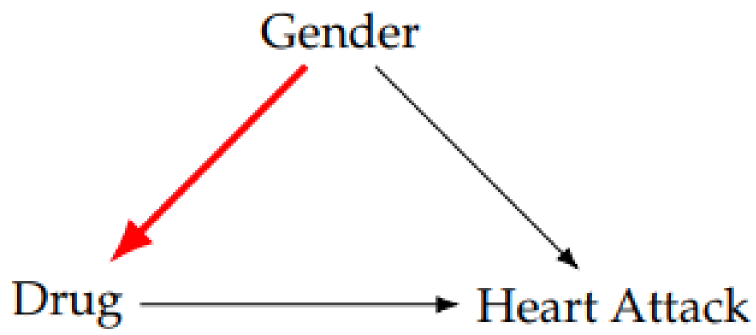
- To deconfound two variables $\boldsymbol{A}$ and $\boldsymbol{Y}$, we need only block every noncausal path between them without blocking or perturbing any causal paths.

- 

- **Backdoor Path**
  - It is any path from $\boldsymbol{A}$ to $\boldsymbol{Y}$ that starts with an arrow pointing into $\boldsymbol{A}$.
- Backdoor Adjustment
  - If we do this by controlling for some set of variables $\boldsymbol{C}$, we also need to make sure that no member of $\boldsymbol{C}$ is a descendant of $\boldsymbol{A}$ on a causal path.

Causal Diagram

- **BBG** causal structure



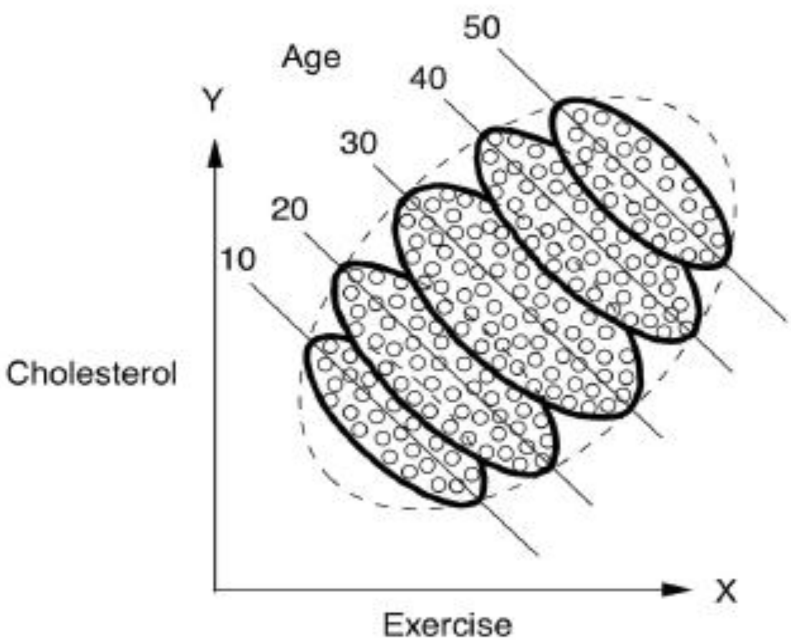
- In the study, women clearly had preference for taking Drug and men preferred not to.
- Thus Gender serves as a confounder/common cause between Drug and Heart Attack.
- For a unbiased estimate of the effect of Drug on Heart Attack, we must adjust for the confounder.
  - Control vs Treatment

