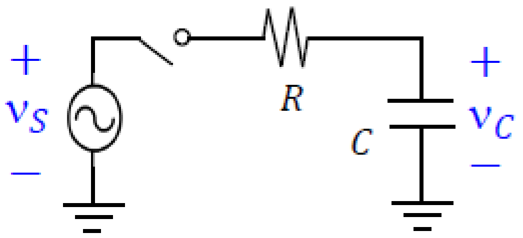
Part III: Frequency Response

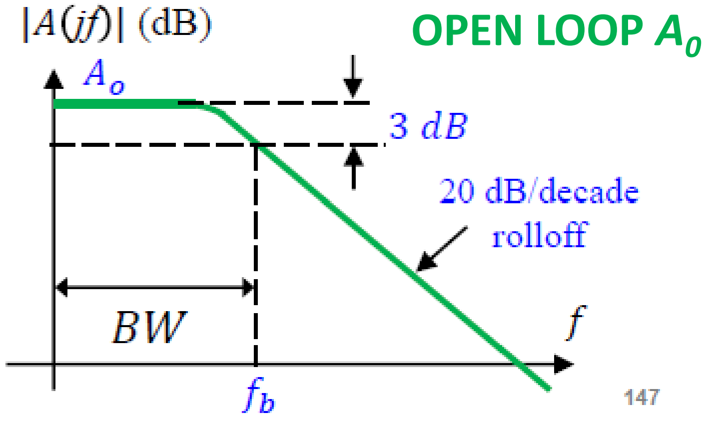
AY23-24s2

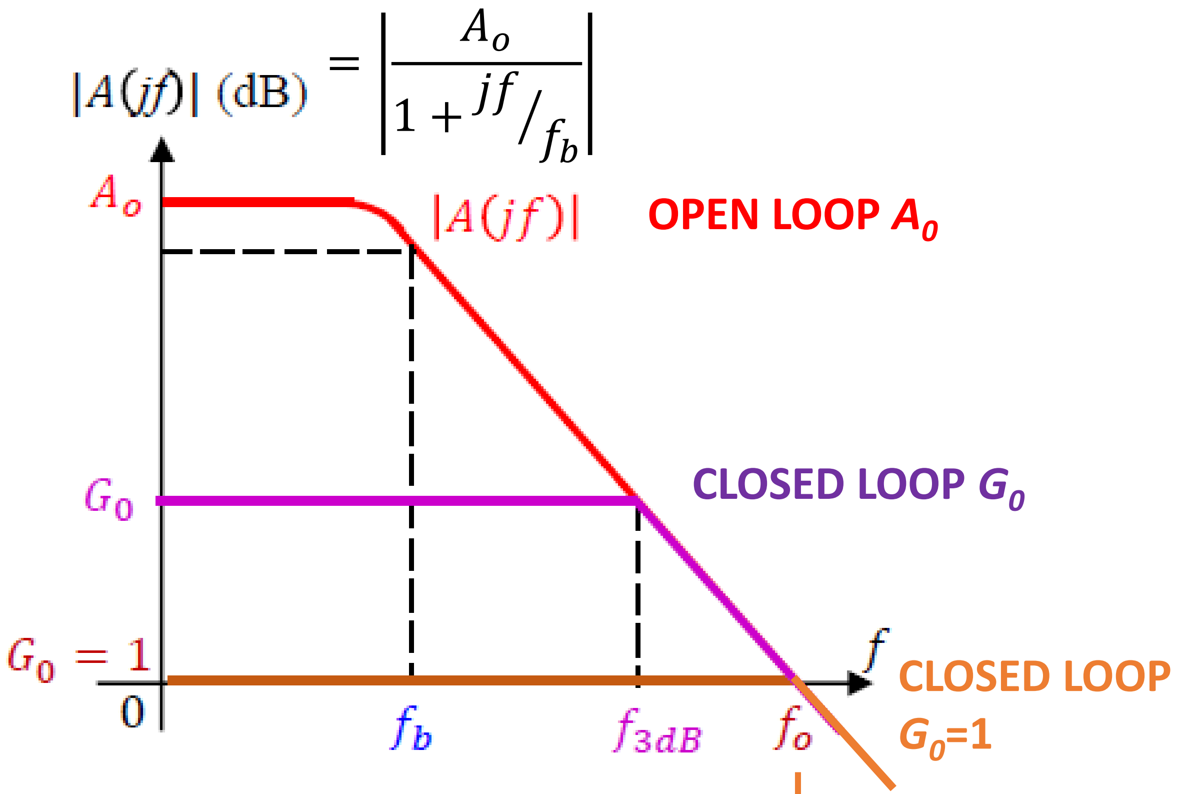
Review from Part I

- The RC circuit has a BW of

$$\boxed{f_{3dB} = \text{BW} = \frac{1}{2\pi RC}}$$







$$G_0 f_{3dB} \approx A_o f_b$$

$$(1) f_o \approx A_o f_b$$

$$f_o \approx A_o f_b$$

= GBWP (const)
e.g. 1MHz for OPA344

finite BW = LOW PASS f_{3dB}

Review from Part II (L2102C)

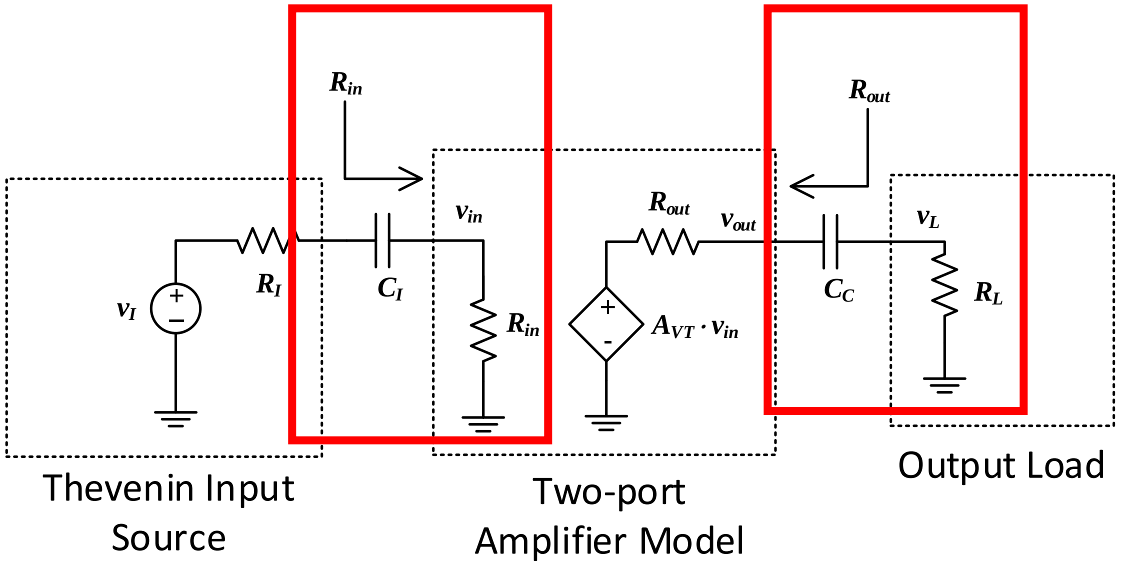
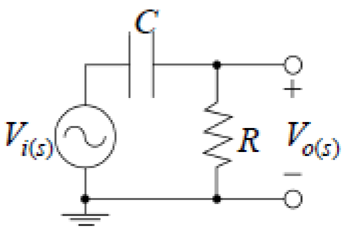


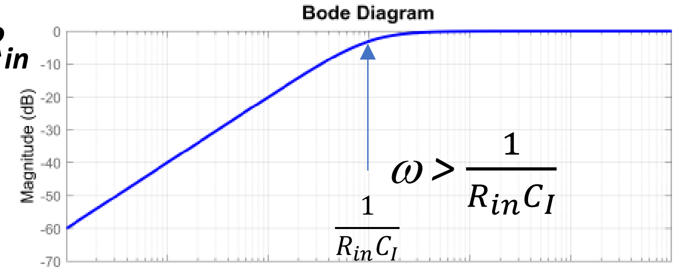
Fig. 9. A general Two-Port Amplifier Model with R_{in}, A_{VT} and R_{out}.

- We had assumed all capacitors had "infinite value"
- That really means = "large enough to be AC short circuits," e.g. 1F
- In practice: how large is enough?

