Testing the Fresnel equations: Measuring the reflectance from a quartz-air interface for linearly polarized light

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Using a simple setup we aim to verify the Fresnel equations experimentally by measuring the intensity of s and p polarized light using a photo-detector which was reflected off a quartz-air interface having passed through a polarizer of known transmission axis. By plotting the angle of incidence versus the reflectance for external and internal reflection we compare the experimentally obtained and theoretically calculated graph curves. A good agreement can be seen between the two in terms of the overall shape and expected trend. The experimental data looks to lay above the theoretical one in the case of external reflection. Whereas, the converse in seen for internal reflection as the experimental graphs lays below. In addition,the critical and Brewster's angle were measured and used to calculate the refractive index of the quartz and compared to the known value where a percent discrepancy of 1.57 and 12.3 is seen for external and internal reflection respectively which is within the expected uncertainty. These results indicate that the Fresnel equations correctly describe the reflectance through an interface between different medium.

I. INTRODUCTION

Light is a transverse electromagnetic wave. Therefore, it has an electric and magnetic field (mutually perpendicular to each other) oscillating along it. When light is emitted from a natural source such as a star it is usually unpolarized meaning that the electric field may get geometrically oriented in any direction with respect to time[7]. When the geometric orientation of the electric field is constant with respect to time,it is called linearly polarized light[2].

From the Laws of optics it is known that the incident beam,reflected/refracted beam and the normal all lie on the same plane[7]. For linearly polarized light, When the beam of light is moving in this plane and the electric field is oscillating perpendicular to the plane, perpendicular or S-type polarized is produced. Whereas if the oscillation of the electric field is along the direction of motion of the beam in the plane, the light is parallel or P-type polarized [2].

The laws of optics mainly describe the angle between the incident and reflected or refracted beam but fail to describe the change in intensity or what percent of the incident light gets reflected or transmitted through.This is described by the Fresnel equations[6] which we aim to verify experimentally using graphs and some special cases defined by it as mentioned below.

When light is incident on a surface it may get reflected, refracted or both depending on the type of surface and the polarization of light. The reflectance R is a ratio of intensities[3] described mathematically as-

$$R_{i,o} = \frac{I_r}{I_i} \tag{1}$$

Where I_r is the reflected intensity and I_i is the incident intensity from the laser. i and o correspond to p-type and s-type polarization respectively. This difference in reflectance based on polarization is due to the fact that reflected light is partially polarized because s-type polarized light is preferentially reflected and p-type is preferentially refracted (the reasons for this are beyond this text and have not been discussed) [4]. For the purpose of this experiment the incident intensity will be taken as highest reflected intensity measured as opposed to directly pointing the laser into the photo-detector which in theory should be the same.Transmittance T is similarly defined to reflectance as ratio of intensities but can be written more usefully when relating to reflectance as written below.The symbols have the same meanings as described previously.

$$T_{i,o} = 1 - R_{i,o} \tag{2}$$

As mentioned before the Fresnel equations describe the Reflectance and transmission of light from a surface and are described mathematically as-

$$R_i = (\frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t})^2 \tag{3}$$

Where R_i denotes the reflectance for p-type polarization and R_o denotes the reflectance for s-type polarization.[5]

$$R_o = (\frac{n_i cos\theta_i - n_t cos\theta_t}{n_i cos\theta_i + n_y cos\theta_t})^2 \tag{4}$$

n_i and n_t denote the refractive indexes of the incident medium and reflected medium. When the beam travels from a rarer to a denser medium (Air to quartz), external reflection takes places. Whereas internal reflection takes place when it goes from a denser to a rarer medium(Quartz to air)[2]. It should also be noted that the Q_i and Q_t are the angles of incidence and refraction that are related using Snell's law for optics which states that "The ratio of sine of angle of incidence to that of refraction is equal to the ratio of the refractive indexes".[7] or-

$$n_i sin\theta_i = n_t sin\theta_t \tag{5}$$

In internal reflection the beam of light bends away from the normal. Therefore with reference to equation 5, as the angle of incidence is increased a point approaches where the refracted beam is perpendicular to the normal. This is called the critical angle and it signifies the point after which all light will be reflected[7]. This is given by-

$$\theta_c = sin^{-1}(\frac{n_t}{n_i}) \tag{6}$$

According to equation 3, when the angle of incidence is increased a special case occurs for p-type polarization called the Brewster's angle. At this point the reflectance $R_i = 0$ which signifies that all the incident light is transmitted. Using equation 5 and the fact that at Brewster's angle the refracted and reflected rays are perpendicular ($sin\theta_t = cos\theta_i$) an equation for Brewster's angle is obtained by solving it using some trigonometry[2]-

$$\theta_B = tan^{-1}\frac{n_t}{n_i} \tag{7}$$

By experimentally measuring the critical and Brewster's angle we determine the refractive index of the quartz and compare it to the tabulated value.Then further draw conclusion about the Fresnel equations.