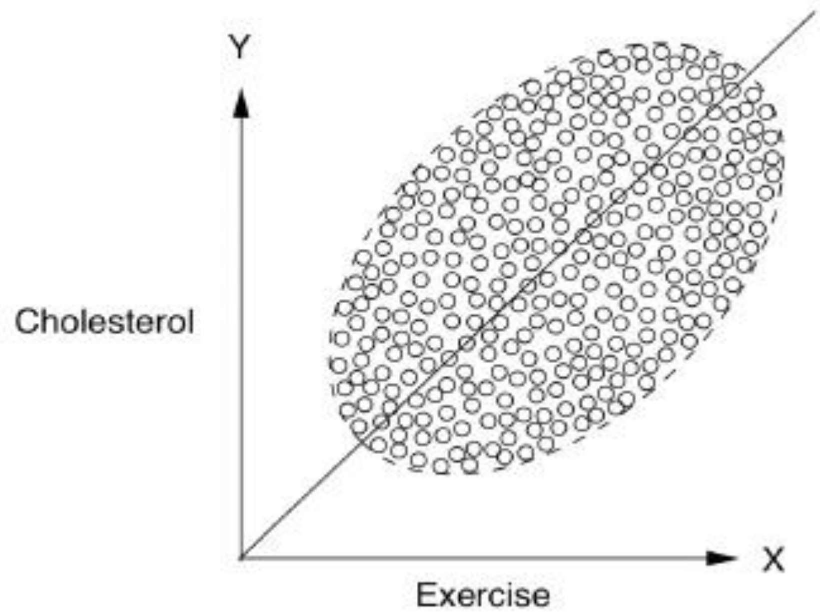
$$\frac{1}{20}+\frac{12}{40}=\frac{14}{40}<\frac{19}{40}=\frac{3}{40}+\frac{8}{20}$$

  - Drug is not **BBG**, it's **BBB**(≙Bad/Bad/Bad).

Paradox Galore - 3

- Consider a study that measures weekly exercise and cholesterol levels in various age groups.

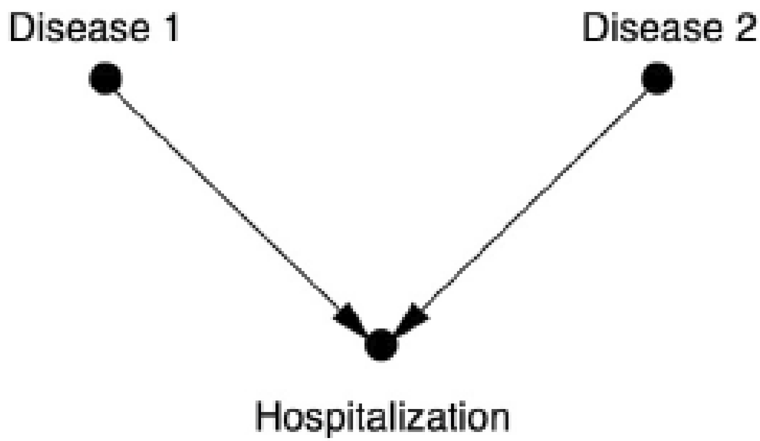




- Once again we seem to have a **BBG** situation
  - It seems to have a beneficial effect in each age group but harmful effect on the population as whole.

Paradox Galore - 4

- Berkson's Paradox



  - It occurs when **selection bias**, also known as collider-stratification bias, creates a spurious association between two independent variables due to conditioning on a common effect (a collider).

	General Population			Hospitalized in Last Six Months		
Respiratory disease? ↓	*Bone disease?* ↓			*Bone disease?* ↓		
	Yes	No	% Yes	Yes	No	% Yes
Yes	17	207	7.6	5	15	25.0
No (control)	184	2,376	7.2	18	219	7.6