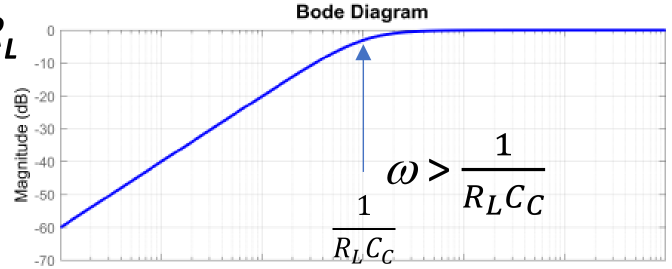
Swap the capacitor with the resistor:



$X_{C_I} < R_{in}$

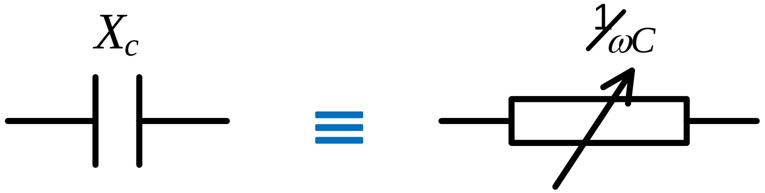


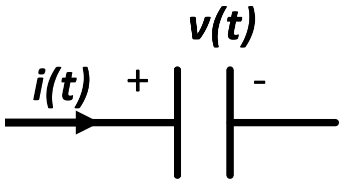
$X_{C_C} < R_L$



AC coupling = HIGH PASS f_{3dB}

Start with the understanding the capacitor





$$\boxed{\begin{array}{l} \dfrac{d}{dt} \xrightarrow{\mathcal{L}} s \\ \displaystyle\int dt \xrightarrow{\mathcal{L}} \frac{1}{s} \end{array}}$$

$$i(t) = C \cdot \frac{dv(t)}{dt} \xrightarrow{\mathcal{L}} I(s) = C \cdot sV(s)$$

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{sC}$$

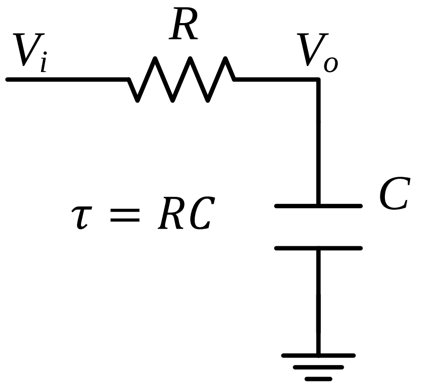
Laplace: s = ℂ-plane

$$v(t) = \int \frac{i(t)}{C}\,dt \xleftarrow{\mathcal{L}^{-1}} V(s) = \frac{1}{sC} \cdot I(s)$$

$$Z(s = j\omega) = \frac{1}{j\omega C}$$

Fourier: $s = j\omega$ (Imag. Axis)
→ Bode plot

Canonical LPF



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/_{sC}}{R + 1/_{sC}} = \frac{1}{RCs + 1}$$

$$H(s) = \frac{1}{\tau s + 1}\bigg|_{\tau = RC}$$

$$H(s) = \frac{1}{s/_{\omega_p} + 1}\bigg|_{\omega_p = \frac{1}{RC}}$$

$\omega_p = 1/\tau$ = Pole *Frequency* (rad/s)

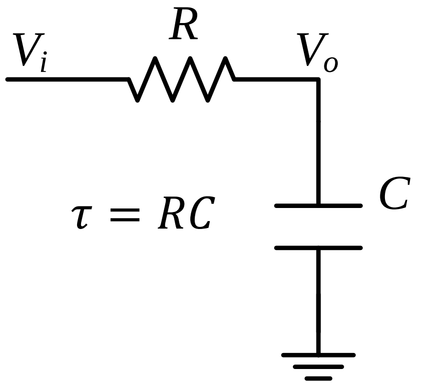
Negative!

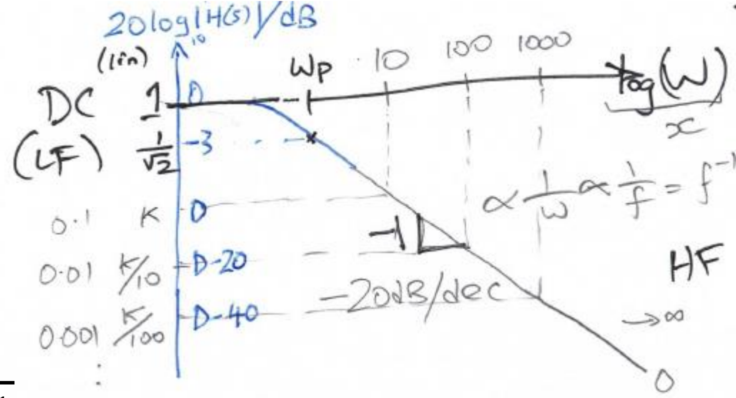
Pole Location s = root of denominator: solve $\left(s/_{\omega_p} + 1\right) = 0$ $\Rightarrow$ $s = -\omega_p$ Laplace: $s = \mathbb{C}$-plane

Pole Frequency $\omega = |s|$: $|s| = |j\omega| = |-\omega_p|$ $\Rightarrow$ $\omega = \omega_p$ Fourier: $s = j\omega$ (Imag. Axis)

Must be Positive! Positive! $\rightarrow$ Bode plot

Canonical LPF





$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs + 1}$$

$$H(s) = \frac{1}{\tau s + 1}\bigg|_{\tau = RC}$$

Canonical form:
($\tau s+1$) terms

$$H(s) = \frac{1}{s/\omega_p + 1}\bigg|_{\omega_p = \frac{1}{RC}}$$

$\omega_p = 1/\tau$ = Pole *Frequency* (rad/s)

DC: $f = 0$ $\quad H(s=0) = 1 \quad Z_C = \frac{1}{sC} = \infty\ (OC)$

HF: $f \to \infty$ $\quad H(s \to \infty) = 0 \quad Z_C = \frac{1}{sC} = 0\ (SC)$

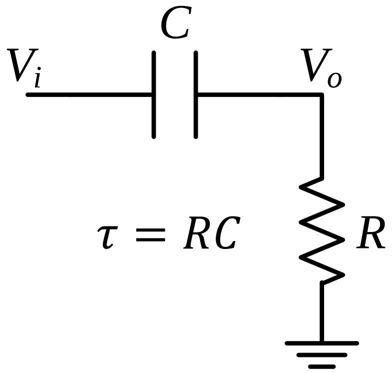
finite BW = LOW PASS f_{3dB}

How about at $\omega = \omega_p$?

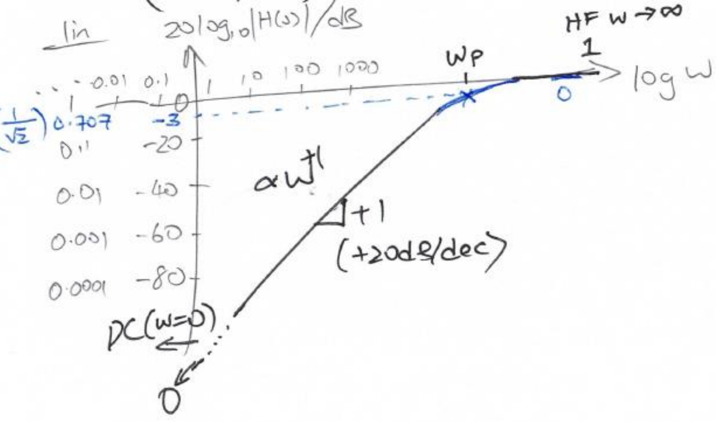
$$|H(s = j\omega_p)| = \left|\frac{1}{s/\omega_p + 1}\right| = \left|\frac{1}{j\omega_p/\omega_p + 1}\right| = \left|\frac{1}{j+1}\right| = \frac{1}{\sqrt{2}} = -3dB$$

Recall: Opamp open-loop $|A(jf)| = \left|\frac{A_o}{1+jf/f_b}\right|$

Canonical HPF



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + {}^{1}\!/_{sC}} = \frac{RCs}{RCs + 1}$$



$$H(s) = \frac{\tau s}{\tau s + 1}\Bigg|_{\tau = RC}$$

Canonical form:
τs and ($\tau s+1$) terms

$$H(s) = \frac{{}^{s}\!/_{\omega_p}}{{}^{s}\!/_{\omega_p} + 1}\Bigg|_{\omega_p = \frac{1}{RC}}$$

$\omega_p = 1/\tau$ = Pole *Frequency* (rad/s)

Zero Location s = root of numerator: solve ${}^{s}\!/_{\omega_p} = 0 \Rightarrow s = 0$

Zero Frequency $\omega = |s|$: $|s| = |j\omega| = |0| \Rightarrow \omega = 0$

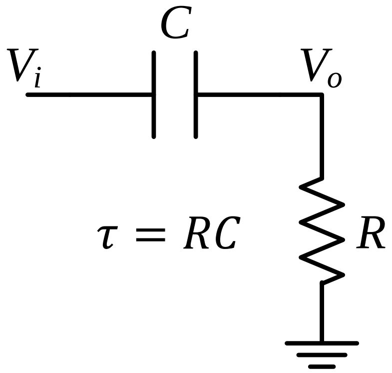
Must be Positive!

