Homework #2 Solutions

1. (5 points) $\mathbf{c} = \begin{bmatrix} 3m & 0 & 4m \end{bmatrix}^T$ (5 points) $\boldsymbol{\omega} = \begin{bmatrix} 0 & 5rad/s & 0 \end{bmatrix}^T$ (5 points) $p = 2m/rad$
   (5 points) The twist is a screw

2. (80 points) Note: These are not arranged in order from smallest to largest natural frequencies

```
eigenvectors =

   0.959117273755343   0.283407662678229   0.180822993645531  -0.502338275926124   0.005839103327805  -0.018800714185216
   0.156571345987761   0.050150435337831  -0.429435530663137   0.832162797196003   0.994615026227860  -0.939971427783985
   0.100786073748209  -0.931474129971036   0.855571384694033   0.186874075090716  -0.000889530911273   0.027492474694423
   0.018763425526247  -0.210843661029664   0.203409984348178   0.021358724477081  -0.034380462776627  -0.327305959865229
   0.020349896309896   0.034362425754472  -0.081841816837110  -0.031149509446643   0.000236303863219  -0.003078860029785
  -0.211318365791563  -0.062330971958312  -0.053028963313899   0.137160229335724   0.097590937840502  -0.090585546245843


natfreq =

   1.0e+04 *

   1.726782762691482                   0                   0                   0                   0                   0
                   0   1.665545068977042                   0                   0                   0                   0
                   0                   0   0.349916557548455                   0                   0                   0
                   0                   0                   0   0.204080367696285                   0                   0
                   0                   0                   0                   0   0.014755147625217                   0
                   0                   0                   0                   0                   0   0.015614056434440
```

The MATLAB code that generated this is:

```
function [eigenvectors,natfreq]=Homework2()
format long;
elements=2; %number of elements
stages=1; %number of s tages
Topology=zeros(elements,14);
RigidBody=zeros(stages,13);
E=6.8e10;
G=2.5e10;
den=2698.9;
%Define Flexure Topology
Topology=[ [(.0025/2) (.05+.15) (.05-(.0025/2))] [0 1 0] [0 0 1] .15 .0025
.0025 E G;
    [(.1-(.0025/2)) (.05+.15) (.05/2)] [0 1 0] [1 0 0] .15 .05 .0025 E G];

Mass=den*.05*.05*.1; %total mass
CenterMass=[.05 (.05+.15+.025) .025]; %center of mass
MI1=Mass*(((.05)^2)+((.05)^2))/12; %moment of inertia about an axis parallel
to the Xaxis about center of mass
MI2=Mass*(((.05)^2)+((.1)^2))/12; %moment of inertia about an axis parallel
to the Yaxis about center of mass
MI3=Mass*(((.05)^2)+((.1)^2))/12; %moment of inertia about an axis parallel
to the Zaxis about center of mass
```

```
%Define Rigid Body
RigidBody=[ CenterMass Mass [1 0 0] [0 1 0] MI1 MI2 MI3 ];

%Construct Stiffness matrix
K=zeros(6,6);
for i=1:elements;
    N=zeros(6,6);
    n3=Topology(i,4:6);
    n3=n3/sqrt(dot(n3,n3));
    n2=Topology(i,7:9);
    n2=n2/sqrt(dot(n2,n2));
    n1=cross(n2,n3);
    N(1:3,1)=transpose(n1);
    N(1:3,2)=transpose(n2);
    N(1:3,3)=transpose(n3);
    N(4:6,4)=transpose(n1);
    N(4:6,5)=transpose(n2);
    N(4:6,6)=transpose(n3);
    N(4:6,1)=transpose(cross(Topology(i,1:3),n1));
    N(4:6,2)=transpose(cross(Topology(i,1:3),n2));
    N(4:6,3)=transpose(cross(Topology(i,1:3),n3));
    NR=zeros(6,6);
    NR(1:6,1:3)=N(1:6,4:6);
    NR(1:6,4:6)=N(1:6,1:3);
    ss=zeros(6,6);
    E=Topology(i,13);
    G=Topology(i,14);
    L=Topology(i,10);
    W=Topology(i,11);
    T=Topology(i,12);
    I1=W*(T^3)/12;
    I2=T*(W^3)/12;
    Temp=0;
    if(W>T)
        for n=1:2:100
            Temp=Temp+(tanh(n*pi*W/(2*T))/(n^5));
        end
            J=((T^3)*W/3)*(1-((192*T/((pi^5)*W))*Temp));
    else
        for n=1:2:100
            Temp=Temp+(tanh(n*pi*T/(2*W))/(n^5));
        end
            J=((W^3)*T/3)*(1-((192*W/((pi^5)*T))*Temp));
    end
    A=W*T;
    ss(1,1)=L/(E*I1);
    ss(1,5)=-(L^2)/(2*E*I1);
    ss(2,2)=L/(E*I2);
    ss(2,4)=(L^2)/(2*E*I2);
    ss(3,3)=L/(G*J);
    ss(4,2)=(L^2)/(2*E*I2);
    ss(4,4)=(L^3)/(3*E*I2);
    ss(5,1)=-(L^2)/(2*E*I1);
    ss(5,5)=(L^3)/(3*E*I1);
    ss(6,6)=L/(E*A);
    S=inv(ss);
    K1=NR*S*inv(N);
```

```
    K=K+K1;
end

%Construct Mass matrix
M=zeros(6,6);
N=zeros(6,6);
n1=RigidBody(1,5:7);
n1=n1/sqrt(dot(n1,n1));
n2=RigidBody(1,8:10);
n2=n2/sqrt(dot(n2,n2));
n3=cross(n1,n2);
N(1:3,1)=transpose(n1);
N(1:3,2)=transpose(n2);
N(1:3,3)=transpose(n3);
N(4:6,4)=transpose(n1);
N(4:6,5)=transpose(n2);
N(4:6,6)=transpose(n3);
N(4:6,1)=transpose(cross(RigidBody(1,1:3),n1));
N(4:6,2)=transpose(cross(RigidBody(1,1:3),n2));
N(4:6,3)=transpose(cross(RigidBody(1,1:3),n3));
NR=zeros(6,6);
NR(1:6,1:3)=N(1:6,4:6);
NR(1:6,4:6)=N(1:6,1:3);
mm=zeros(6,6);
mm(1,1)=RigidBody(1,11);
mm(2,2)=RigidBody(1,12);
mm(3,3)=RigidBody(1,13);
mm(4,4)=RigidBody(1,4);
mm(5,5)=RigidBody(1,4);
mm(6,6)=RigidBody(1,4);
M=NR*mm*inv(N);

[eigenvectors eigenvalues]=eig(inv(M)*K);
natfreq=sqrt(eigenvalues);
end
```