

CFD

Source/Vortex Panel Method

- ✈ **Literature Review**
- ✈ **Panel Method Geometry**
- ✈ **Source Panel Method**
- ✈ **Source/Vortex Panel Method**
- ✈ **Flow Chart**

Literature Review

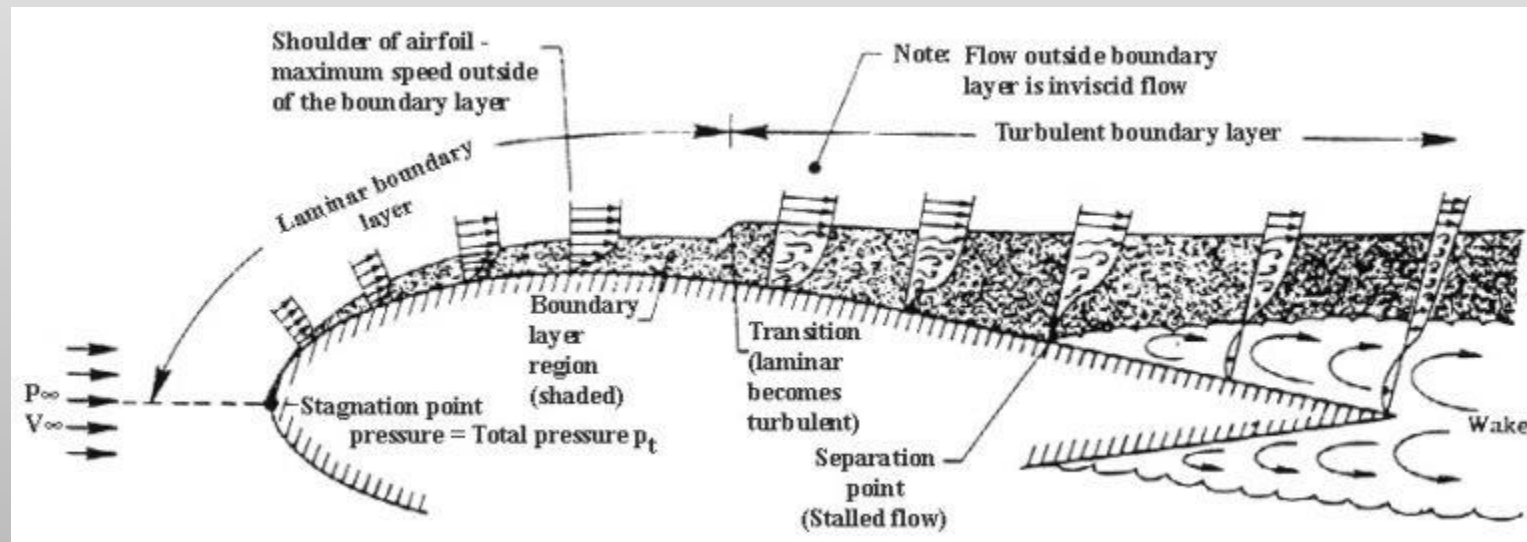
Potential Flow

비회전·비압축성의 경우에서 스칼라 함수(ϕ)는 속도장에 대하여 식(1)의 관계로 표현이 가능하다

$$\begin{aligned} U &= \nabla \phi \\ \omega &= \nabla \times U = 0 \end{aligned} \quad (1)$$

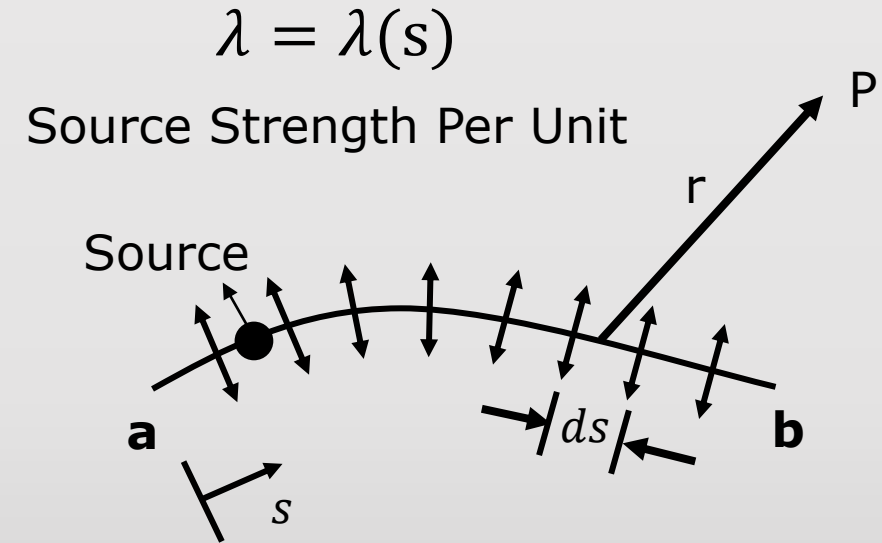
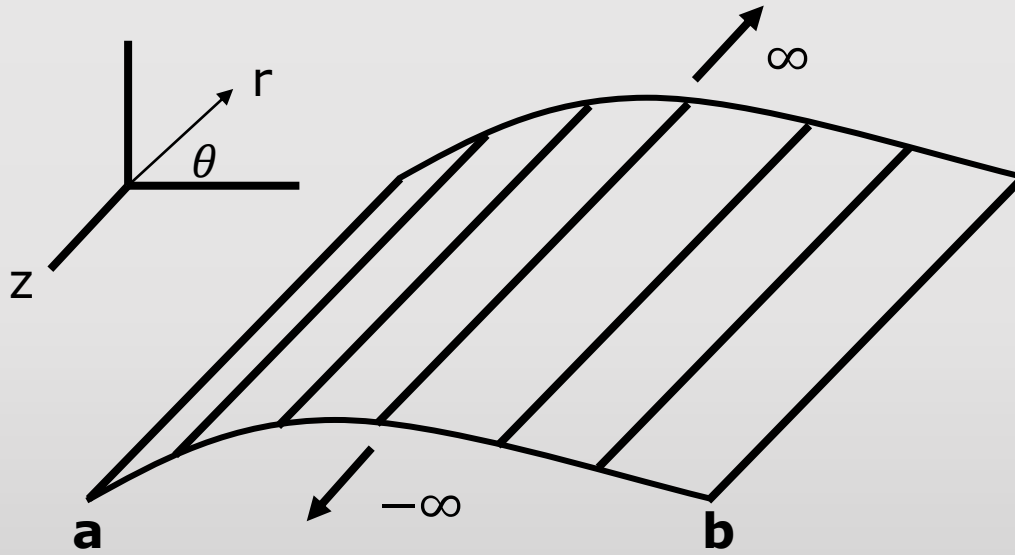
$$C_p = 1 - \frac{U^2}{U_\infty^2} \quad (2)$$

포텐셜 함수와 식(2)을 구하면 물체의 표면에 압력장을 구할 수 있으며, 실제 유동의 경계층은 매우 얇아 비점성 유동과 유사하다



Literature Review

Source Sheet



$$\lambda = \lambda(s)$$

Source Strength Per Unit

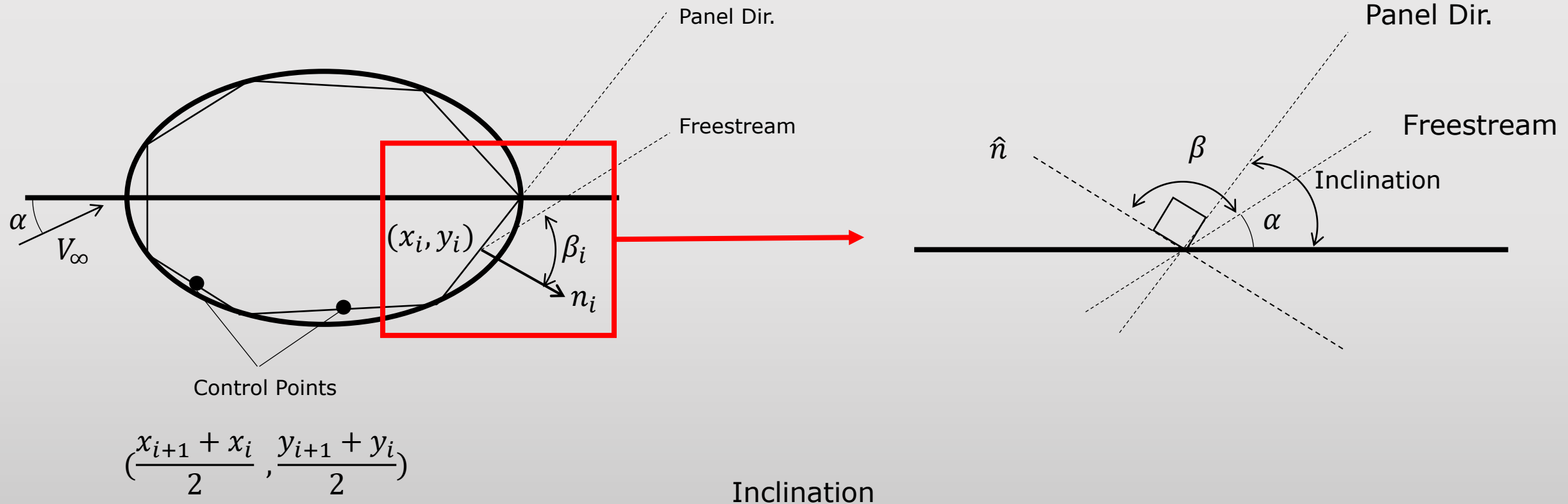
$$\lambda ds = d\phi = \frac{\lambda ds}{2\pi} \ln r \quad (1)$$