Laplace: $s = \mathbb{C}$-plane

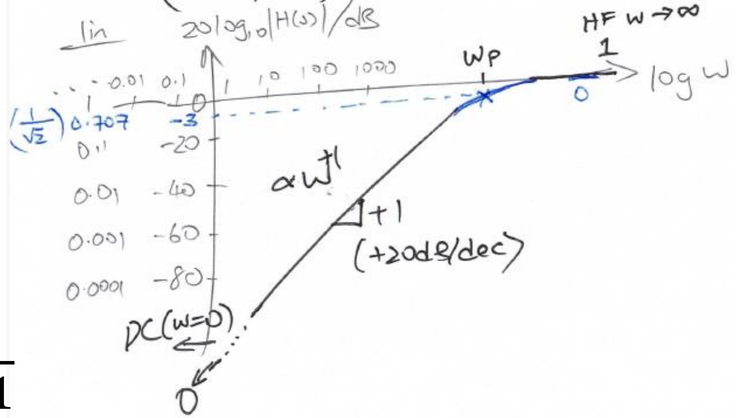
Fourier: $s = j\omega$ (Imag. Axis)

$\rightarrow$ Bode plot

Canonical HPF



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + {}^{1}/_{sC}} = \frac{RCs}{RCs + 1}$$



$$H(s) = \frac{\tau s}{\tau s + 1}\bigg|_{\tau = RC}$$

Canonical form:
τs and ($\tau s+1$) terms

$$H(s) = \frac{{}^{s}/_{\omega_p}}{{}^{s}/_{\omega_p} + 1}\Bigg|_{\omega_p = \frac{1}{RC}}$$

$\omega_p = 1/\tau$ = Pole *Frequency* (rad/s)

DC: $f = 0$ $\quad H(s = 0) = 0$ $\quad Z_C = \frac{1}{sC} = \infty\ (OC)$

HF: $f \to \infty$ $\quad H(s \to \infty) = 1$ $\quad Z_C = \frac{1}{sC} = 0\ (SC)$

AC coupling = HIGH PASS f_{3dB}

How about at $\omega = \omega_p$?

$$|H(s = j\omega_p)| = \left|\frac{{}^{s}/_{\omega_p}}{{}^{s}/_{\omega_p} + 1}\right| = \left|\frac{{}^{j\omega_p}/_{\omega_p}}{{}^{j\omega_p}/_{\omega_p} + 1}\right| = \left|\frac{j}{j+1}\right| = \frac{1}{\sqrt{2}} = -3dB$$

Bode Plots of Single Pole System: Amplitude Response

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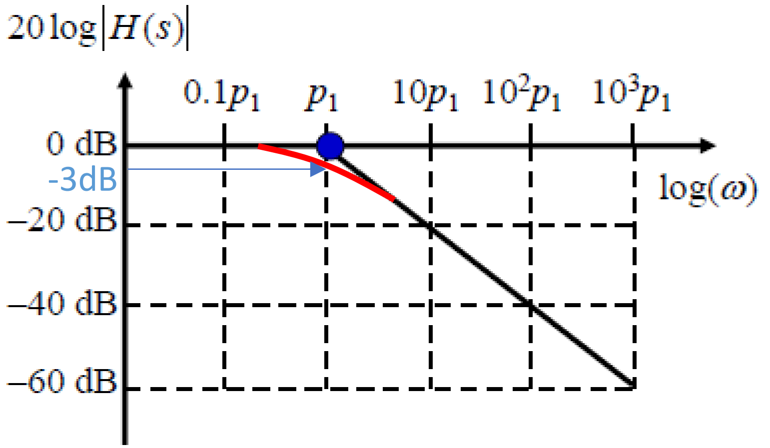
$$H(s)=\frac{1}{1+\frac{s}{p_1}} \Rightarrow |H(s)|=\frac{1}{\sqrt{1+\left(\frac{\omega}{p_1}\right)^2}}$$

Canonical form:
($\tau s+1$) terms

$$dB(H)=20\log\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{p_1}\right)^2}}\right)=-10\log\left[1+\left(\frac{\omega}{p_1}\right)^2\right]$$

$$dB(H)=\begin{cases}0 & \text{when } \omega \ll p_1\\ -20\log\omega+20\log p_1 & \text{when } \omega \gg p_1\end{cases}$$



Low pass

Try some frequency $\omega_1 \gg p_1$, dB[$H(\omega_1)$],
$= -20\log\omega_1 + 20\log p_1$

Try another frequency $\omega_2 = 10^* \omega_1$, dB[$H(\omega_2)$],
$= -20\log\omega_1 - 20 + 20\log p_1$

When we hit a pole, p_1, the Bode magnitude falls with a slope of -20 dB/dec.

Bode Plots of Single Zero System: Amplitude Response

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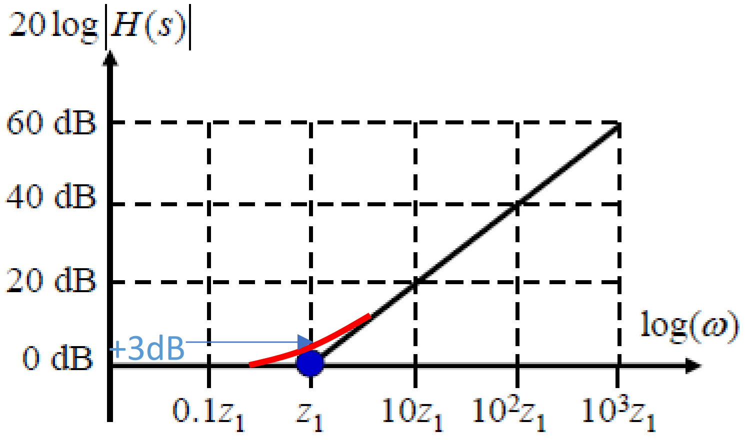
$$H(s) = 1 + \frac{s}{z_1} \Rightarrow |H(s)| = |H(j\omega)| = \sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}$$

Canonical form: ($\tau s+1$) terms

$$dB(H) = 20\log\left(\sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}\right) = 10\log\left[1 + \left(\frac{\omega}{z_1}\right)^2\right]$$

$$dB(H) = \begin{cases} 0 & \text{when } \omega << z_1 \\ 20\log\omega - 20\log z_1 & \text{when } \omega >> z_1 \end{cases}$$



High pass

Try some frequency $\omega_1 >> z_1$, $\mathrm{dB}[H(\omega_1)]$, = $20 \log \omega_1 - 20 \log z_1$

Try another frequency $\omega_2 = 10^* \omega_1$, $\mathrm{dB}[H(\omega_2)]$, = $-20 \log \omega_1 + 20 + 20 \log z_1$

When we hit a zero, z_1, the Bode magnitude rises with a slope of +20 dB/dec.

Many-Pole and Many-Zero System's Bode Plots

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Now you can deal with a system with many poles and/ or many zeros using the complex number's knowledge:

gain

high pass zeroes
(AC feedthrough)

$$H(s) = \frac{H_0\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)}$$

Canonical form:
(*τs+1*) terms

low pass poles

Amplitude: $\mathrm{dB}(H) = 20\log(H_0) + dB\left(1+\frac{s}{z_1}\right) + \ldots dB\left(1+\frac{s}{z_m}\right) + dB\left(\frac{1}{1+\frac{s}{p_1}}\right) + ..dB\left(\frac{1}{1+\frac{s}{p_n}}\right)$

How About a Very Complicated Circuit?

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$$H(s)=\frac{b_0+b_1s+b_2s^2+\ldots+b_{m\text{-}1}s^{m\text{-}1}+b_ms^m}{a_0+a_1s+a_2s^2+\ldots+a_{n\text{-}1}s^{n\text{-}1}+a_ns^n}=\frac{H_0\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)}$$

Canonical form:
($\tau s+1$) terms

Many-pole and Many-zero System: Example *(a BAD one!)*

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Given: $H(s)=\dfrac{10^8 s^2}{(s-10)(s-10^3)(s-10^6)}$

2 Zeros: $z_1=z_2=0$

3 Poles: $p_1=10,\ p_2=10^3,\ p_3=10^6$

1. *Unstable **positive pole locations, $\sigma > 0$***
2. *Non-canonical form*

Re-express this in Canonical form!

