Subject : .... No. : 0

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謬誤

ξ Causal Argument & Causal Fallacies

A cause Y

reason conclusion

Good causal arguments rest on the application of 2 important principles:

1) The Principle of Agreement:
   If A is common factor in multiple occurrences of Y, then A is a cause of Y
2) The Principle of Difference:
   If A is a difference between situations where Y occurs and situations where Y does not occur, then A is a cause of Y.

☆ Post Hoc Fallacy

Def. Asserting that A is a cause of B just because B occurs after A.

(Post Hoc is Latin, and short for "post hoc ergo propter hoc" which means "after this, therefore because of this".
Simply, If Y comes after A, then A caused Y.)

☆ Reversing Cause and Effect

Def. Claiming that A is a cause of B when the evidence suggests or is compatible w/ B being a cause of A.

$A_1 \rightarrow$ Effect
$A_2 \rightarrow$ Effect
$\vdots$
$A_n \rightarrow$ Effect

☆ Causal Oversimplification

Def. Falsely assuming that only one cause is present while there are actually multiple causes is to commit the causal oversimplification fallacy.

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★ Neglecting a Common Cause

Def. Claiming that multiple events have distinct causes when the evidence suggests or is compatible w/ all the events having the same cause.

★ Causal Series

$$A_1 \longrightarrow A_2 \longrightarrow \cdots \longrightarrow A_n$$

災難性

(Innocuous Starting Pt $\longrightarrow$ Disastrous End Pt)

One event leads to another event, and that leads to another event, etc. Regardless, each step needs to be justify the series as a whole.
The "slippery slope fallacy" occurs when a causal series is not justified.

★ The Gambler's Fallacy

Def. Assuming that a random event is due because it hasn't happened in a while.

(Gambler erroneously thinks there's a dependent causal link between the long-run statistical dist^{n} and his individual chances.)

★ Causal Determinism Fallacy

Def. Asserting or denying a causal relationship based on the fact that the proposed cause does not immediately, absolutely, or uniquely determine the effect.

(Deterministic = A absolutely determine B
Non-Deterministic = All things being equal, B is more "probable" when A occurs than when A does not occur)

Double A

Causal Effect (Human)

Outcome $\begin{cases} \text{take action (有做)} \\ \text{vs} \\ \text{not take action (沒做)} \end{cases}$

If the two outcome differ, we say action has causal effect.

A → 因 : Action (Exposure → 暴露 / Risk factor / intervention → 介入 / cause / treatment)

Y → 果 : Outcome of interest

$C = (C_M, C_U)$: confounder / confounding variable / common cause

C_U: Unmeasured

C_M: Measured

C → 干擾因子 / 混淆因子

Variable
- discrete ↔ categorical
  - binary / dichotomous $\begin{cases} 1 & \text{treated} \\ 0 & \text{not treated} \end{cases}$
  - nominal
  - ordinal
- conti

Outcome
- Binary $\begin{cases} 1 & \text{death} \\ 0 & \text{survival} \end{cases}$
- Time to event ⟺ Counting proc.

★ $A = \begin{cases} 1 & \text{smoke} \\ 0 & \text{non-smoke} \end{cases}$, $Y = \begin{cases} 1 & \text{lung cancer} \\ 0 & \text{no cancer} \end{cases}$

	$A=1$	$A=0$
$Y=1$	C_{11}	C_{10}
$Y=0$	C_{01}	C_{00}

Risk → 風險 $\Rightarrow$ $R_a = P(Y=1 \mid A=a)$ $a = 0, 1$

Odds → 勝算比 $\Rightarrow$ $Odd_a = R_a / (1 - R_a)$

關聯

A vs Y : association

(Risk Ratio) $RR = R_1 / R_0$ $\quad\quad$ $RR = 1$

$\Rightarrow$ $\quad\quad$ $\Updownarrow$

(Risk Difference) $RD = R_1 - R_0$ $\quad\quad$ $RD = 0$ $\left(\begin{matrix} R_1 = R_0 \\ Odd_{,1} = Odd_{,0} \end{matrix} \right)$