$$\phi(x, y) = \int_a^b \frac{\lambda \ln r}{2\pi} ds \quad (2)$$

패널에 존재하는 Source Strength(1)를 이용하여 Velocity Potential(2)을 구하여 속도장을 구하는 것이 목표

Literature Review

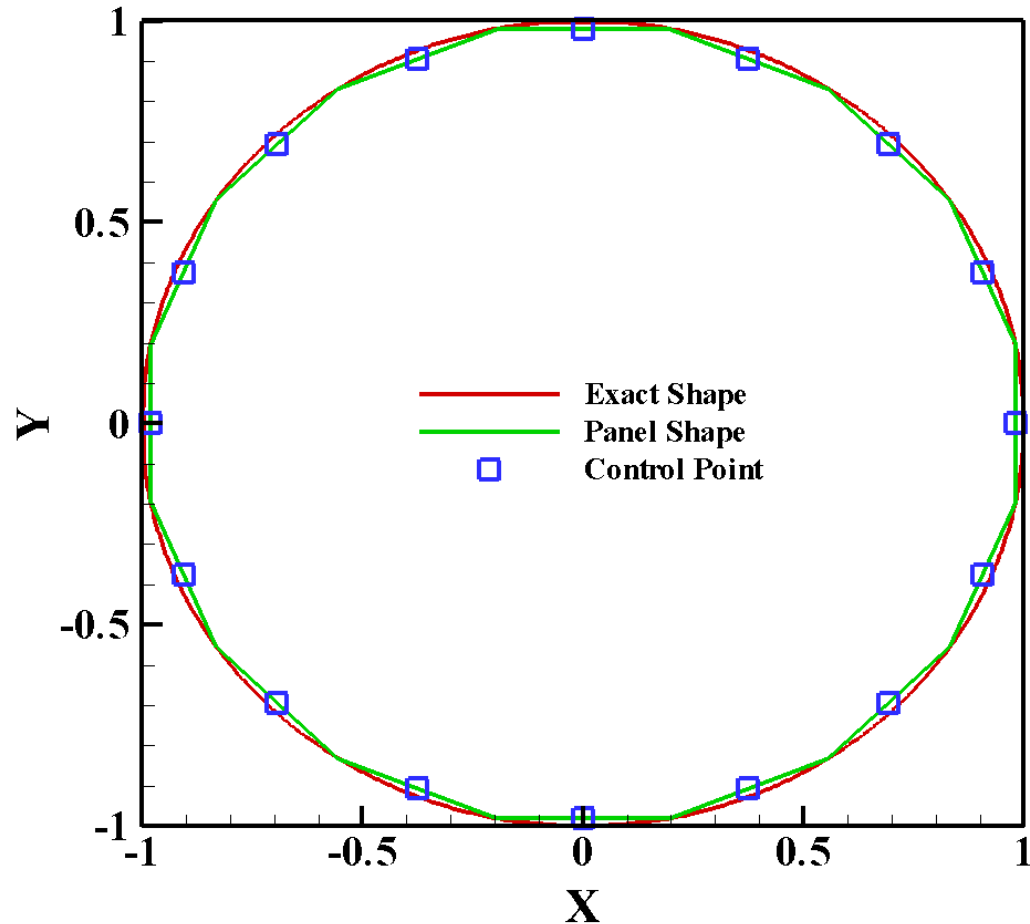
Panel Geometry



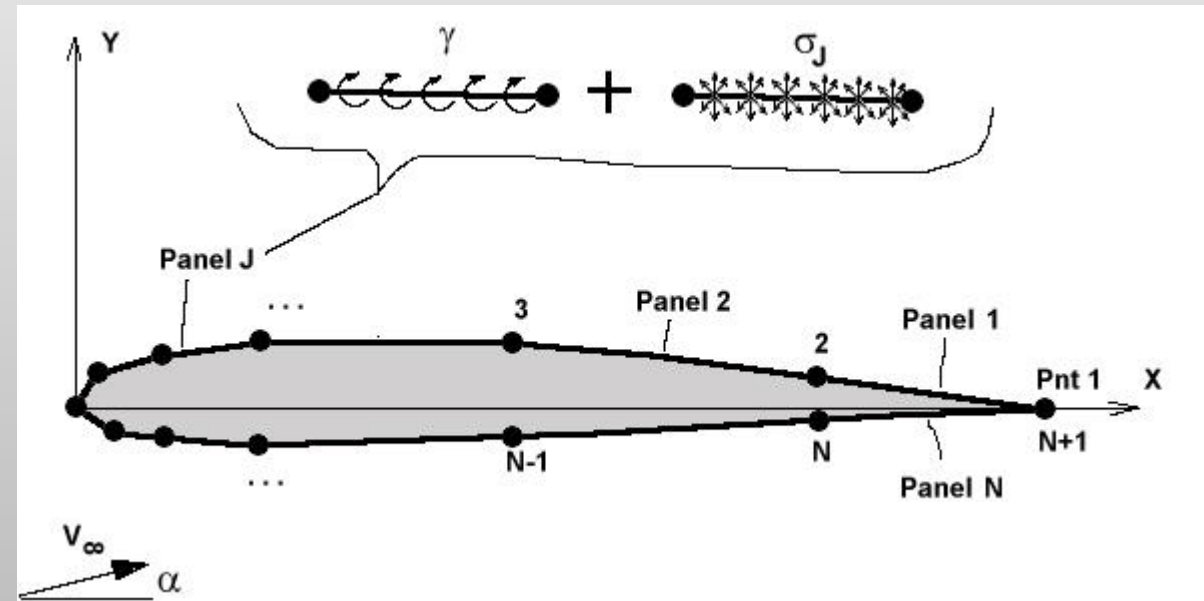
$$\beta_i = -\alpha + \boxed{\text{atan} \left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right)} + \frac{\pi}{2}$$

Literature Review

Panel Geometry

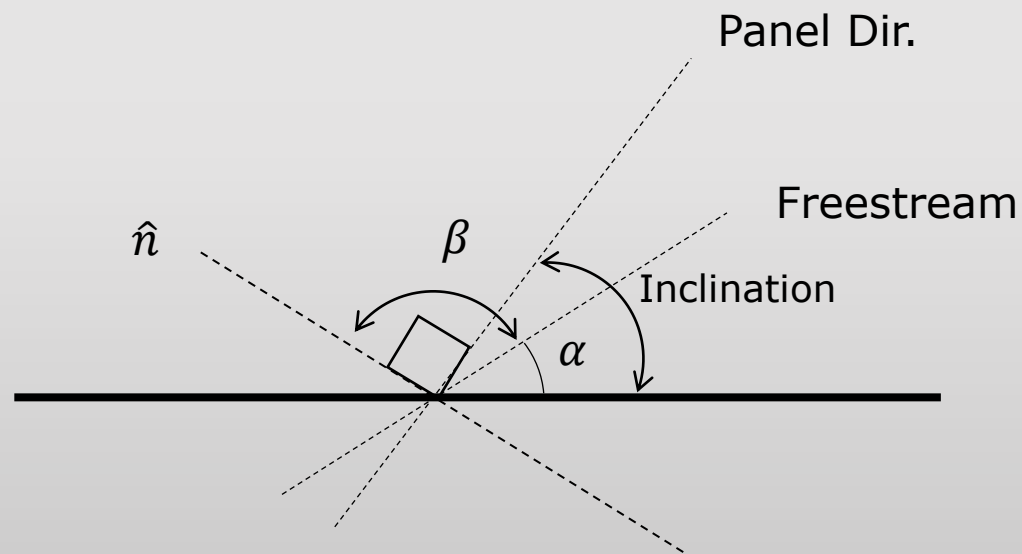


간단한 원 모양에서의 Panel을 배치하는 알고리즘에 대한 분석이 끝났으며, Airfoil에 대한 Panel 배치에 대한 공부가 필요할 것으로 보임



Literature Review

Mathematical Problem



$$V_n = \overset{(1)}{\frac{\lambda_i}{2}} + \overset{(2)}{\sum_{\substack{i=1 \\ j \neq i}}^n \frac{\lambda_i}{2} \int_j \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j} + \overset{(3)}{V_\infty \cos \beta_i}$$

The Total Normal Velocity Eq.

- (1) The Normal Component of Velocity Induced by the Source Panels ($i = j$)
- (2) The Normal Component of Velocity Induced by the Source Panels ($i \neq j$)
- (3) The Component of V_∞ normal to i^{th} Panel

Literature Review

1. <https://kr.mathworks.com/matlabcentral/fileexchange/69443-numerical-implementation-of-source-panel-method>

2. Anderson Jr, John David, "Fundamentals of aerodynamics," Tata McGraw-Hill Education, 2010.

Mathematical Problem

The Surface Velocity at the i^{th} Eq.

$$V_{s,i} = \sum_{\substack{j=1 \\ j \neq i}}^n \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial s} (\ln r_{ij}) ds_j + V_{\infty} \sin \beta_i$$

No – Penetration Boundary Conditions

$$0 = \frac{\lambda_i}{2} + \sum_{\substack{i=1 \\ j \neq i}}^n \frac{\lambda_i}{2\pi} \int_j \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j + V_{\infty} \cos \beta_i$$

The Pressure Coefficient

$$C_{p,i} = 1 - \left(\frac{V_i}{V_{\infty}} \right)^2$$

Literature Review

1. <https://kr.mathworks.com/matlabcentral/fileexchange/69443-numerical-implementation-of-source-panel-method>