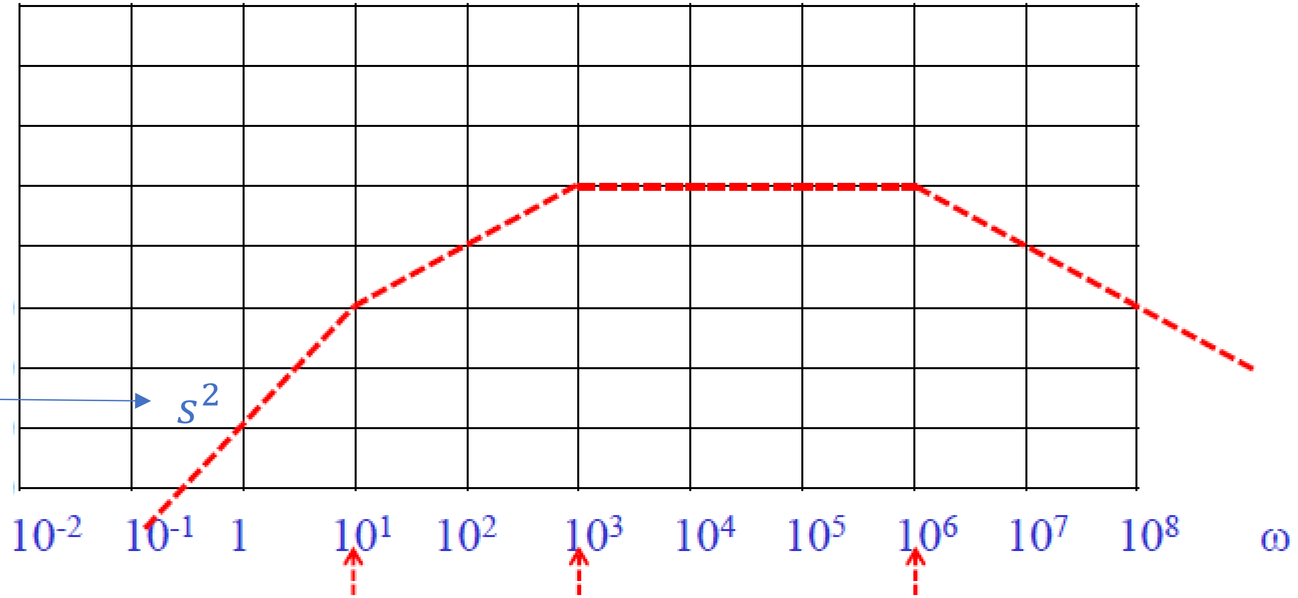
1. Two high pass filters
2. One low pass filter
3. gain H_0

Step 1:
Shape

Non-canonical form:

$$H(s) = \frac{10^8 s^2}{(s+10)(s+10^3)(s+10^6)}$$

s^2



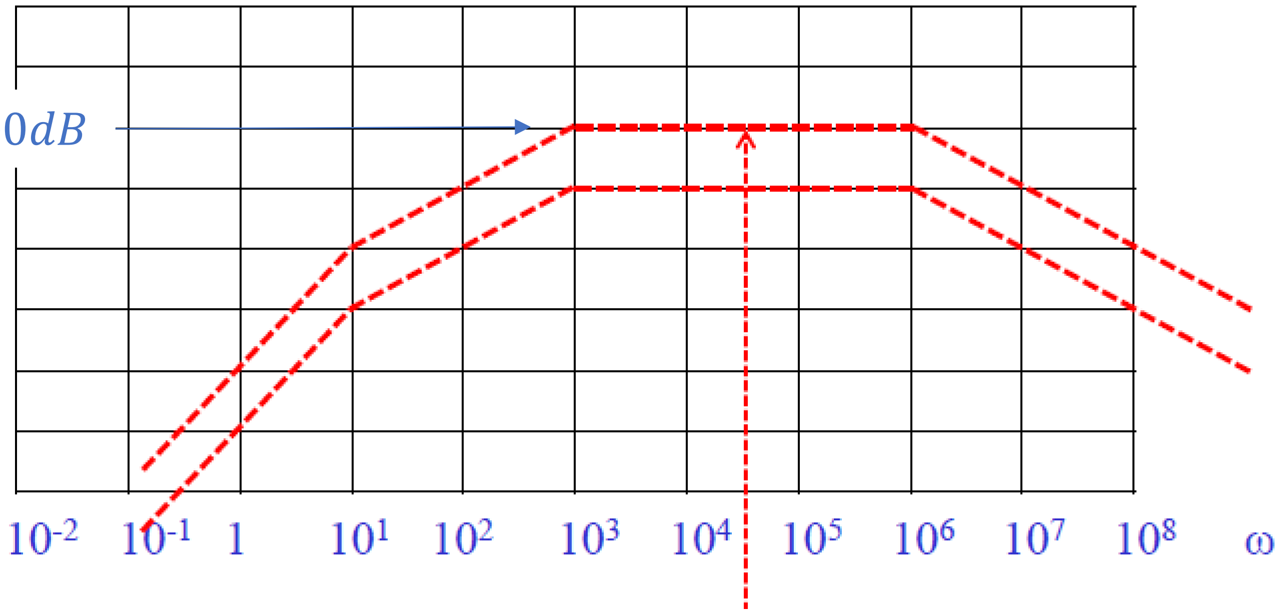
→ *Stable* ***negative pole locations, σ < 0***

Pole Frequencies remain positive:

$$\boldsymbol{\omega_p = 10^1, 10^3, 10^6}$$

Step 2:
Height

$H_0 = 40dB$



Canonical form:

$$H(s) = \frac{10^{-2} s^2}{(s/10+1)(s/10^3+1)(s/10^6+1)}$$

$$= \frac{10^2 \cdot s/10 \cdot s/10^3}{(s/10+1)(s/10^3+1)(s/10^6+1)}$$

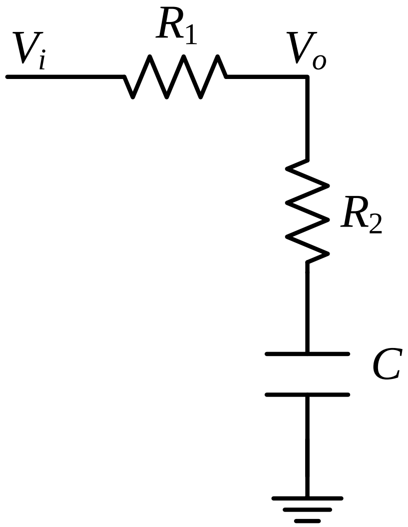
τs and (*τs+1*) terms

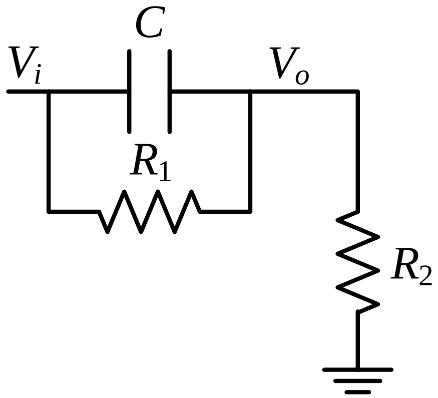
$$= H_0 \cdot HPF_{\omega_p=10} \cdot HPF_{\omega_p=10^3} \cdot LPF_{\omega_p=10^6}$$

Try any specific frequency you prefer, for example, 10^4:

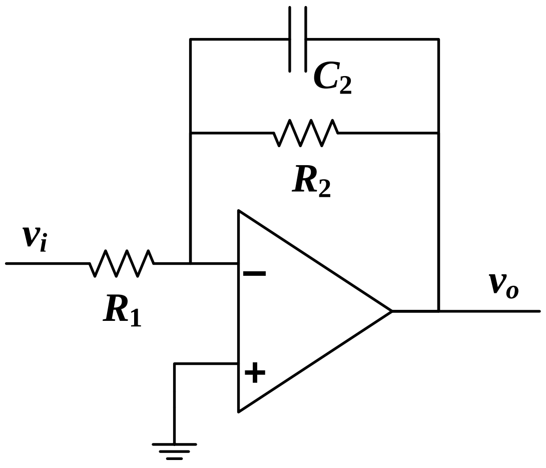
$$20\log|H(s=j10^4)| = 20\log\left|\frac{10^8(j10^4)^2}{(j10^4-10)(j10^4-10^3)(j10^4-10^6)}\right| = 40$$