$\Updownarrow$

(Odds Ratio) $OR = Odd_{,1} / Odd_{,0}$ $\quad\quad$ $OR = 1$

§ Causal Structure

有向非循環圖

DAG

- D ← Directed vs undirected
- A ← Acyclic vs cyclic
- G ← Graph

ancestor/parent

$A \longrightarrow Y$

child/descendant

Def. A directed graph $G = (V, E)$

V: set of vertices/nodes $\Leftrightarrow$ r.v

$E = V \times V$: set of ordered pair of edges/arcs $\Leftrightarrow$ relation

Def. A path in a graph is a seq of vertices $\{v_1, \ldots, v_N\}$ s.t. $(v_i, v_{i+1}) \in E$ for $1 \le i < N$

$v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_N$

Def. A cycle in directed graph is a path of length at least 1 s.t. $v_1 = v_N$

Def. A directed graph is $\begin{cases} \text{acyclic} \\ \text{cyclic} \end{cases}$ if it has $\begin{cases} \text{no cycles.} \\ \text{at least one cycle.} \end{cases}$

Ex $A \xrightarrow[\text{cause}]{} Y$

因 果
parent child

— 早 - - - - - 晚 → Time

B肝 ⟶ 肝癌
↓
3~6月 • 1 cm
↓
∘ 3 cm
↓
◯ 5cm

☆ Temporality
(時序性)

⇒ The effect has to occur after cause.
(and if there is an expected delay between the cause and expected effect, then the effect must occur after that delay)

☆ Wrong Notations

$A \longleftrightarrow Y$ (bi-directed)

$A \text{———} Y$ (undirected)

$$A \text{ — } \begin{cases} \text{——} Y_1 \\ \text{——} Y_2 \\ \text{——} Y_3 \end{cases}$$

$$A \xrightarrow[C]{\quad\quad} Y$$

☆ Acyclic (X)

$A \longrightarrow Y$

$$\begin{tikzcd} \text{chicken} \arrow[r, bend left] & \text{egg} \arrow[l, bend left] \end{tikzcd}$$

obs $A \longrightarrow Y \longrightarrow A \longrightarrow Y$ ------

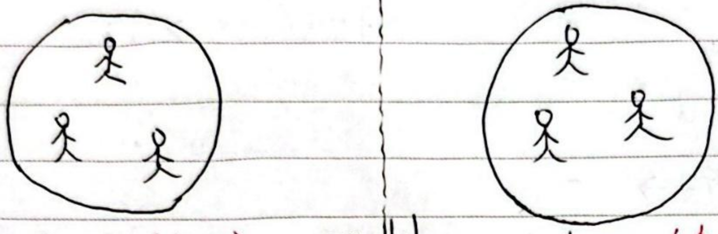
t ⊢ 1 ⊢ 2 ⊢ 3 ⊢ 4 → Time

$\Rightarrow A_{(1)} \longrightarrow Y_{(1)} \longrightarrow A_{(2)} \longrightarrow Y_{(2)}$ ------

Subject : 反事實　潛在結果　

Date : ..../......../.........

ξ Contourfactual / Potential Outcome

$Y(1)$  $Y(0)$

$A=1$ (介入)　parallel universe　$A=0$ (介入)

parenthetical notation $Y(a)$ — Treatment assign

☆ other notation
$P(Y^{A=a}=1)$
$P(Y=1 \mid do(A=a))$

Def. For each individual i (unit)

$Y_i(1)$: the outcome if individual i received the treatment

$(1 \leftrightarrow 0$, individual $\leftrightarrow$ unit, received $\leftrightarrow$ not recieved$)$

Def. The individual-level causal effect is defined as contrast between $Y_i(1)$ and $Y_i(0)$

Ex : $Y_i(1) - Y_i(0)$

(Only one treatment is applied to each individual
Individual-level causal effect is not possible to estimate)

☆ Let $E[Y(1)]$ be the mean of counterfactual outcome had all individuals in the population received treatment. ($1 \leftrightarrow 0$, received $\leftrightarrow$ not received)

for Y is
$$\begin{cases} \text{discrete}, & E(Y(a)) = \sum_y y\,P(Y(a)=y) \\ \text{binary} & P(Y(a)=1) \\ \text{conti} & \int y\, f_{Y(a)}(y)\,dy \end{cases}$$

☆ Other contrast of functionals

Ex. median, variance, hazard, CDF ......

