A Book

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Hello World

Hi.

Bye.

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Governing Equations of Electrodynamics

2.1 Lorentz Force

A charged particle with a velocity v and charge q experiences the force given by

$$f = q\vec{E}_s + q\vec{E}_i + q(\vec{b} \times \vec{b})$$

* $\vec{E}_s$ = Static Electric Field * $\vec{E}_i$ = Force from Varying Magnetic Field * $q(v \times B)$ = Lorentz Force

The static electric field is Irrotational and has a fixed divergence.

$$\nabla \times \vec{E}_s = 0 \quad \nabla \cdot \vec{E}_s = \frac{\rho_e}{\epsilon_0}$$

* ρ_e = Total Charge Density * ϵ_0 = Permitivity of Free Space

The Electrostatic Potential V is given by

$$\vec{E}_s = -\nabla V \quad \nabla^2 V = \frac{\rho_e}{\epsilon_0}$$

The total electric field given by $\vec{E}_s + \vec{E}_i = \vec{E}$ is subject to the following constraints.

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} \quad \text{Gauss Law} \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \quad \text{Faraday's Law}$$

THe magnetic field $\vec{B}$ is a pseudovector, which will have consequences for Dynamo Theory.

2.1.1 Volumetric Lorentz Force

Fluid Dynamics are written with Volumetric Equations, and thus to incorporate Electrodynamics in them, we have to write Electrodynamics Equations also in a Volumetric formulation.

Let $\vec{J}$ represent current density.

$$\vec{J} = \sigma\vec{E}$$

In a moving frame, the same equation can be written as

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

We can identify this as the Volumetric Gauss Law

The Electrostatic Lorentz force is given by

$\vec{F} = q(\vec{E} + \vec{v} \times B)$

Dividing this by the volume, we get

$$\vec{f} = \rho_l\vec{E} + \vec{J} \times B$$

This is the volumetric Lorentz Force Equation.

Claim: In non Relativistic Frames, the leading $\rho_l\vec{E}$ term is negligible.

Proof: We can write the continuity equation of charge to be of the form

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_l}{\partial t}$$

This simply says that the rate at which charge is decreasing inside a small volume must equal the rate at which charge flows out across the surface of that volume. Taking divergence on both sides of the volumetric gauss law, and substituting it in the continuity of charge equation, we get

$$\frac{\partial \rho_l}{\partial t} + \frac{\rho_l}{\tau_l} + \sigma\nabla \cdot (\vec{v} \times \vec{B}) = 0$$

where τ_l represents the relaxation time of the system and is a very small quantity. So when we observe at timescales much larger than τ_l,

$$\frac{\partial \rho_l}{\partial t} << \frac{\rho_l}{\tau_l}$$

and we can neglect the term $\frac{\partial \rho_l}{\partial t}$.Rearranging the terms we get that $\rho_l = -\epsilon_0 \tau_l \nabla \cdot ((\vec{v} \times \vec{B})$ is very small and can be neglected as well (because τ_l is small) Going back to the volumetric Lorentz Force equation, we find under these assumptions of Non-Relativistic frames, the equation reduces to

$$\vec{f} = \vec{J} \times \vec{B}$$