Circuits with positive zero frequencies $\omega_Z > 0$

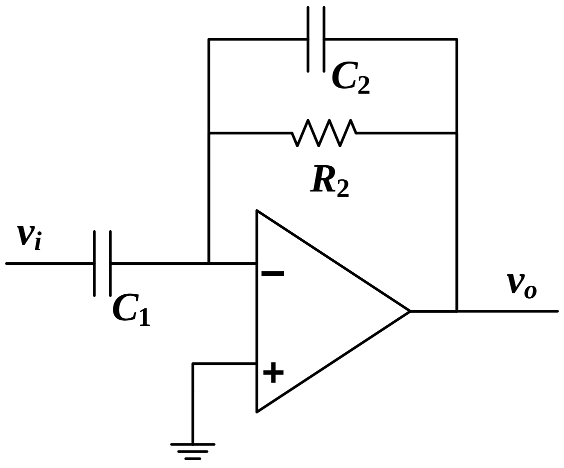




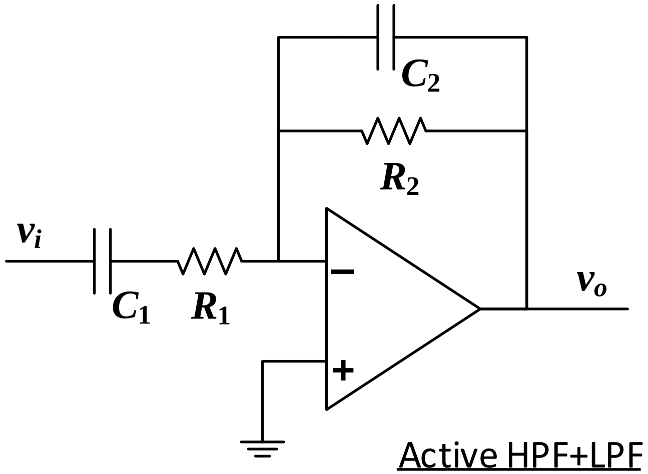
Active Filters (i.e. with Gain>1)



Active LPF



Active HPF



Active HPF+LPF

Lecture Milestones

- The basics of signal processing and analysis are covered.
- Math ←→ Circuits, toolkit: impedance, KVL, KCL
- Study frequency response of a few examples in order to account the behavior of a circuit operating at different frequencies.
- Study time constants for approximating the response of amplifiers.

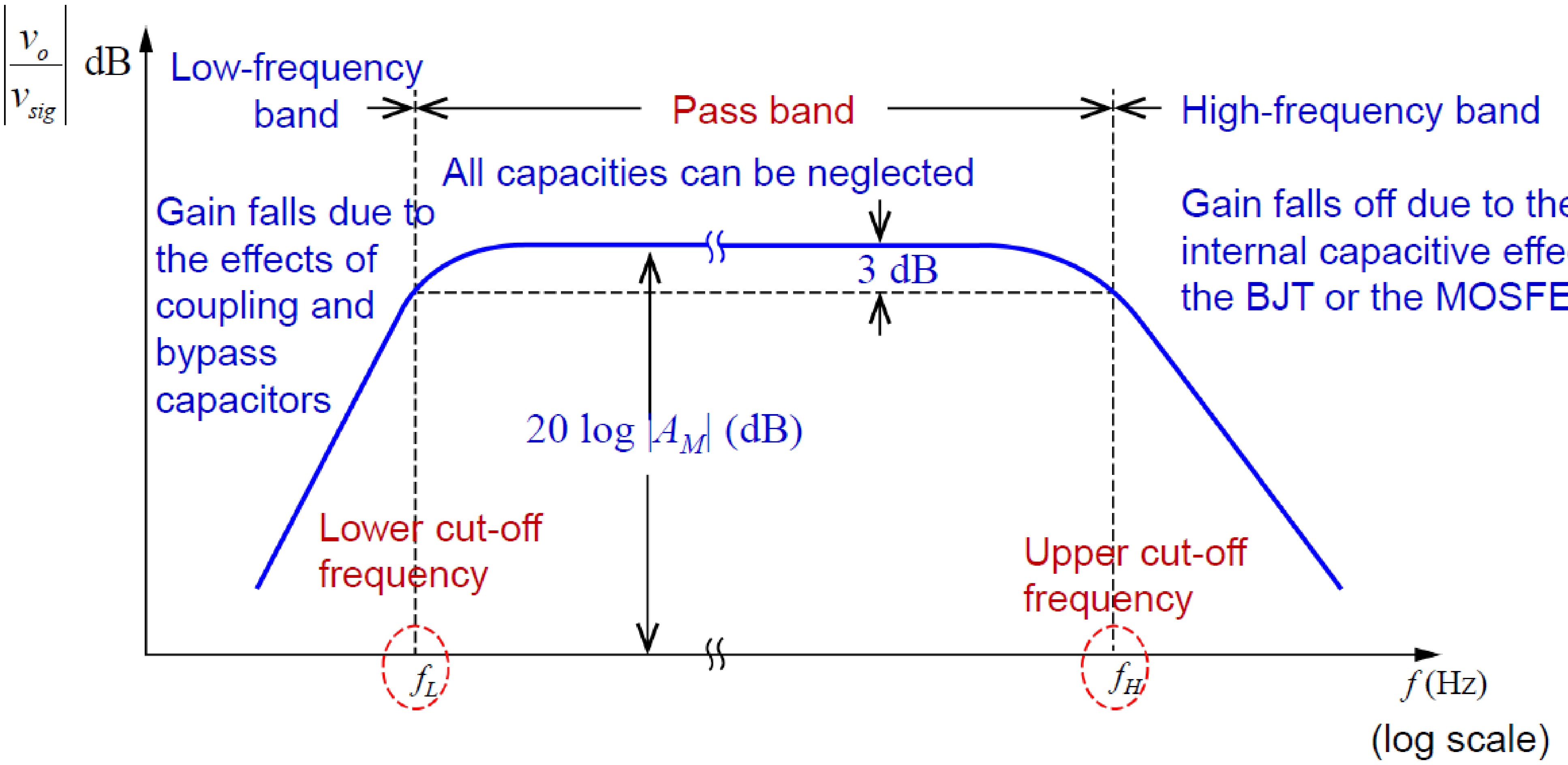
Understand bode plot, pole, zero, the method to identify the pole/zero elements in real circuits.

How About a Very Complicated Circuit?

$$H(s)=\frac{b_0+b_1s+b_2s^2+\ldots+b_{m\text{-}1}s^{m\text{-}1}+b_ms^m}{a_0+a_1s+a_2s^2+\ldots+a_{n\text{-}1}s^{n\text{-}1}+a_ns^n}=\frac{H_0\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)}$$

- However, for multistage amplifier with many capacitive elements, explicit computation (by hand) of the frequency response (i.e. transfer function) is generally **impractical**.
- Machine computation is cheap and getting cheaper all the time, so perhaps the analysis of networks doesn't present much of a problem. However, we are interested in developing design insight so that if a simulator tells us that there is a problem, we have some idea of what to do about it.
- In fact, accurate calculation on the frequency response may not be required but only a very rough estimation to predict the performance is sufficient. In that case, the -3 dB frequency is the most important parameter.

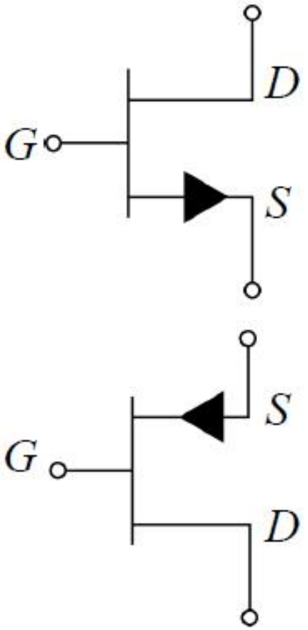
Find the Sources of the Lower and Higher Cut-off Frequencies



Identify capacitors that contribute to the cut-off frequency.

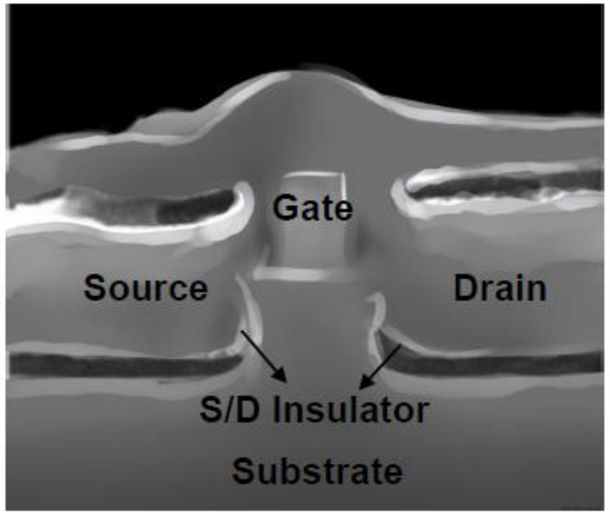
Where do Capacitors come from?