In general, a population causal effect can be defined as contrast of any functional of marginal dist^n of $Y(a)$ and $Y(a^*)$ (different actions or treatments)

Recall DAGs

$$G = (\underset{\text{vertex}}{V}, \underset{\text{edge}}{E})$$

where $V = \{A, C, Y\}$

$E = \{(C, A), (C, Y), (A, Y)\}$

$$\begin{tikzcd} & C \arrow[dl] \arrow[dr] & \\ A \arrow[rr] & & Y \end{tikzcd}$$

(We adopt convention that time flow "left" to "right"
(C, A) mean $C \longrightarrow A$)

Thus C is temporally prior to A and Y

Graphical approach $\longrightarrow$ are intimately linked
Counterfactual approach $\nearrow$

$\Updownarrow$

NPSEM

§ NPSEM (Non-parametric Structual Equation Modeling)

$$\begin{tikzcd} X_1 \arrow[r] \arrow[rr, bend left] \arrow[rrr, bend left] & X_2 \arrow[r] \arrow[rr, bend right] & X_3 \arrow[r] & X_4 \\ \varepsilon_1 \arrow[u] & \varepsilon_2 \arrow[u] & \varepsilon_3 \arrow[u] & \varepsilon_4 \arrow[u] \end{tikzcd}$$

$X_1 = f_1(\varepsilon_1)$

$X_2 = f_2(X_1, \varepsilon_2) = f_2(f_1(\varepsilon_1), \varepsilon_2)$

$X_3 = f_3(X_2, X_1, \varepsilon_3)$

$X_4 = f_4(X_3, X_2, X_1, \varepsilon_4)$

1. $$\begin{tikzcd} A \arrow[r] & Y \\ \varepsilon_A \arrow[u] & \varepsilon_Y \arrow[u] \end{tikzcd}$$

$\varepsilon_A \perp\!\!\!\perp \varepsilon_Y$

$A = g_A(\varepsilon_A)$

$Y = g_Y(A, \varepsilon_Y) = g_Y(g_A(\varepsilon_A), \varepsilon_Y) = f(\varepsilon_A, \varepsilon_Y)$

$\Rightarrow A \not\!\perp\!\!\!\perp Y$

2. $$\begin{tikzcd} A & Y \\ \varepsilon_A \arrow[u] & \varepsilon_Y \arrow[u] \end{tikzcd}$$

$\varepsilon_A \perp\!\!\!\perp \varepsilon_Y$

$A = g_A(\varepsilon_A)$

$Y = g_Y(\varepsilon_Y)$ $\Rightarrow A \perp\!\!\!\perp Y$

3. $$\begin{tikzcd} A \arrow[r] & M \arrow[r] & Y \\ \varepsilon_A \arrow[u] & \varepsilon_M \arrow[u] & \varepsilon_Y \arrow[u] \end{tikzcd}$$

$\varepsilon_A \perp\!\!\!\perp \varepsilon_M$, $\varepsilon_A \perp\!\!\!\perp \varepsilon_M$, $\varepsilon_M \perp\!\!\!\perp \varepsilon_Y$

$A = g_A(\varepsilon_A)$

$M = g_M(A, \varepsilon_M) = g_M(g_A(\varepsilon_A), \varepsilon_M)$

$Y = g_Y(M, \varepsilon_Y) = g_Y(g_M(g_A(\varepsilon_A), \varepsilon_M), \varepsilon_Y)$

$= f(\varepsilon_A, \varepsilon_M, \varepsilon_Y) \Rightarrow A \not\perp\!\!\!\perp Y$

Control (restriction) $M=m$

4. $$\begin{tikzcd} A \arrow[r] & \boxed{M} \arrow[r] & Y \\ \varepsilon_A \arrow[u] & \varepsilon_M \arrow[u] & \varepsilon_Y \arrow[u] \end{tikzcd}$$

$A = g_A(\varepsilon_A)$

$Y = g_Y(m, \varepsilon_A) = f(\varepsilon_Y)$

$\Rightarrow Y \perp\!\!\!\perp A \mid M=m$

5. $$\begin{tikzcd} & & \varepsilon_A \arrow[d] \\ \varepsilon_C \arrow[r] & C \arrow[r] \arrow[dr] & A \\ & & Y \\ & & \varepsilon_Y \arrow[u] \end{tikzcd}$$

$\varepsilon_C \perp\!\!\!\perp \varepsilon_A$, $\varepsilon_C \perp\!\!\!\perp \varepsilon_Y$, $\varepsilon_A \perp\!\!\!\perp \varepsilon_Y$