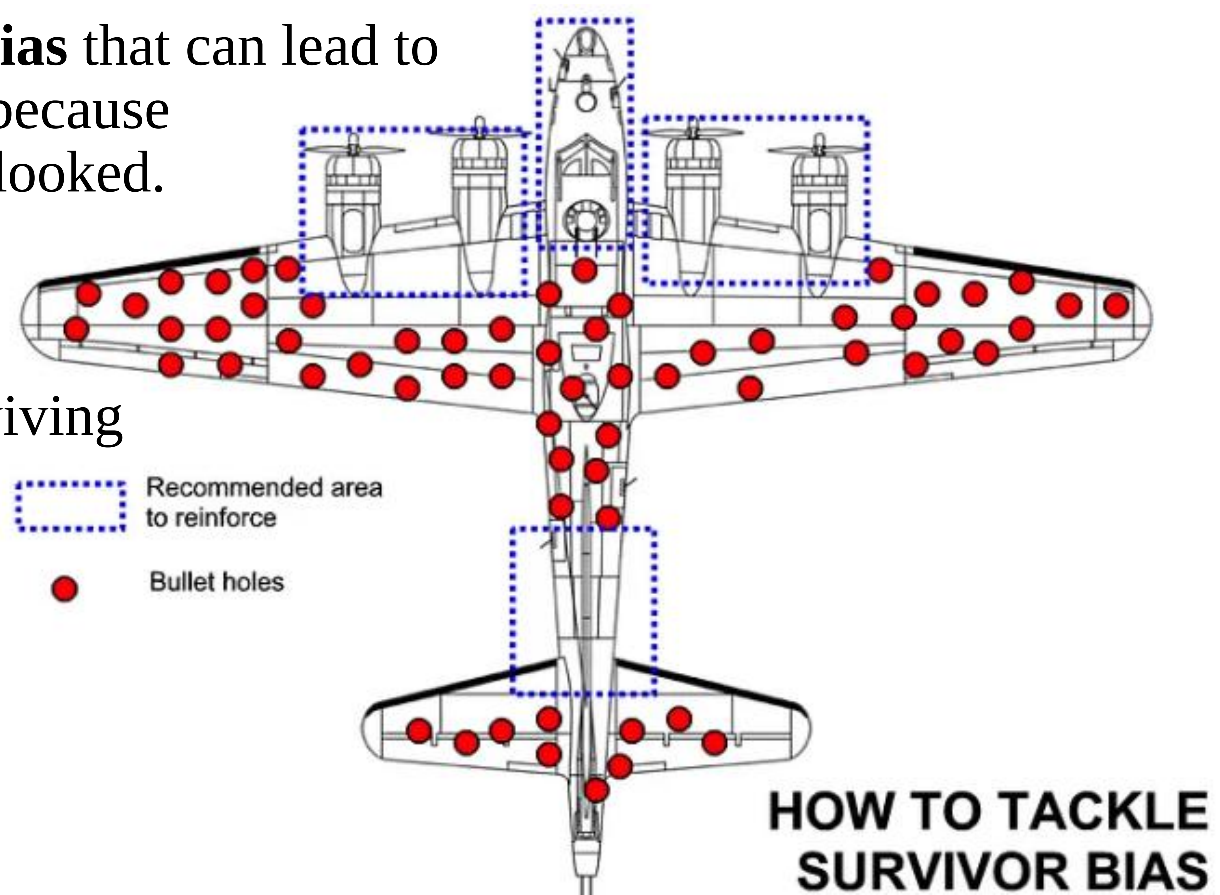
Survivorship Bias

Dead men tell no tales

- It is the logical error of concentrating on entities that passed a selection process while overlooking those that did not.

- It is a form of **selection bias** that can lead to overly optimistic beliefs because multiple failures are overlooked.

- This hypothetical pattern of damage of surviving aircraft shows locations where they can sustain damage and still return home.



Cobra Effect

Unintended Consequences Everywhere

- It refers to a situation when an attempted solution to a problem makes the problem worse.
- When we try to make a single change within a complex system, we often end up causing unintended consequences.

Be Careful what you wish for ~

Source of Bias

- **Causal Bias** =
  - + **Systematic Bias (Hidden Bias)**
    - ∗ Confounding Bias (Simpson's paradox)
    - ∗ Selection Bias (Berkson's paradox)
    - ∗ Measurement Bias
  - + **Misspecification bias**
    - ∗ Due to parametric modeling
  - + **Random Variability**
    - ∗ Finite sample bias

- The distinction between association and causation primarily arises from the need to account for potential systematic biases (hidden biases).