Clean Schematic



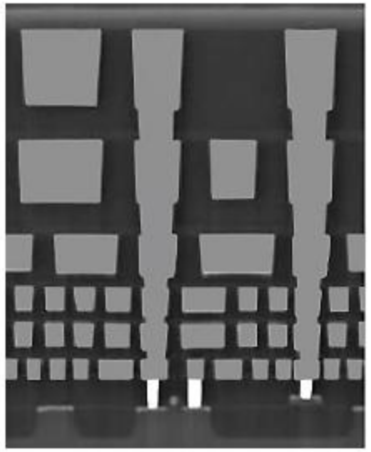
Transistor symbol

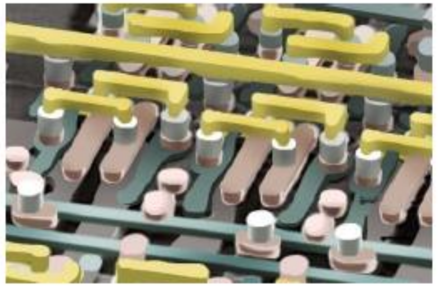
Complicated Real Silicon



Cross section

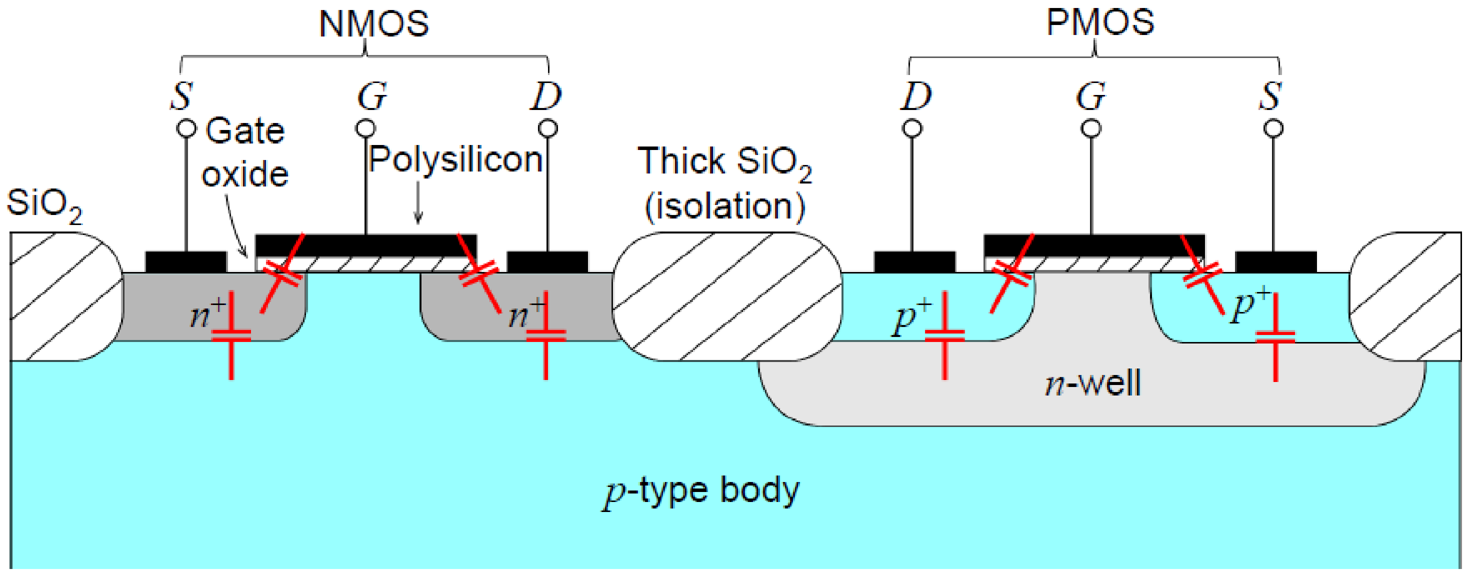
Interconnection





Some capacitors come from the capacitive structures from active devices (transistors), some come from the coupling effect between interconnections.

Identify Parasitic Capacitors

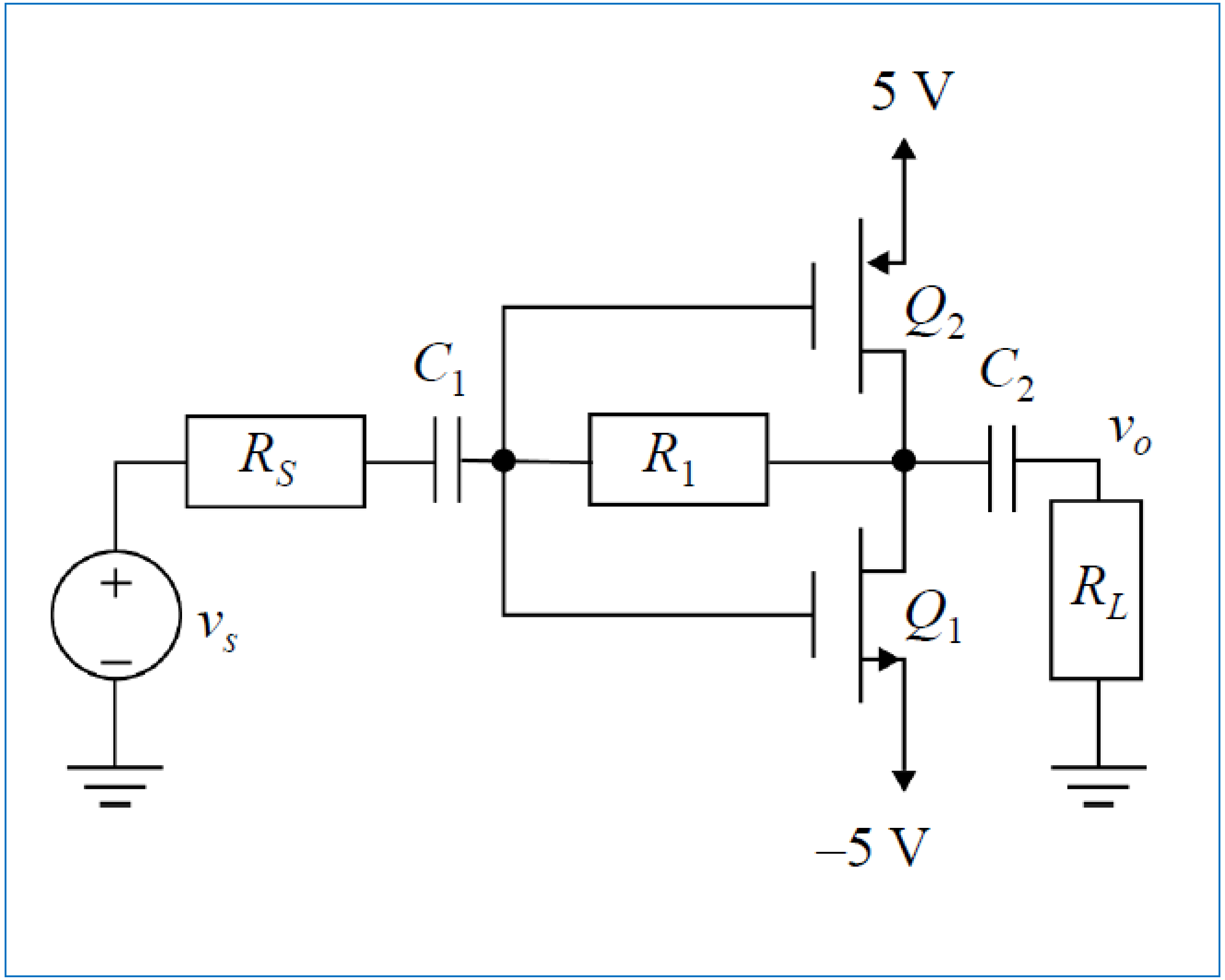


1. Capacitance between **Gate/Source** and **Gate/Drain** due to the overlap of gate electrode (Parallel-plate capacitor)

2. Junction capacitance between **Source/Body** and **Drain/Body** (Reverse-bias junction)

Note: The body of NMOS is automatically connected to the lowest voltage in the system; the body of PMOS is automatically connected to the highest voltage in the system.