2. Anderson Jr, John David, "Fundamentals of aerodynamics," Tata McGraw-Hill Education, 2010.

Mathematical Problem

Critical Integrals

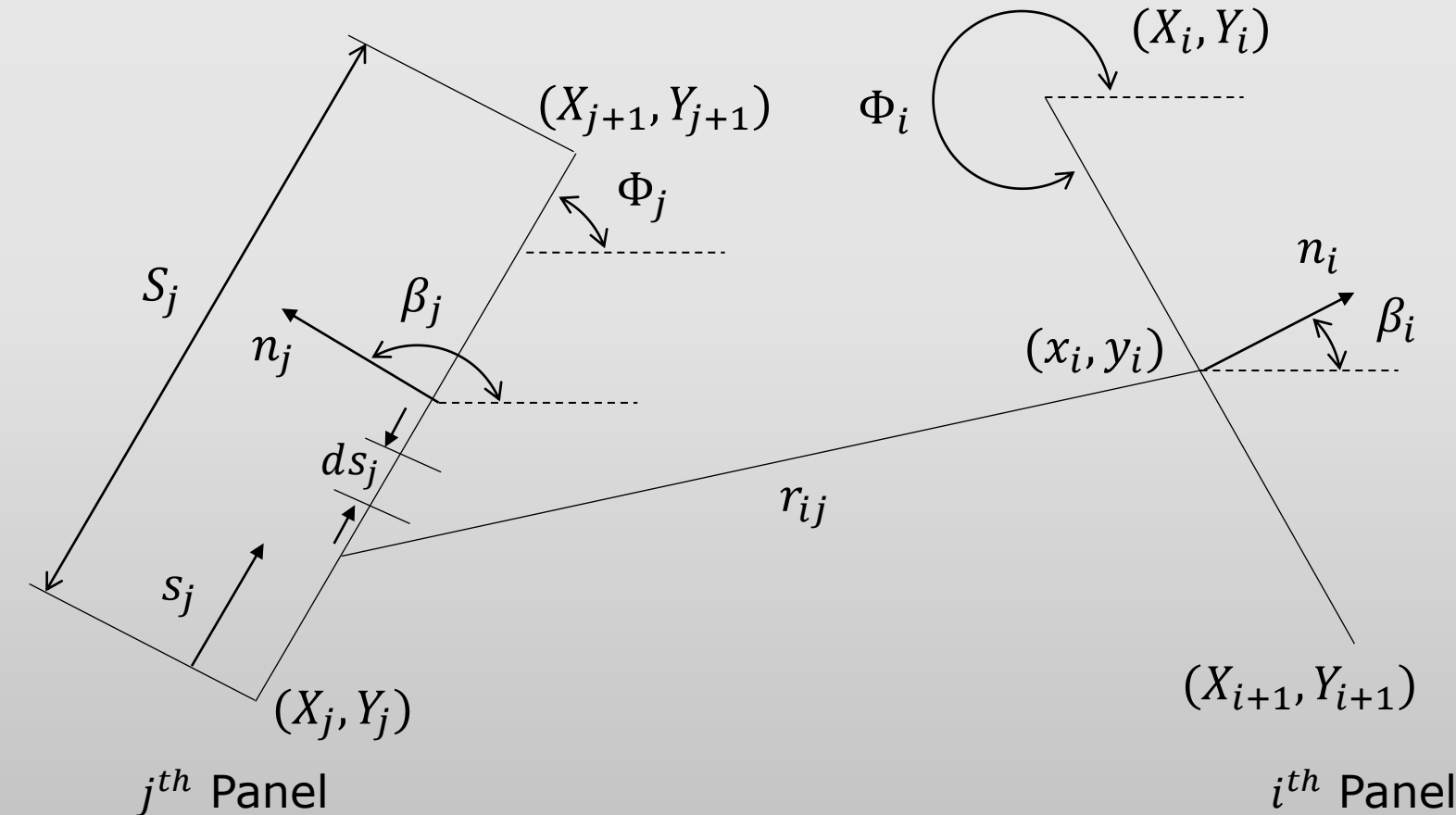
$$I_{i,j} = \int_j \boxed{\frac{\partial}{\partial n_i} (\ln r_{ij})} ds_j$$

$$\begin{aligned} \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j &= \frac{1}{r_{ij}} \frac{\partial r_{ij}}{\partial n_i} \\ &= \frac{1}{r_{ij}} \frac{1}{2} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{-0.5} \cdot 2 \left[(x_i - x_j) \frac{dx_i}{dn_i} + (y_i - y_j) \frac{dy_i}{dn_i} \right] \\ &= \frac{(x_i - x_j) \cos \beta_i + (y_i - y_j) \sin \beta_i}{(x_i - x_j)^2 + (y_i - y_j)^2} \quad [1.a] \end{aligned}$$

Literature Review

Mathematical Problem

Critical Integrals



$$\beta_i = \Phi_i + \frac{\pi}{2}$$

$$x_j = X_j + s_j \cos \Phi_j$$

$$y_j = Y_j + s_j \sin \Phi_j$$

Therefore, (Eq.1.a) is

$$I_{i,j} = \int_j \frac{Cs_j + D}{s_j^2 + 2As_j + B} ds_j$$

$$A = -(x_i - X_i) \cos \Phi_j - (y_i - Y_i) \sin \Phi_j$$

$$B = (x_i - X_j)^2 + (y_i - Y_j)^2$$

$$C = \sin(\Phi_i - \Phi_j)$$

$$D = (y_i - Y_j) \cos \Phi_i - (x_i - X_j) \sin \Phi_i$$

(1.b)

Literature Review

Mathematical Problem

Critical Integrals

Also, We can obtain Expression (Eq.1.b) is

$$I_{i,j} = \frac{C}{2} \ln \left(\frac{S_j^2 + 2AS_j + B}{B} \right) + \frac{D - AC}{E} \left(\tan^{-1} \frac{S_j + A}{E} - \tan^{-1} \frac{A}{E} \right) \quad (1.c)$$

$$A = -(x_i - X_i) \cos \Phi_j - (y_i - Y_i) \sin \Phi_j$$

$$B = (x_i - X_j)^2 + (y_i - Y_j)^2$$

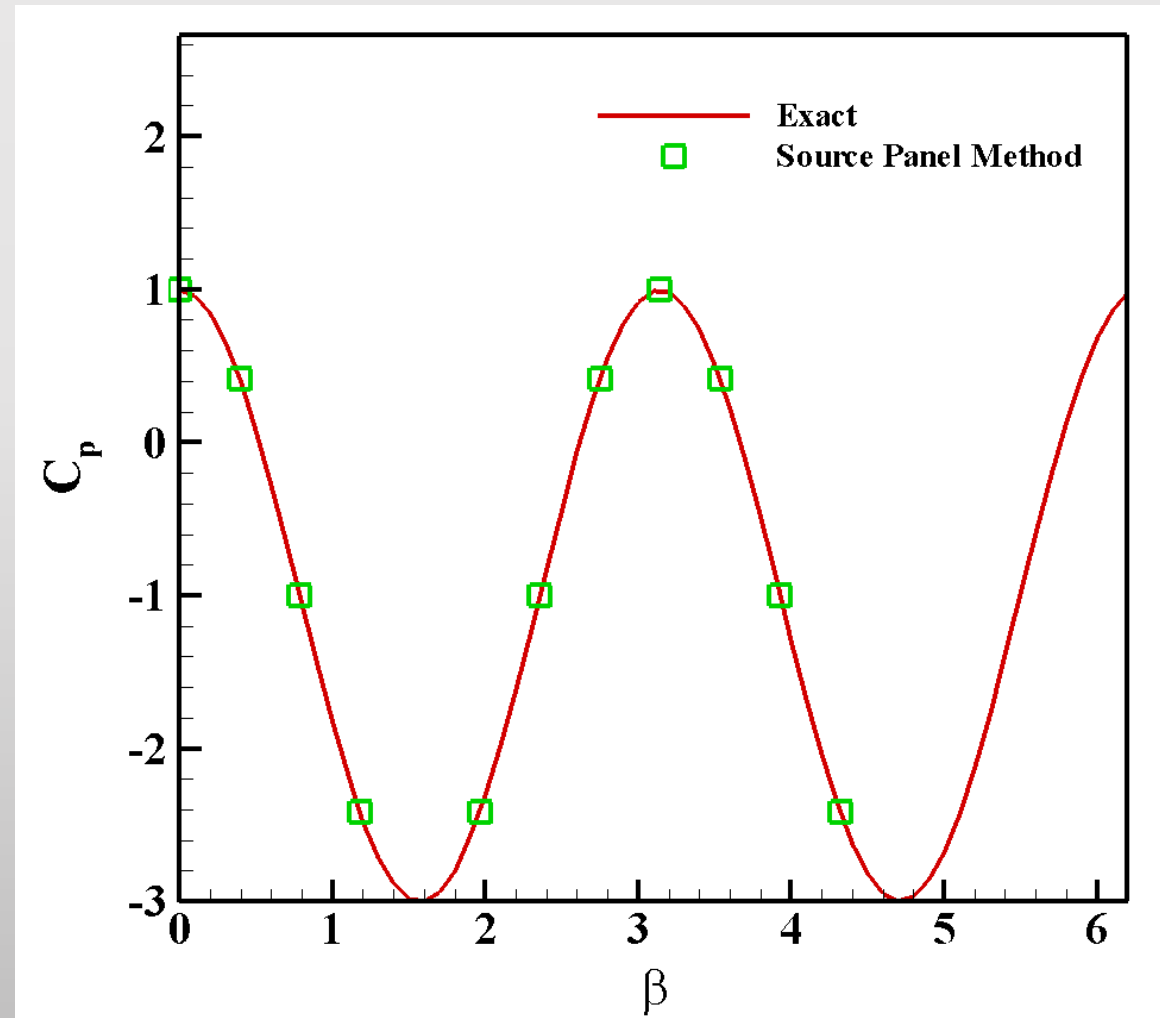
$$C = \sin(\Phi_i - \Phi_j)$$

$$D = (y_i - Y_j) \cos \Phi_i - (x_i - X_j) \sin \Phi_i$$

$$E = \sqrt{B - A^2}$$

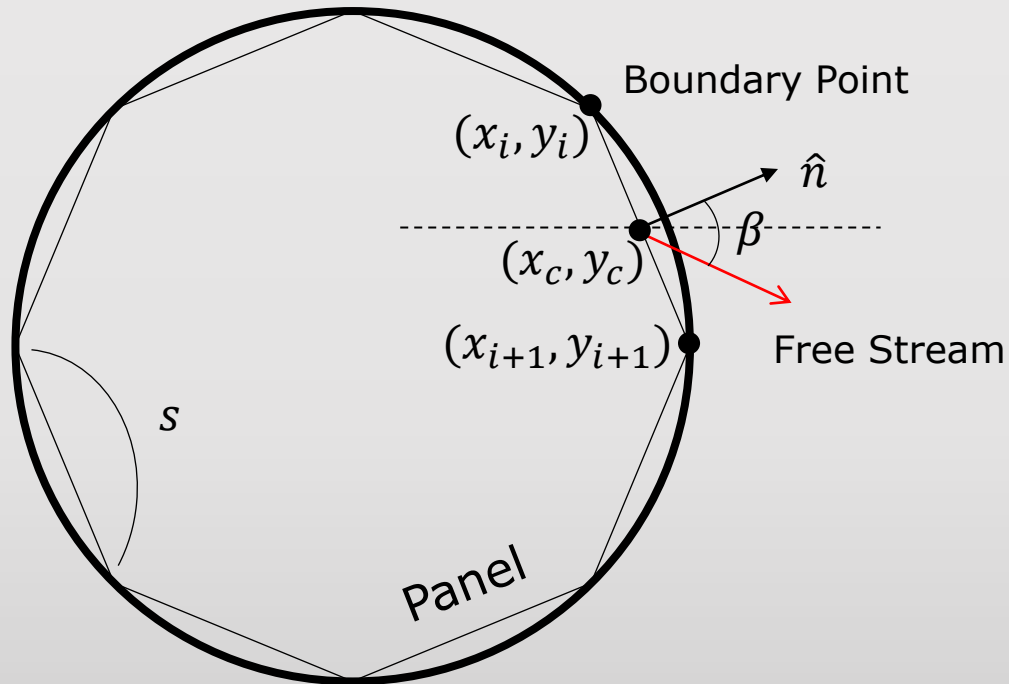
Literature Review

Coefficient of Pressure (2D Cylinder)