$C = g_C(\varepsilon_C)$

$A = g_A(C, \varepsilon_A) = g_A(g_C(\varepsilon_C), \varepsilon_A)$

$Y = g_Y(C, \varepsilon_Y) = g_Y(g_C(\varepsilon_C), \varepsilon_Y)$

$\Rightarrow A \not\perp\!\!\!\perp Y$

6. $$\begin{tikzcd} & A \\ \boxed{C} \arrow[ur] \arrow[dr] & \\ & Y \end{tikzcd}$$

$C = c$

$A = g_A(c, \varepsilon_A)$

$Y = g_Y(c, \varepsilon_Y) \Rightarrow A \perp\!\!\!\perp Y$

7. $$\begin{tikzcd} A \arrow[dr] & \varepsilon_A \arrow[l] & \\ & S & \varepsilon_S \arrow[l] \\ Y \arrow[ur] & \varepsilon_Y \arrow[l] & \end{tikzcd}$$

$\varepsilon_A, \varepsilon_Y, \varepsilon_M$ mutually Indep

$A = g_A(\varepsilon_A)$

$Y = g_Y(\varepsilon_Y)$

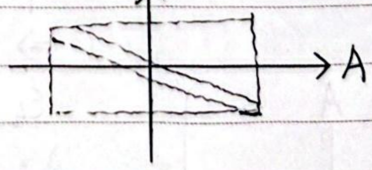
$S = g_S(A, Y, \varepsilon_S) \Rightarrow A \perp\!\!\!\perp Y$

8. $$\begin{tikzcd} A \arrow[dr] & \\ & \boxed{S} \\ Y \arrow[ur] & \end{tikzcd}$$

$\varepsilon_S \sim N(0, 0.1)$

$A \sim U(-3, 3)$

$Y \sim U(-3, 3)$



$s = Y + A + \varepsilon_S \Rightarrow A \not\perp\!\!\!\perp Y \mid S = s$

& d-separation Rule ⇐
↳ directional

NPSEM

To infer associational statements from causal diagrams were formalized.

A : Exposure   C : Confounder
M : Mediator   S : Collider
Y : Outcome

	DAGs	Causality	Association	Independence
1.	$A \longrightarrow Y$	O	O	$A \not\perp\!\!\!\perp Y$
2.	$A \quad Y$	X	X	$A \perp\!\!\!\perp Y$
3.	$A \rightarrow M \rightarrow Y$	O	O	$A \not\perp\!\!\!\perp Y$
4	$A \rightarrow \boxed{M} \rightarrow Y$	O	X	$A \perp\!\!\!\perp Y \mid M$
5.	$A \leftarrow C \rightarrow Y$	X	O	$A \not\perp\!\!\!\perp Y$
6.	$A \leftarrow \boxed{C} \rightarrow Y$	X	X	$A \perp\!\!\!\perp Y \mid C$
7.	$A \rightarrow S \leftarrow Y$	X	X	$A \perp\!\!\!\perp Y$
8.	$A \rightarrow \boxed{S} \leftarrow Y$	X	O	$A \not\perp\!\!\!\perp Y \mid S$

5*

$\varepsilon_C \rightarrow C$, $C \rightarrow A \leftarrow \varepsilon_A$, $C \rightarrow Y \leftarrow \varepsilon_Y$, $C \rightarrow \boxed{Z}$

$\varepsilon_C, \varepsilon_A, \varepsilon_Y, \varepsilon_Z \perp\!\!\!\perp$

$C = g_C(\varepsilon_C)$

$A = g_A(C, \varepsilon_A) = f_A(\varepsilon_C, \varepsilon_A) \Rightarrow A \not\perp\!\!\!\perp Y \mid Z$