Identify Parasitic Capacitors:



Answer: Tutorial 10, Q2

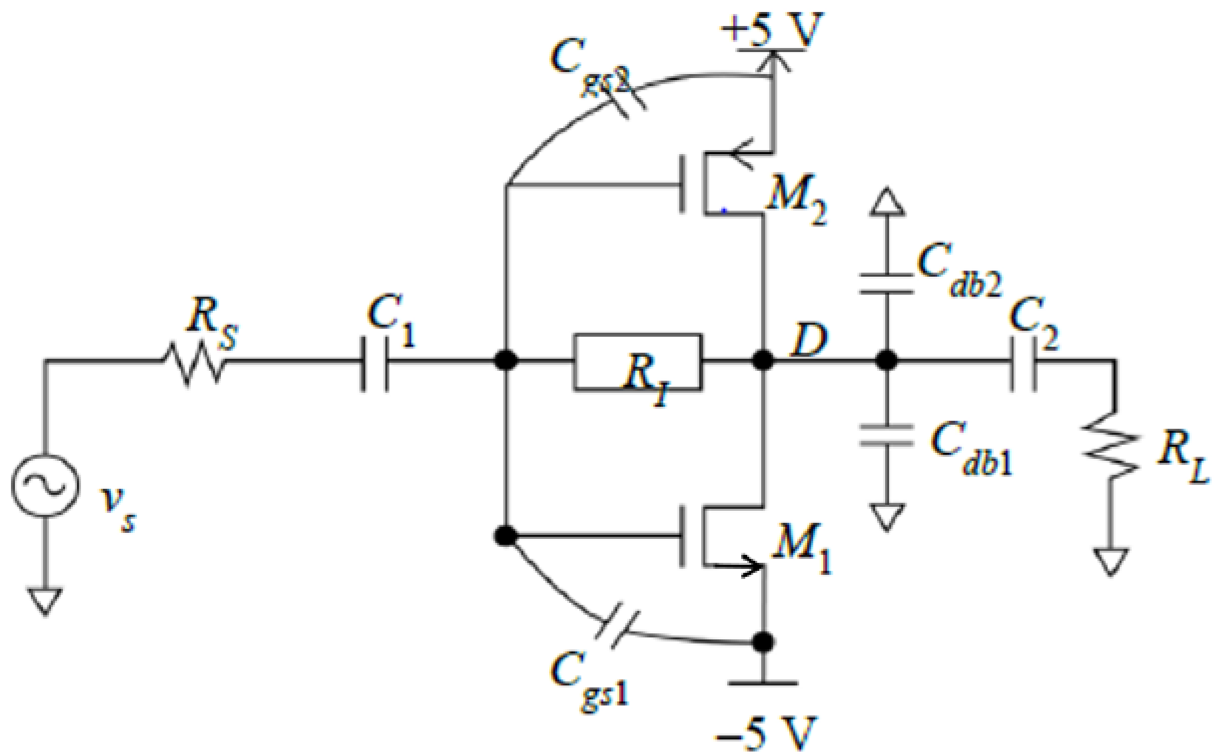


Figure 2

SCTC and OCTC Methods

4-step Standard Procedure

- Identify OPEN CIRCUIT CAPACITORS = LOW PASS
- Identify SHORT CIRCUIT CAPACITORS = HIGH PASS

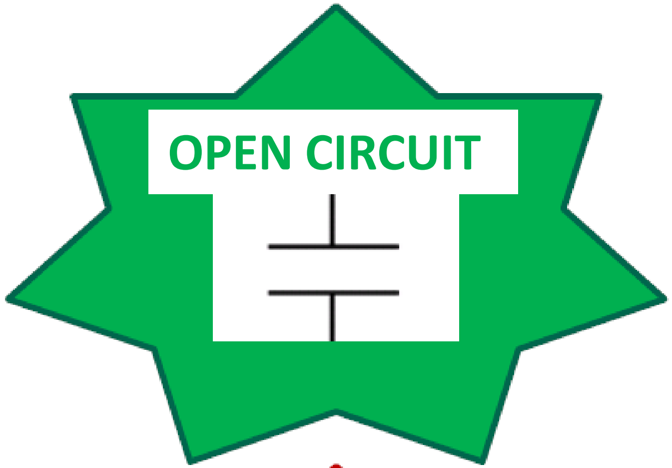
Disable all sources → Identify OC/SC Capacitors → Find R_x seen by each C_x to get $\tau_x = R_x C_x$ (keeping all other caps OC/SC) →

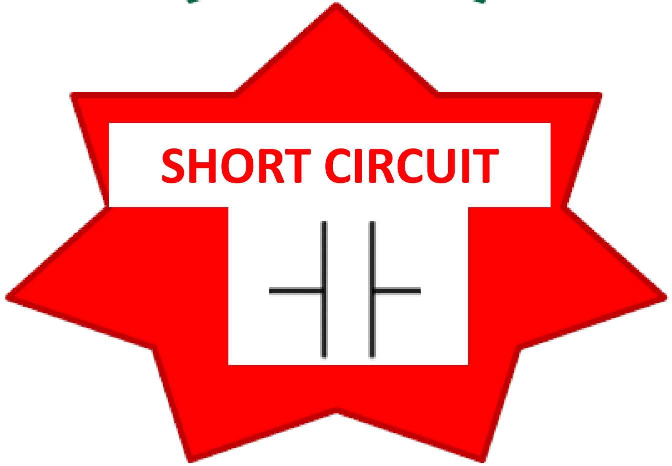
OCTC: $\omega_H = \frac{1}{\sum \tau_i} = \frac{1}{\tau_1+\tau_2+\cdots}$

$= \omega_1 // \omega_2 // \cdots$

SCTC: $\omega_L = \sum \frac{1}{\tau_i} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \ldots$

$= \omega_1 + \omega_2 + \cdots$



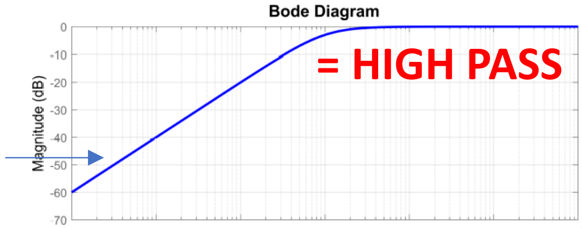


Q1: How to Classify Capacitors?

A Game of Short/ Open

- Short a cap ➔ Higher Gain ➔ **SHORT CIRCUIT CAPACITOR**

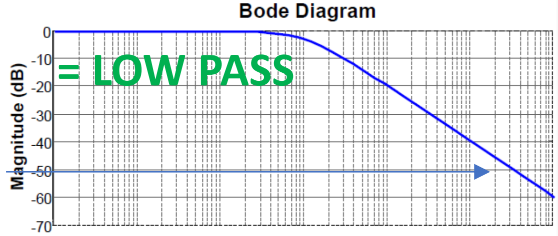
  ⟺ **If OPEN, it would kill the signal! (@LF)**



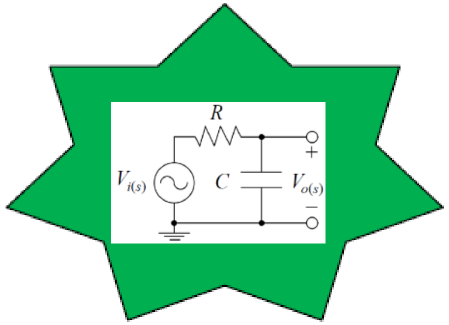
= HIGH PASS

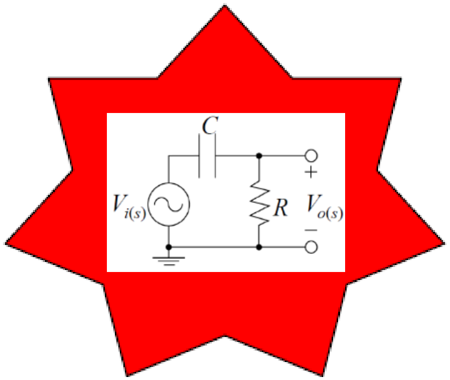
- Open a cap ➔ Higher Gain ➔ **OPEN CIRCUIT CAPACITOR**

  ⟺ **If SHORT, it would kill the signal! (@HF)**



= LOW PASS





The OCTC / SCTC song
(sung to the tune of "The Gambler" by Kenny Rogers)

- Every gambler knows that the secret to survivin'
  Is knowin' what to throw away and knowing what to keep
  'Cause every hand's a winner and every hand's a loser

- Every Analog guy knows, that the secret to survivin' (*EE2102*)
- is knowing which cap's an **OPEN**, and knowing which cap's a **SHORT**
- 'Cause every cap's a helper, and every cap's a troublemaker!
  (at some frequency) *(at some frequency)*

- ~~And~~ [If not] the best that you can hope for is to die in your sleep!

(Just Joking!!)