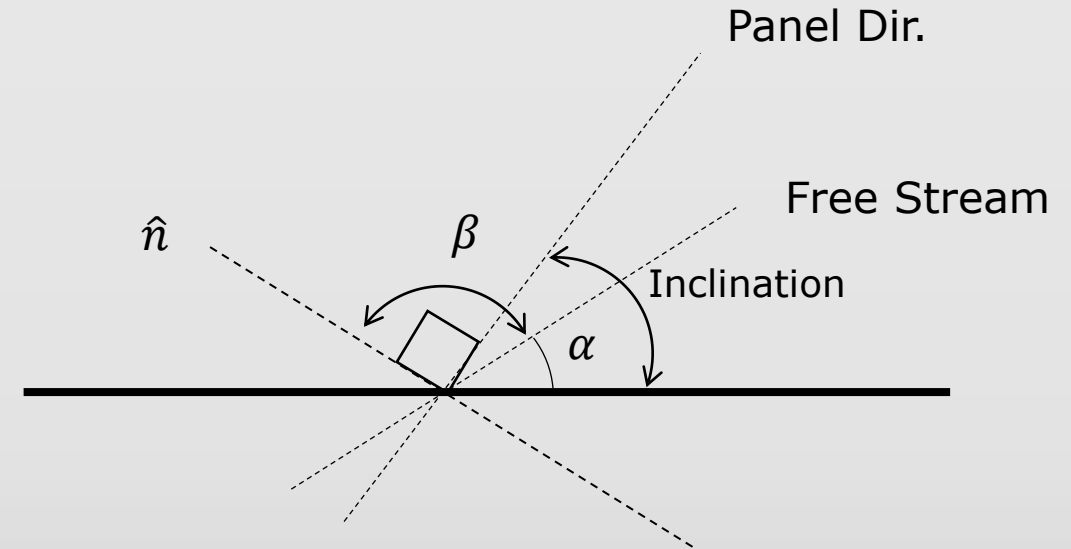


Panel Method Geometry

Panel Geometry



$$(x_c, y_c) = \left(\frac{x_i + x_{i+1}}{2}, \frac{y_i + y_{i+1}}{2} \right)$$

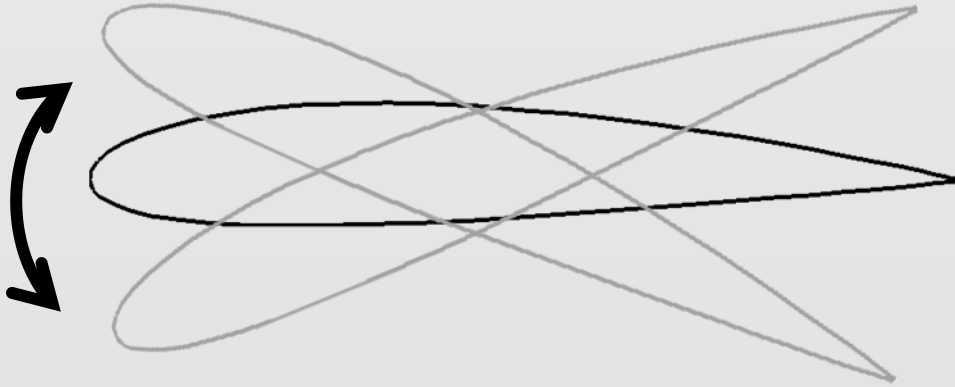


$$\beta_i = -\alpha + \boxed{\text{atan} \left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right)} + \frac{\pi}{2}$$

Inclination

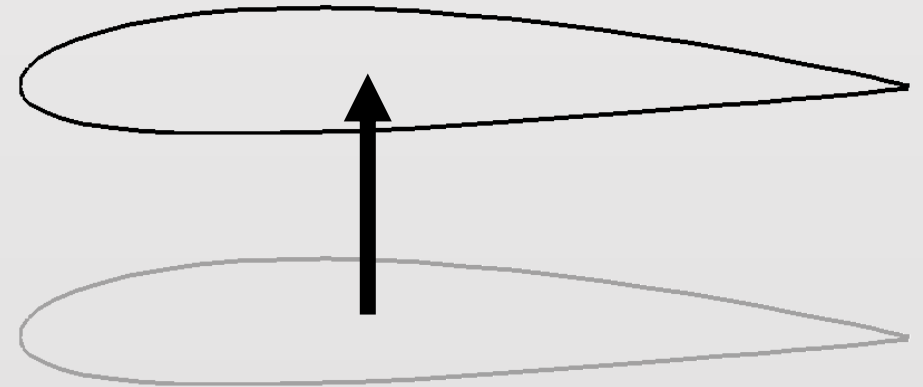
Panel Method Geometry

Airfoil Geometry



$$A = \begin{pmatrix} a & A \\ b & B \\ \vdots & \vdots \end{pmatrix} \quad R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$V = R * A'$$



$$A = \begin{pmatrix} a & A \\ b & B \\ \vdots & \vdots \end{pmatrix}$$

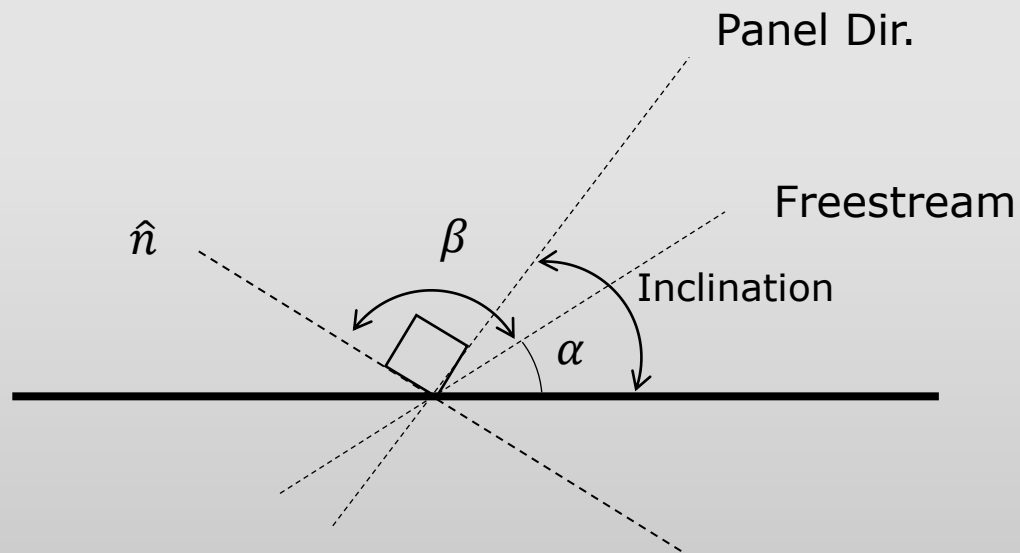
$$V = C + A$$

A : Airfoil Geometry Dat. R : Rotataion Mat. C : Constant Mat.

Source Panel Method

Mathematical Problem

The Normal Velocity at the i^{th} Eq.



$$V_n = \overset{(1)}{\frac{\lambda_i}{2}} + \overset{(2)}{\sum_{\substack{i=1 \\ j \neq i}}^n \frac{\lambda_i}{2} \int_j \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j} + \overset{(3)}{V_\infty \cos \beta_i}$$

The Total Normal Velocity Eq.

- (1) The Normal Component of Velocity Induced by the Source Panels ($i = j$)
- (2) The Normal Component of Velocity Induced by the Source Panels ($i \neq j$)
- (3) The Component of V_∞ normal to i^{th} Panel

Source Panel Method

Mathematical Problem

The Surface Velocity at the i^{th} Eq.

$$V_{s,i} = \sum_{\substack{j=1 \\ j \neq i}}^n \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial s} (\ln r_{ij}) ds_j + V_\infty \sin \beta_i$$

No – Penetration Boundary Conditions

The Pressure Coefficient

$$0 = \frac{\lambda_i}{2} + \sum_{\substack{i=1 \\ j \neq i}}^n \frac{\lambda_i}{2\pi} \int_j \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j + V_\infty \cos \beta_i$$