$Y = g_Y(C, \varepsilon_Y) = f_Y(\varepsilon_C, \varepsilon_Y)$

$Z = g_Z(C, \varepsilon_Z) = f_Z(\varepsilon_C, \varepsilon_Z)$

8*

$$\begin{tikzcd} & \varepsilon_A \arrow[d] & & & \\ & A \arrow[r] & S \arrow[r] & \boxed{L} & \\ & & \varepsilon_S \arrow[u] & \varepsilon_L \arrow[u] & \\ & Y \arrow[uur] & & & \\ & \varepsilon_Y \arrow[u] & & & \end{tikzcd}$$

$$\begin{aligned} A &= g_A(\varepsilon_A) \\ Y &= g_Y(\varepsilon_Y) \\ S &= g_S(A, Y, \varepsilon_S) \\ L &= g_L(S, \varepsilon_L) \\ &= g_L(g_S(A, Y, \varepsilon_S), \varepsilon_L) \\ &= f(A, Y, \varepsilon_S, \varepsilon_L) \\ \Rightarrow\ & A \not\!\perp\!\!\!\perp Y \mid L \end{aligned}$$

	DAGs	Causality	Association	Independence
7*	$A \to S \to L$, $Y \to S$	X	X	$A \perp\!\!\!\perp Y$
8*	$A \to S \to \boxed{L}$, $Y \to S$	X	O	$A \not\!\perp\!\!\!\perp Y \mid L$
5*	$C \to A$, $C \to Y$, $C \to \boxed{Z}$	X	O	$A \not\!\perp\!\!\!\perp Y \mid Z$

3. $A \longrightarrow M \longrightarrow Y$

+	+	+
+	−	−
−	+	−
−	−	+

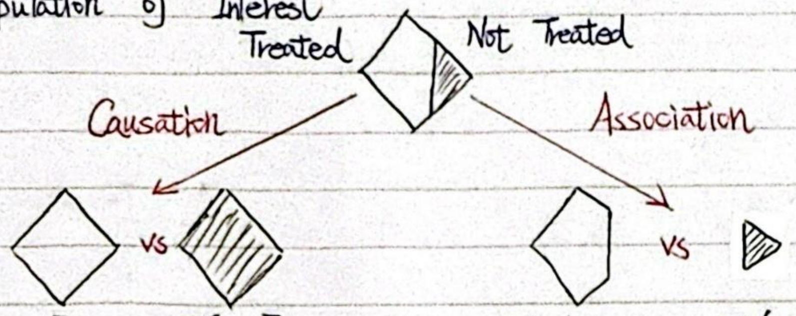
5. $A \xleftarrow{\text{sign1}} C \xrightarrow{\text{sign2}} Y$

sign 1	sign 2.	A vs Y
+	+	+
+	−	−
−	+	−
−	−	+

8. $A \xrightarrow{\text{sign 1}} S \xleftarrow{\text{sign2}} Y$

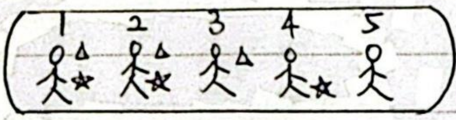
sign 1	sign 2	A vs Y
+	+	−
+	−	+
−	+	+
−	−	−

★ Population of Interest



$\mathbb{E}[Y(1)]$ $\mathbb{E}[Y(0)]$ $\mathbb{E}(Y|A=1)$ $\mathbb{E}(Y|A=0)$

Let $Y \begin{cases} 1 & \text{Survival} \\ 0 & \text{Death} \end{cases}$ and $A = \begin{cases} 1 & \text{Vaccine} \\ 0 & \text{No.} \end{cases}$



$A=1$ (★)
$Y=1$ (△)

★ Causal Types

1. Doomed, $Y(1)=Y(0)=0$ (註定会死)
2. Immune. $Y(1)=Y(0)=1$ (天選之人)
3. Harmed $Y(1)=0, Y(0)=1$ (不作死就不会死)
4. Responsive $Y(1)=1, Y(0)=0$ (有拜有保庇)

id	A (treatment state)	Y (← observed outcome)	Y(1)	Y(0)	Type	CE.
1	1	1	1	? 1	Immune	×
2	1	1	1	? 0	Responsive	△
3	0	1	0 ?	1	Harmed	▽
4	1	0	0	? 0	Doomed	×
5	0	0	0 ?	0	Doomed	×

Actual (My Table) — A, Y
Counterfactual (God Table) — Y(1), Y(0)

Causal Effect $A \longrightarrow Y$

$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$
$= P(Y(1)=1) - P(Y(0)=1) = \frac{2}{5} - \frac{2}{5} = 0$

Association

$\mathbb{E}(Y|A=1) - \mathbb{E}(Y|A=0)$
$= P(Y=1|A=1) - P(Y=1|A=0) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$