SCTC and OCTC Methods

Developed in the mid-1960s at MIT. Procedure is as follows:

1. **Disable** all independent sources (voltage sources → Short Circuit; current sources → Open Circuit); **Do not** remove or "disable" dependent sources!
2. **Identify** capacitors contributing to the frequency of interest, i.e., lower of higher cut-off.

higher cut-off

3. **Idealise** irrelevant capacitors by **short circuit** (because at high f, cap → short)
4. For each contributing capacitor C_i, set all other capacitors (other than the one you are looking at) **removed** (i.e. **Open Circuits**) and determine the resistance, R_i seen by C_i
5. Higher cut-off frequency is estimated as:

$$\omega_{H-3\text{dB}} \approx \frac{1}{\sum_i C_i R_i}$$

lower cut-off

3. **Idealise** irrelevant capacitors by **open circuit** (because at low f, cap → open)
4. For each contributing capacitor C_i, set all other capacitors (other than the one you are looking at) **removed** (i.e. **Short Circuits**) and determine the resistance, R_i seen by C_i
5. Lower cut-off frequency is estimated as:

$$\omega_{L-3\text{dB}} \approx \sum_i \frac{1}{C_i R_i}$$

SCTC and OCTC Methods

Developed in the mid-1960s at MIT. Procedure is as follows:

1. **Disable** all independent sources (voltage sources → Short Circuit; current sources → Open Circuit); **Do not** remove or "disable" dependent sources!
2. **Identify** capacitors as **OPEN CIRCUIT** or **SHORT CIRCUIT** = higher or lower cut-off, respectively

higher cut-off **(OCTC)**

3. Keep **SHORT CIRCUIT caps SHORT**
   They are irrelevant to the **OCTC!**
4. Keep **OPEN CIRCUIT caps OPEN**
   Each **open circuit** C_i will contribute to the **OCTC**!

   determine the resistance, R_i seen by C_i
5. Higher cut-off frequency is estimated as:

$$\omega_{H-3\mathrm{dB}} \approx \frac{1}{\sum_i C_i R_i}$$

(SCTC) **lower cut-off**

3. Keep **OPEN CIRCUIT caps OPEN**
   They are irrelevant to the **SCTC!**
4. Keep **SHORT CIRCUIT caps SHORT**
   Each **short circuit** C_i will contribute to the **SCTC**!

   determine the resistance, R_i seen by C_i
5. Lower cut-off frequency is estimated as:

$$\omega_{L-3\mathrm{dB}} \approx \sum_i \frac{1}{C_i R_i}$$

Learn by Case Study: Frequency Analysis of Common-Source Amplifier

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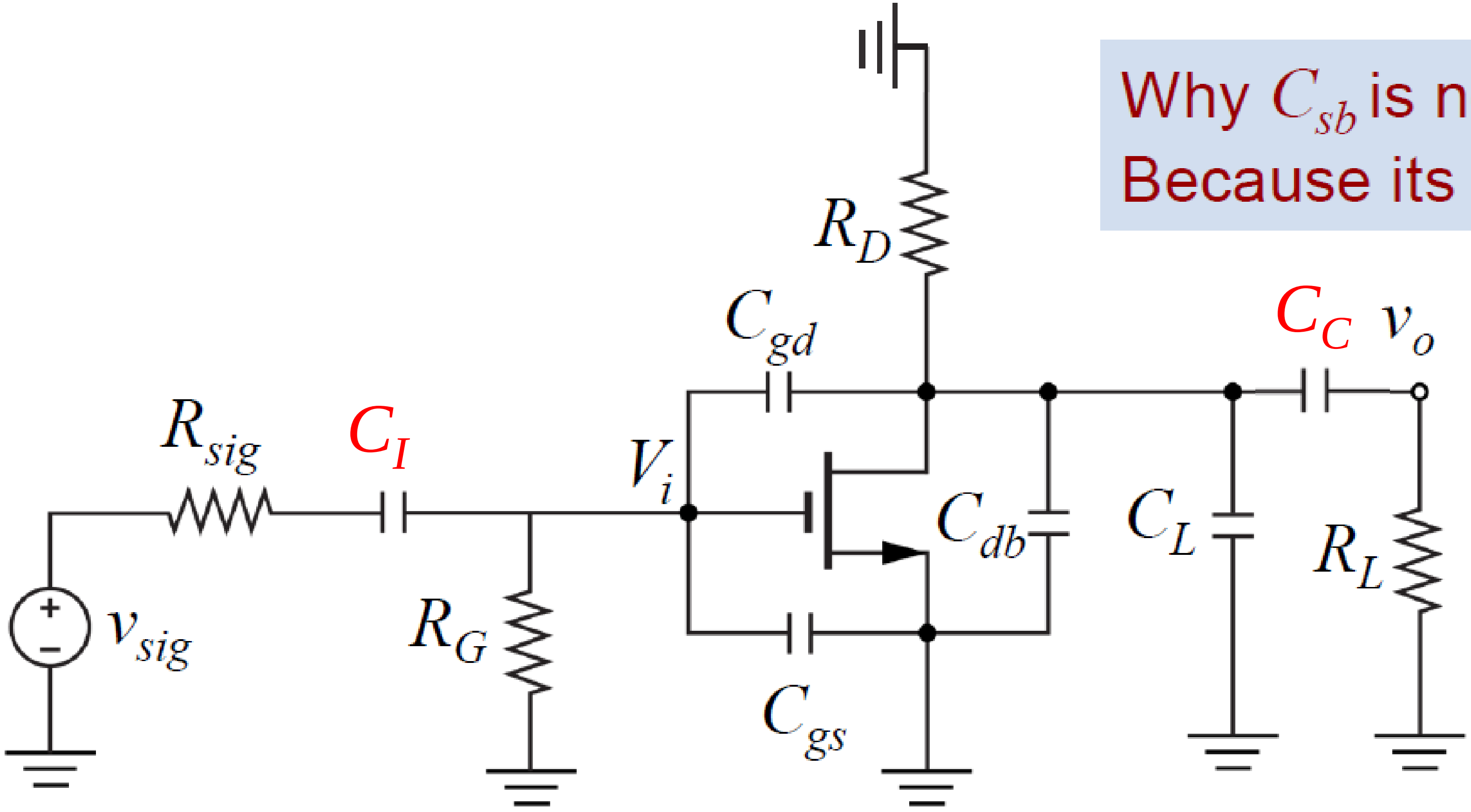
Step 1: Label parasitic capacitors



Why C_{sb} is not labeled?
Because its body → $-V_{ss}$.

Learn by Case Study: Frequency Analysis of Common-Source Amplifier

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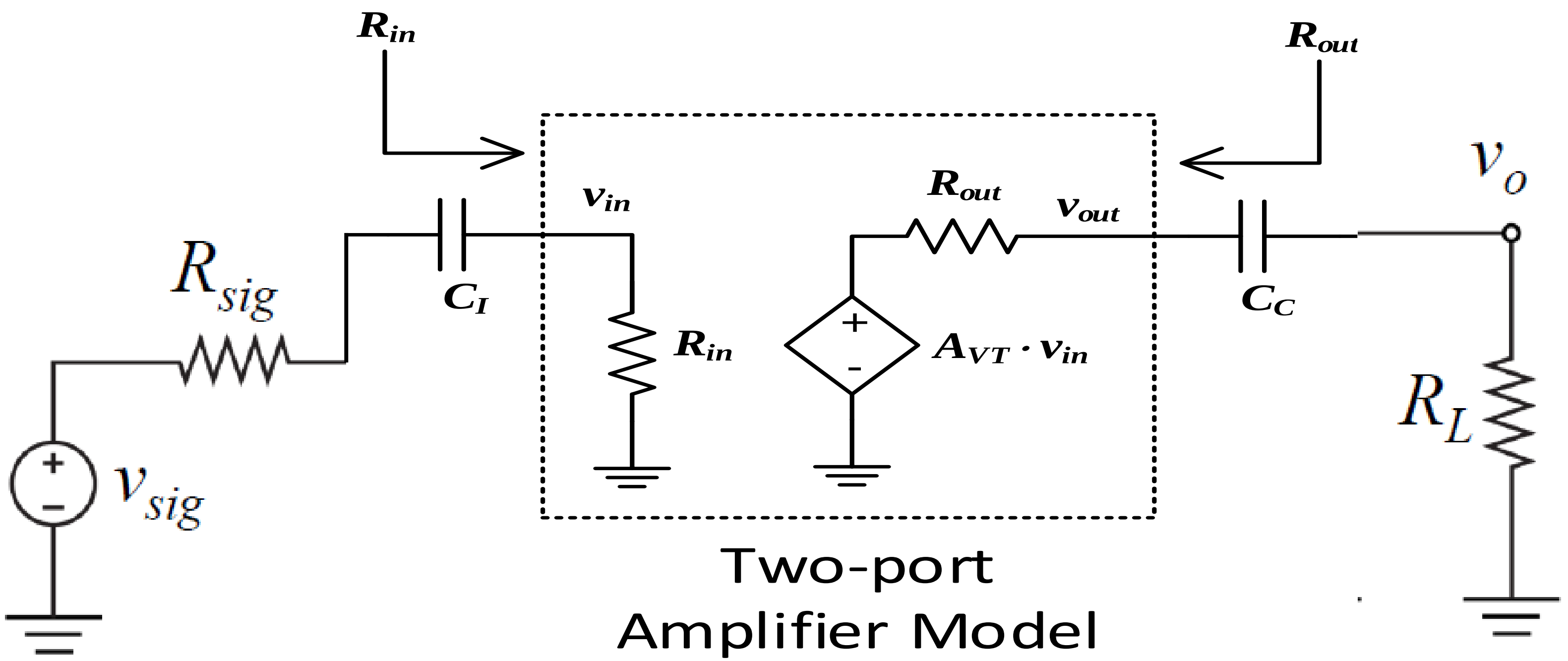
STEP 2: Use the TWO-PORT MODEL to simplify each τ_i



Two-port
Amplifier Model