$$C_{p,i} = 1 - \left(\frac{V_i}{V_\infty} \right)^2$$

Source Panel Method

Mathematical Problem

Critical Integrals

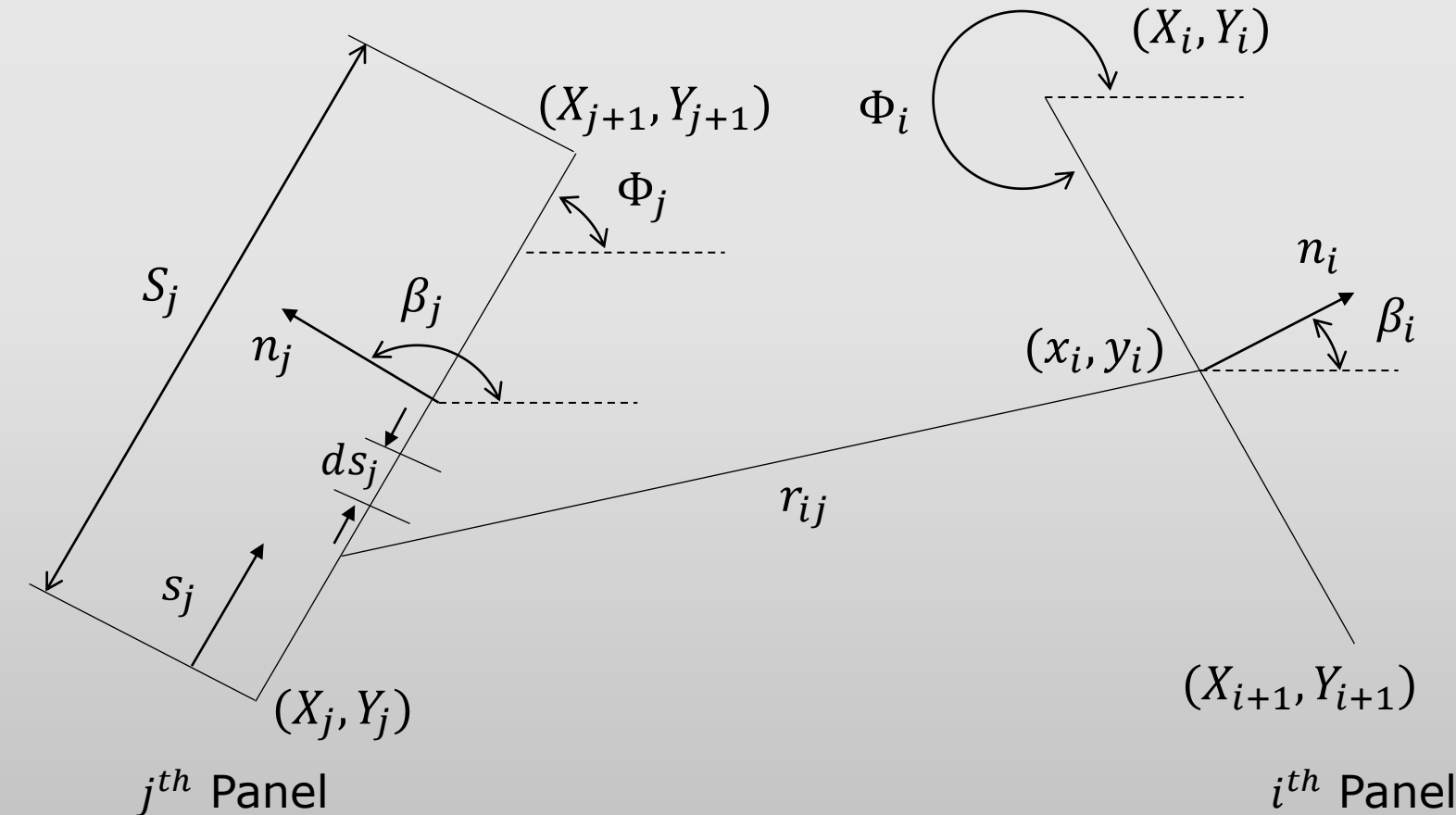
$$I_{i,j} = \int_j \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j$$

$$\begin{aligned} \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j &= \frac{1}{r_{ij}} \frac{\partial r_{ij}}{\partial n_i} \\ &= \frac{1}{r_{ij}} \frac{1}{2} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{-0.5} \cdot 2 \left[(x_i - x_j) \frac{dx_i}{dn_i} + (y_i - y_j) \frac{dy_i}{dn_i} \right] \\ &= \frac{(x_i - x_j) \cos \beta_i + (y_i - y_j) \sin \beta_i}{(x_i - x_j)^2 + (y_i - y_j)^2} \quad \text{[1.a]} \end{aligned}$$

Source Panel Method

Mathematical Problem

Critical Integrals



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$$x_j = X_j + s_j \cos \Phi_j$$

$$y_j = Y_j + s_j \sin \Phi_j$$

Therefore, **[1.a]** is

$$I_{i,j} = \int_j \frac{Cs_j + D}{s_j^2 + 2As_j + B} ds_j$$

$$A = -(x_i - X_i) \cos \Phi_j - (y_i - Y_i) \sin \Phi_j$$

$$B = (x_i - X_j)^2 + (y_i - Y_j)^2$$

$$C = \sin(\Phi_i - \Phi_j)$$

$$D = (y_i - Y_j) \cos \Phi_i - (x_i - X_j) \sin \Phi_i$$

[1.b]

Source Panel Method

Mathematical Problem

Critical Integrals

Also, We can obtain Expression **[1.b]** is

$$I_{i,j} = \frac{C}{2} \ln \left(\frac{S_j^2 + 2AS_j + B}{B} \right) + \frac{D - AC}{E} \left(\tan^{-1} \frac{S_j + A}{E} - \tan^{-1} \frac{A}{E} \right) \quad \mathbf{[1.c]}$$

$$A = -(x_i - X_i) \cos \Phi_j - (y_i - Y_i) \sin \Phi_j$$

$$B = (x_i - X_j)^2 + (y_i - Y_j)^2$$

$$C = \sin(\Phi_i - \Phi_j)$$

$$D = (y_i - Y_j) \cos \Phi_i - (x_i - X_j) \sin \Phi_i$$

$$E = \sqrt{B - A^2}$$

Source Panel Method

Mathematical Problem

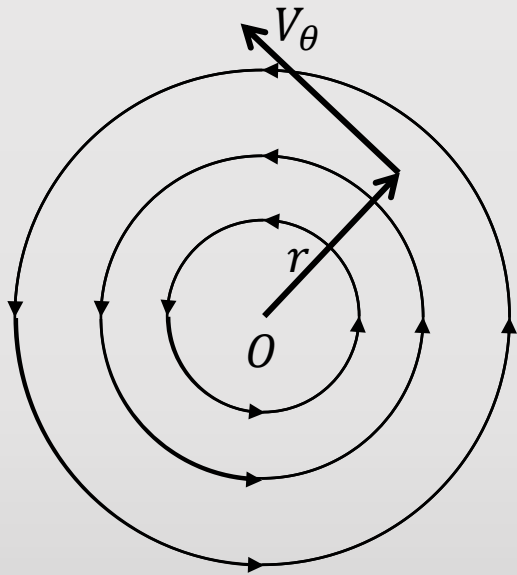
About Matrix is for N Panels

$$V_{s,i} = \sum_{\substack{j=1 \\ j \neq i}}^n \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial s} (\ln r_{ij}) ds_j + V_\infty \sin \beta_i$$

$$\begin{array}{c}
 \text{Panel Geo.} \\
 \begin{bmatrix} \pi & I_{12} & \cdots & I_{1N} \\ I_{21} & \pi & \cdots & I_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ I_{N1} & I_{N2} & \cdots & \pi \end{bmatrix}
 \end{array}
 \begin{array}{c}
 \text{Strength} \\
 \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{bmatrix}
 \end{array}
 =
 \begin{array}{c}
 \text{Free Stream} \\
 \begin{bmatrix} -V_\infty 2\pi \cos \beta_1 \\ -V_\infty 2\pi \cos \beta_2 \\ \vdots \\ -V_\infty 2\pi \cos \beta_N \end{bmatrix}
 \end{array}$$

Vortex Panel Method

Vortex Flow



If $\nabla \cdot V = 0$ and $\nabla \times V = 0$, We can obtain following Eq.

$$\Gamma = - \oint_c V \cdot ds = -V_\theta (2\pi r)$$

$$V_r = \frac{\partial \phi}{\partial r} = 0$$

$$V_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{\Gamma}{2\pi r}$$



$$\phi = -\frac{\Gamma}{2\pi} \theta$$

Stream Function

$$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

$$V_\theta = -\frac{\Gamma}{2\pi r} = -\frac{\partial \psi}{\partial r}$$

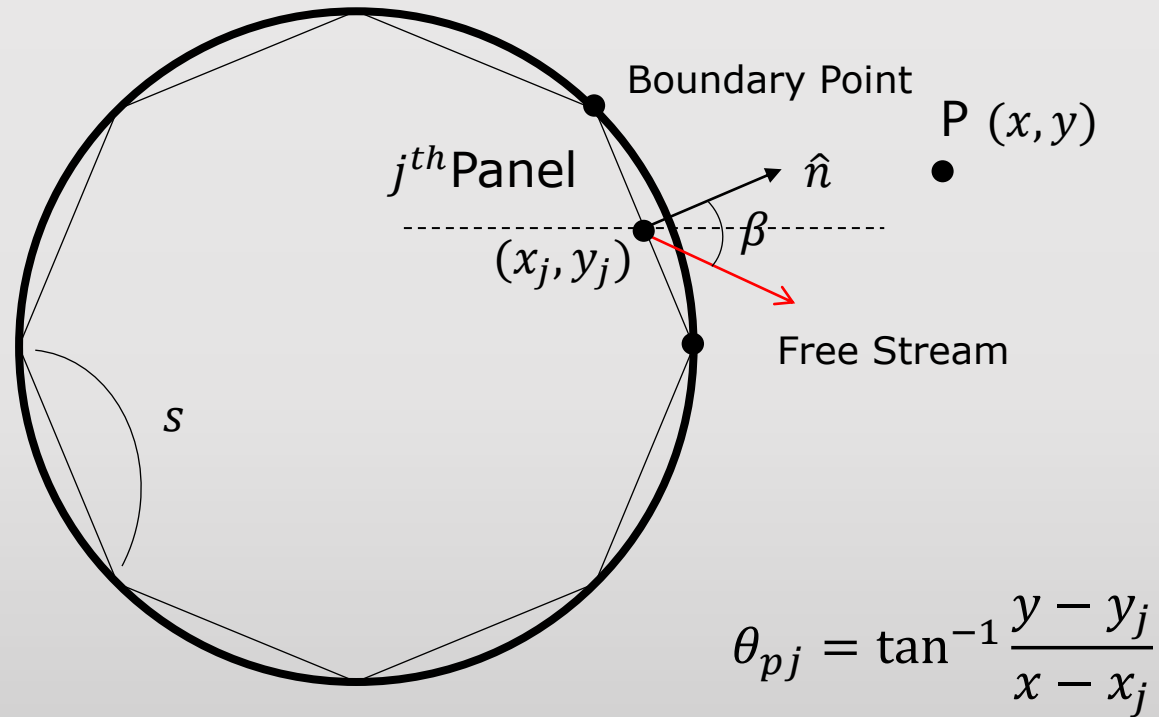


$$\psi = \frac{\Gamma}{2\pi} \ln r$$

ϕ : Velocity Potential
 ψ : Stream Function

Vortex Panel Method

Panel Geometry



The Velocity Potential Induced at P due to the j^{th} Panel

$$\Delta\phi_j = -\frac{1}{2\pi} \int_j \theta_{pj} \gamma_j ds_j$$

Vortex Panel Method

Mathematical Problem

The Normal Velocity at the i^{th} Eq.