II. EXPERIMENTAL SETUP AND METHOD

A beam of light from a laser (∼650 nm wavelength) which was passed through a rotatable polarizer was incident on a quartz crystal (fused silica) of known refractive index. The reflected light from the crystal was measured using a photo-detector as shown in the diagram below.

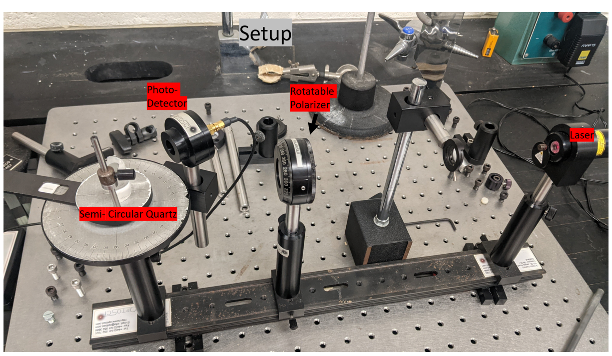

FIG. 1. Experimental setup

A semi-cylindrical quartz was used because the reflection always takes place at the flat surface (as the ray enters perpendicular to the curved surface irrespective of the incidence angle). Therefore, both internal and external reflection from the same flat surface is measured. The beam of light first enters the curved surface but the main reflection only takes place at the flat surface. Thus, we measure light going from a denser to a rarer medium (internal reflection). Similarly, when the normal to the flat surface faces towards the laser we measure external reflection. There is some unwanted loss of intensity due to a very minimal reflection at the curved surface. This does not affect the measurements done for external reflection in any way as we do not measure the intensity of the transmitted beam which has a loss after the reflection at the flat surface. Whereas, for internal reflection as the light enters the quartz through the curved surface a small loss takes place before the beam reaches the flat surface. Given the simple nature of the setup of this experiment this negligible loss at the curved surface has not been taken into account and is considered insignificant.

The rotatable polarizer shown in the figure was first calibrated using the fact that the light reflected off the hallway floor is preferentially p-polarized given that the normal plane is perpendicular to the surface. Rotating the dial on the polarizer to a point of minimum visible light (normal plane parallel to the floor), s-type polarization was found. This calibration was confirmed using an analyzer of which the transmission axis was known. A beam of light was passed through the analyzer of p-type polarization which then passed through the rotatable polarizer before being incident on a sheet of paper. By slightly adjusting the dial to a see a point of minimum light on the sheet of paper the calibration of the rotatable polarizer was confirmed. After data was recorded for s-type polarization the dial was rotated 90 degrees clock-wise from the arbitrary mark of 52 degrees on the polarizer to get p-type polarization for which the data was then recorded.

The procedure mainly consisted of increasing the angle of incidence by about 5 degrees and recording the corresponding reflectance as measured by the photo-detector in a dark room without any ambient light incident on the photo-detector. This procedure was done for s and p-type polarizations in internal and external reflection (4 data sets).

III. RESULTS

The results table summarises the finding on the special cases of critical and Brewster's angle. By using the experimentally found value of Brewster's angle we calculate the refractive index of the quartz using equation 7 and compare this to the tabulated value which is 1.457 [1] and good agreement can be seen as both are within the expected uncertainty range given simple setup of this experiment.

In the graph for external reflection, it can be seen that the theoretical plot reaches 100 percent reflectance at 90

Polarization and reflection	θ_B (Exp.)	θ_B (Theo.)	n (Calc.)	% dsp. (n)	θ_c (Calc.)	% dsp. (θ_c)
P-type in Ext. reflection	56	55	1.48	1.57	-	-
P-type in Int. reflection	39	35	1.28	12.3	46.2	6.59

TABLE I. Results table. Please note Dsp. stands for Discrepancy. As seen above the tabulated and experimental values are within the expected values of each other.