$$V_n = \sum_{j=1}^N \overset{(1)}{\frac{-\gamma}{2\pi} \int_j \frac{\partial \theta_{ij}}{\partial n_i} ds_j} + \overset{(2)}{V_\infty \cos \beta_i}$$

(1) The Normal Component of Velocity Induced by the Vortex Panels

(2) The Component of V_∞ normal to i^{th} Panel

Source/Vortex Panel Method

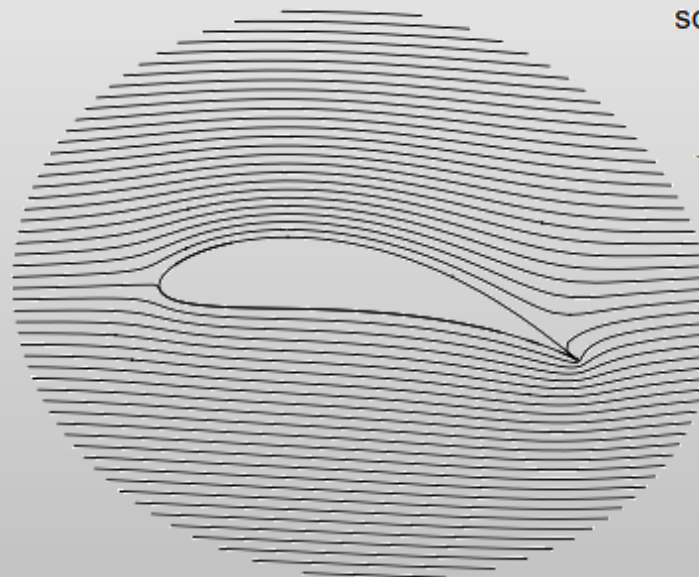
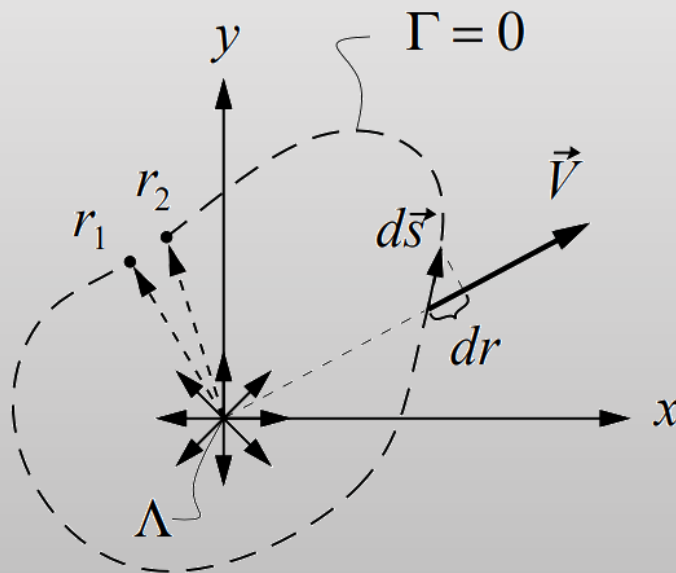
Limitation of Source Sheets

Prediction of Lift

$$\Gamma \equiv - \oint \vec{V} \cdot d\vec{s} = - \oint V_r dr = - \int_1^2 \frac{\Lambda}{2\pi r} dr = - \frac{\Lambda}{2\pi r} (\ln r_2 - \ln r_1) = 0$$

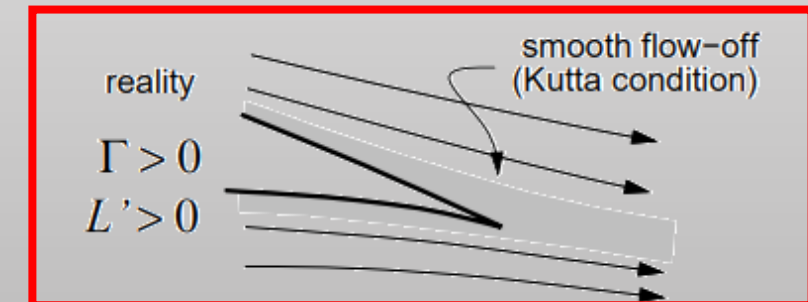
$r_1 = r_2$ for any closed circuit

Need vortices in some manners in flow representation



source sheet model

$\Gamma = 0$
 $L' = 0$



On real airfoils the flow always flow smoothly off the sharp trailing edge

Source/Vortex Panel Method

Mathematical Problem

The Normal Velocity at the i^{th} Eq.

$$V_{n,1} = \sum_{j=1}^N \frac{\lambda_i}{2\pi} \int_j \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j + \sum_{j=1}^N \frac{-\gamma}{2\pi} \int_j \frac{\partial \theta_{ij}}{\partial n_i} ds_j + V_{\infty} \cos \beta_i$$

(1) \diagdown
(2) \diagdown
(3) /

The Normal Velocity Eq.

(1) The Normal Component of Velocity Induced by the Source Panels

(2) The Normal Component of Velocity Induced by the Vortex Panels

(3) The Component of V_{∞} normal to i^{th} Panel

Source/Vortex Panel Method

Mathematical Problem

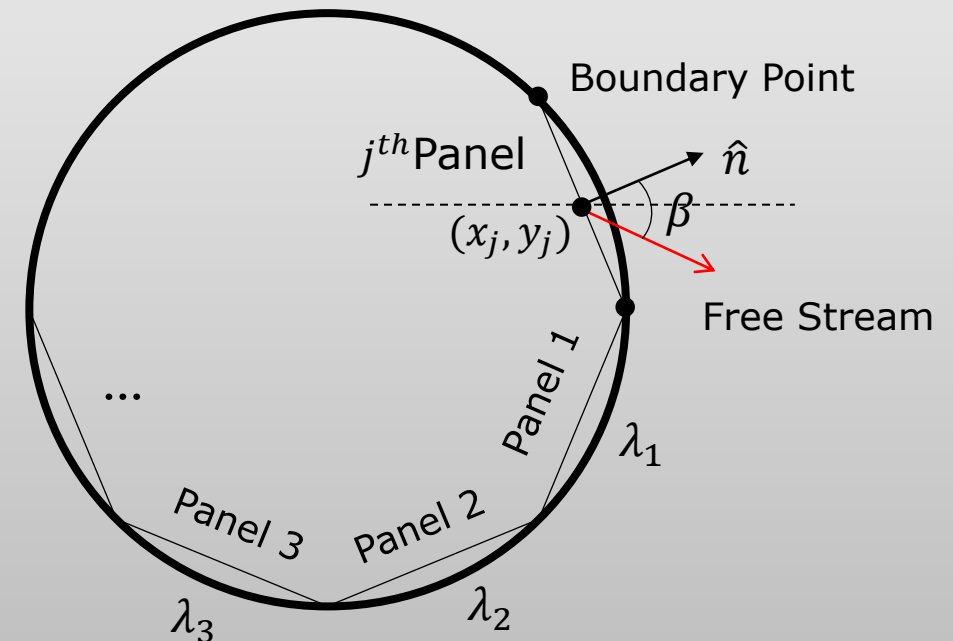
The Normal Velocity at the i^{th} Eq.

$$V_{n,1} = \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\lambda_j I_{ij}}{2\pi} + \frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ j \neq i}}^N \frac{-\gamma K_{ij}}{2\pi} + 0 + V_\infty \cos \beta_i = 0$$

$$\sum_{j=i} \frac{\lambda_j I_{ij}}{2\pi} = \frac{\lambda_i}{2} \qquad \sum_{j=i} \frac{-\gamma K_{ij}}{2\pi} = 0$$

The Normal Velocity Eq. for Panel Geometry

$$\begin{aligned} \lambda_1 \pi + (\lambda_2 I_{12} + \lambda_3 I_{13} + \dots) - \gamma(K_{12} + K_{13} + \dots) &= -V_\infty 2\pi \cos \beta_1 \\ \lambda_2 \pi + (\lambda_1 I_{21} + \lambda_3 I_{23} + \dots) - \gamma(K_{21} + K_{23} + \dots) &= -V_\infty 2\pi \cos \beta_2 \\ \lambda_3 \pi + (\lambda_1 I_{31} + \lambda_2 I_{32} + \dots) - \gamma(K_{31} + K_{32} + \dots) &= -V_\infty 2\pi \cos \beta_3 \\ &\vdots \end{aligned}$$



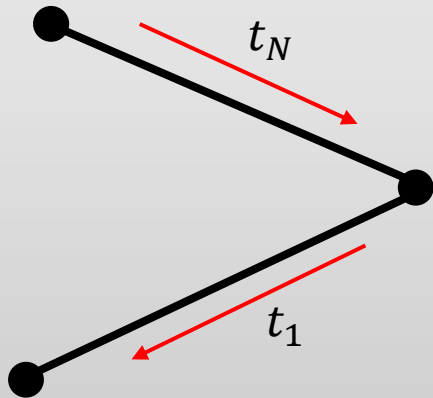
Source/Vortex Panel Method

Mathematical Problem

Kutta Condition

Approximate Kutta Condition by setting First and Last panel velocity equal to each other

$$V_{t,N} = -V_{t,1}$$



Each Panel's tangential velocity

$$V_{t,1} = V_{\infty} \sin \beta_1 + \sum_{j=2}^N \frac{\lambda_j J_{1j}}{2\pi} + \frac{1}{2} \gamma_1 - \sum_{j=2}^N \frac{\gamma L_{1j}}{2\pi}$$

$$V_{t,N} = V_{\infty} \sin \beta_N + \sum_{j=1}^{N-1} \frac{\lambda_j J_{Nj}}{2\pi} + \frac{1}{2} \gamma_N - \sum_{j=1}^{N-1} \frac{\gamma L_{Nj}}{2\pi}$$

Source/Vortex Panel Method

Mathematical Problem

The Normal Velocity Mat. for Panel Geometry

$$\begin{array}{c} \text{Mat. A} \end{array} \begin{pmatrix} \pi & I_{12} & I_{13} & -(K_{12} + K_{13} + \dots) \\ I_{21} & \pi & I_{23} & -(K_{21} + K_{23} + \dots) \\ I_{31} & I_{32} & \pi & -(K_{31} + K_{32} + \dots) \\ \dots & \dots & \dots & \dots \\ J_{N1} & A & J_{1N} & B \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \dots \\ \gamma \end{pmatrix} = \begin{pmatrix} -V_\infty 2\pi \cos \beta_1 \\ -V_\infty 2\pi \cos \beta_2 \\ -V_\infty 2\pi \cos \beta_3 \\ \dots \\ -V_\infty 2\pi (\sin \beta_1 + \sin \beta_N) \end{pmatrix}$$

Kutta Condition

$$A = J_{N2} + \dots + J_{N(N-1)} + J_{12} + \dots + J_{1(N-1)}$$

$$B = -(L_{12} + \dots + L_{1N} + L_{N1} + \dots + L_{N(N-1)}) + 2\pi$$

Source/Vortex Panel Method

Mathematical Problem

Circulation Calculation

$$\Gamma = - \oint_c \vec{V} \cdot d\vec{s}$$

\vec{V} : Velocity Vector
 $d\vec{s}$: Contour Vector

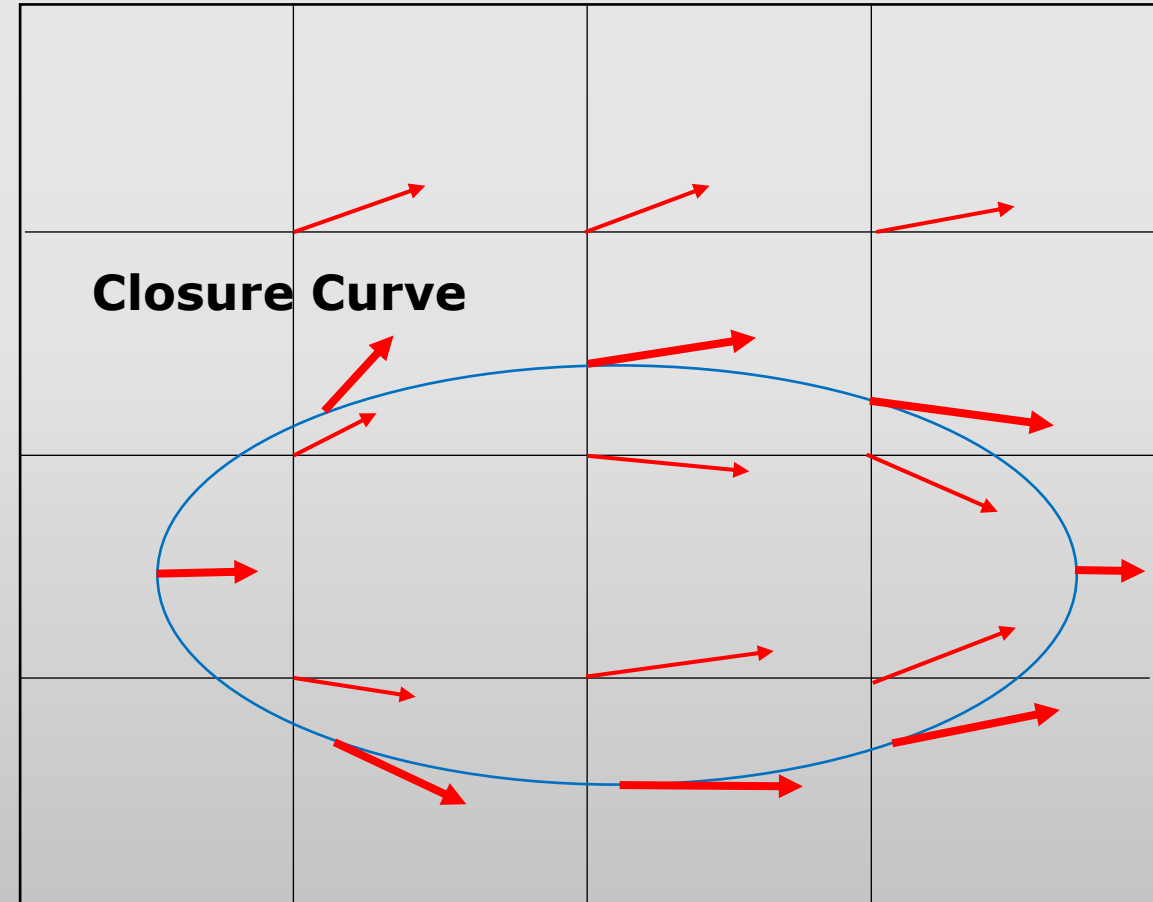
$$L' = \rho V \Gamma$$

L' : Lift per unit Span

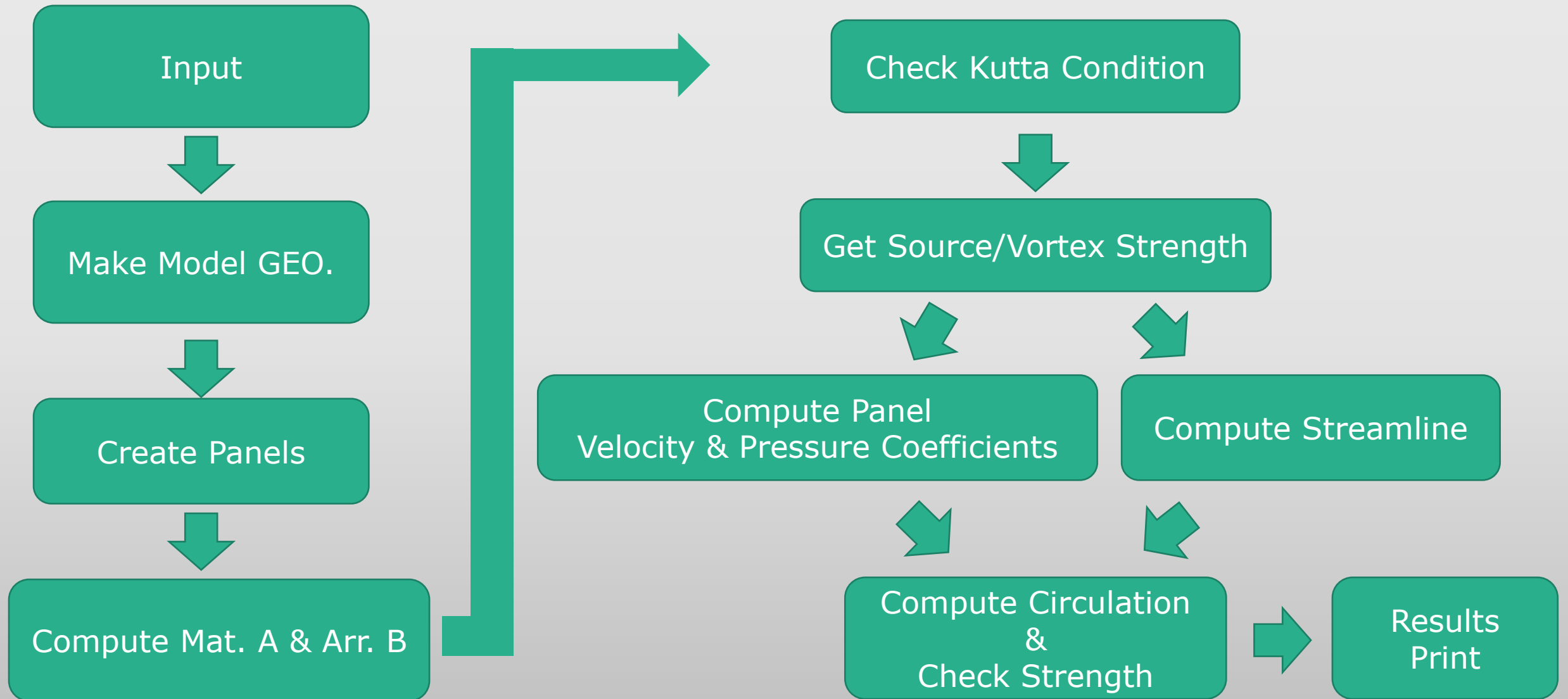
Write Integral as Sum of x and y

$$\begin{aligned} \Gamma &= - \oint_c \vec{V} \cdot d\vec{s} \\ &= - \oint_c (V_x \hat{i} + V_y \hat{j}) \cdot (dx \hat{i} + dy \hat{j}) \\ &= - \oint_c V_x dx - \oint_c V_y dy \end{aligned}$$