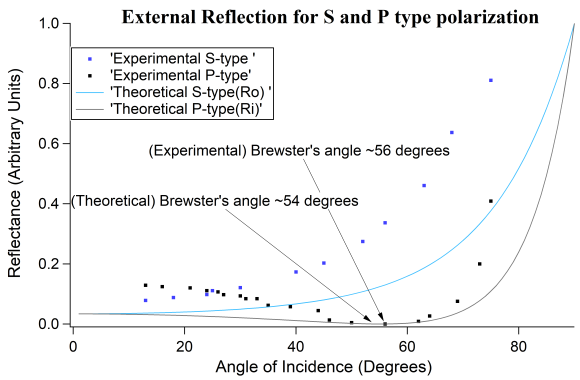

FIG. 2. External reflection (rarer to denser medium) curves for both s and p polarization.Blue color indicates s-type polarization and black/grey indicates p-type.The dots show the experimental curve and the lines are the theoretical curves given by equation 3 and 4. As seen above the experimental data lays above the theoretical curve but similar monotonically increasing trend for s-type is seen and expected shape for p-type polarization is seen.

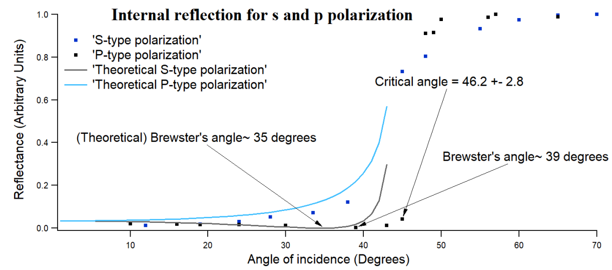

FIG. 3. Internal reflection (denser to rarer medium) curves for both s and p polarization.Blue color indicates s-type polarization and black/grey indicates p-type. The dots show the experimental curve and the lines are the theoretical curves given by equation 3 and 4. As seen the experimental data lays below the theoretical curve but similar monotonically increasing trend for s-type polarization is seen and expected corresponding shapes for both polarizations are seen.

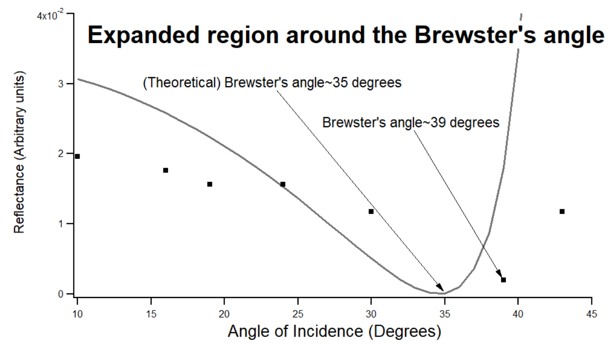

FIG. 4. Brewster's angle for p-polarized light in internal reflection. The reflectance approaching 0 then increasing demonstrates the correctness of equations 3 and 4.

degrees whereas the experimental one is near 70 degrees. This "offset" is due to the fact that angles greater than 70 degrees could not be measured as the quartz stand obstructed the reflected beam. As mentioned earlier the incident Intensity was taken as the maximum reflected intensity measured. This does not affect our results as evident from the overall similar shape of the experimental and theoretical graph curves.Therefore, it can be concluded that equations 3 and 4 correctly predict the behavior for external reflection.

For Internal reflection. In s-type polarization we can see a good agreement between the two plots as the monotonic trend up till the critical angle matches the expected theoretical plot shown. As mentioned in theory total internal reflection takes place after the critical angle and the so the points have only been plotted up till this for the theoretical plot. For p-type polarization, the expanded region shows the reflectance going to zero and the corresponding points have been analyzed in the results table. The overall trend of the graph is seen to be in trend until the critical angle. However, the region after the critical angle should be a straight horizontal line at 100 percent reflectance. As seen in the graph this is not what was observed. The probable reason for explaining this is the beam gets somewhat dispersed before reaching the photodetector and the intensity decreases slightly. At higher angles such as those greater than 60 degrees this was corrected for by placing a lens in front of the detector and as seen the expected horizontal line was observed.

IV. CONCLUSION

An experiment done to test the Fresnel equations for s and p polarization in internal and external reflection using a quartz crystal of known refractive index as an interface shows that the reflectance of light behaves as predicted theoretically by equation 3 and 4. The relevant experimentally plotted and theoretically plotted graphs follow the same trend and are in agreement. The cases of Brewster's and critical angle are within the experimental uncertainty bounds as expected given the simple setup of the experiment as evident in the results table. Therefore, it can be concluded that the Fresnel equation correctly describe the intensity of reflected and refracted light at an interface between mediums of different refractive index.

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