Interpolate Velocity to Ellipse Points



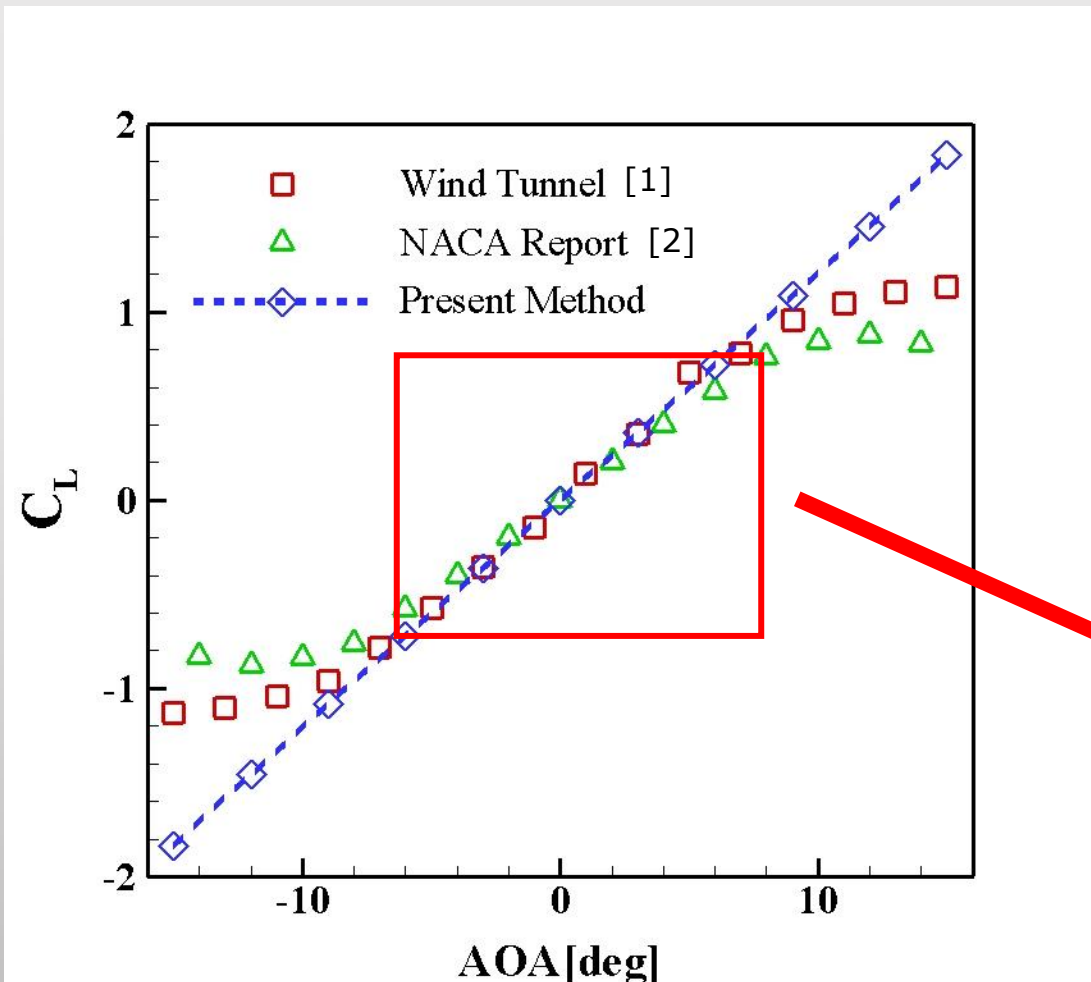
Flow Chart



Validation

1. ADAM BILČÍK and ROBERT POPELA, "COMPARISON OF PANEL CODES FOR AERODYNAMIC ANALYSIS OF AIRFOILS," BRNO UNIVERSITY OF TECHNOLOGY ,2014
2. NACA Report No.586

Comparison Present Method with REF[1-2]



Convergence according to Panel No.

Panel Num.	Cl[G]	Cl[Circ]	AOA[deg]
150	-1.8405042	-1.8405048	-15
175	-1.837074	-1.8370746	-15
200	-1.8339829	-1.8339834	-15

$$Cl[G] = \frac{\rho V \oint \gamma}{0.5 \rho V^2 C}$$

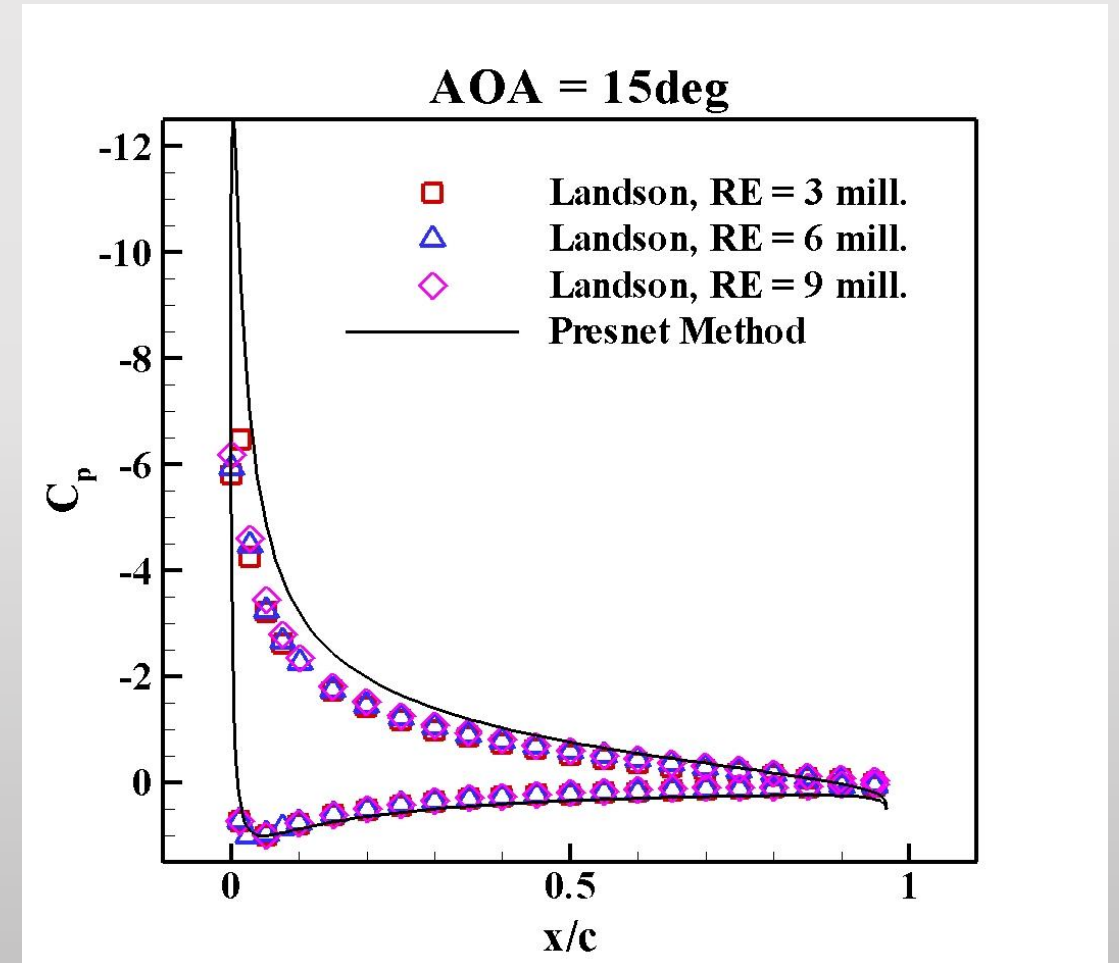
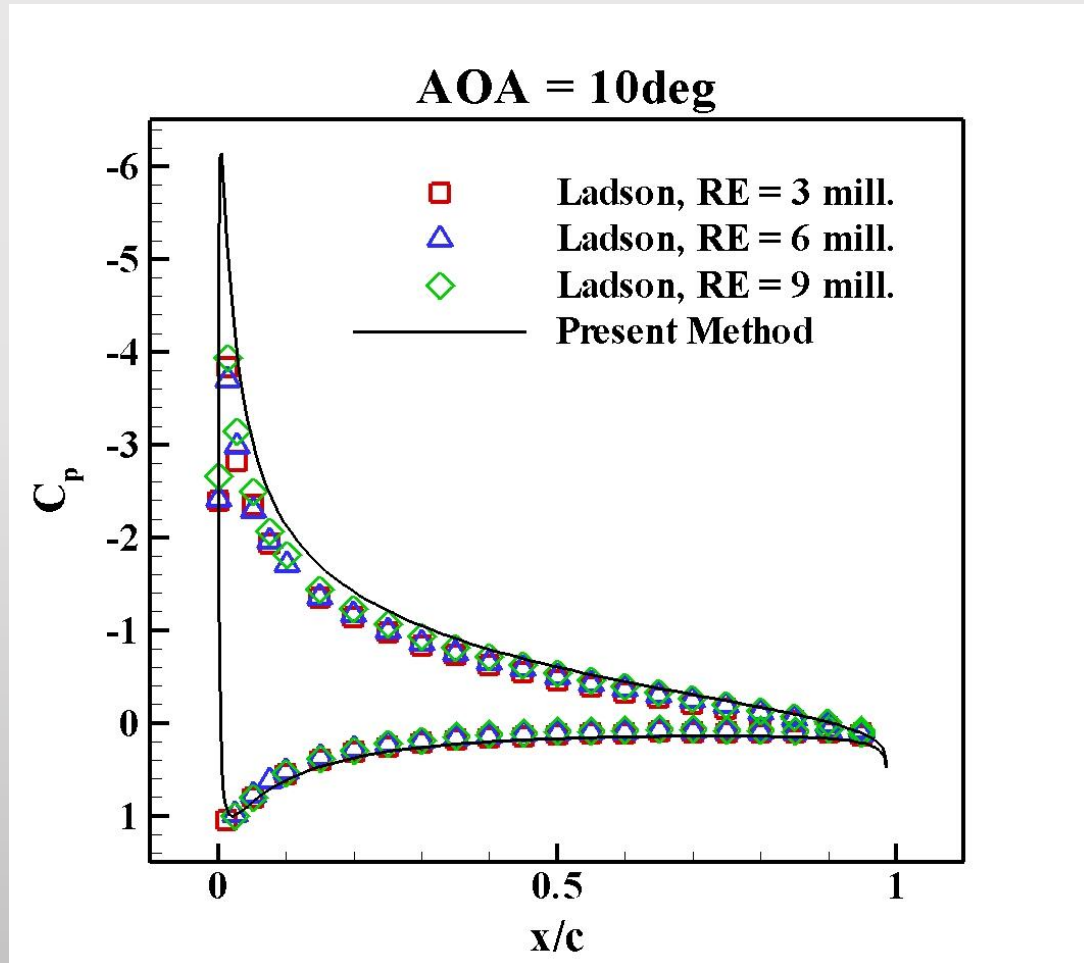
$$Cl[Circ] = \frac{\Gamma}{0.5 \rho V^2 C}$$

AOA – Cl Data Table

AOA	Cl[1]	Cl[2]	Cl[G]	Cl[Circ]
-5	-0.57316	-0.49	-0.5992518	-0.5994532
-4	-0.45743	-0.4	-0.4789728	-0.4791165
-3	-0.35207	-0.3	-0.3589819	-0.3590791
-2	-0.240864	-0.2	-0.2392049	-0.2392646
-1	-0.144169	-0.1	-0.1195687	-0.1195969
0	0.049622	0	0	0
1	0.144169	0.1	0.1195687	0.1195969
2	0.240864	0.2	0.2392049	0.2392646
3	0.352074	0.3	0.3589819	0.3590791
4	0.457429	0.4	0.4789728	0.4791165
5	0.678328	0.49	0.5992518	0.5994532

Validation

Comparison Present Method with REF[1]



GUI Design

