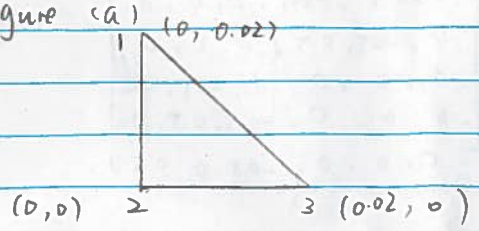
ECSE 543
Assignment 2

Question 1

In figure (a)



$$A = \frac{0.02 \times 0.02}{2} = 0.0002\ m^2$$

We want to minimize the energy in this triangle

$$W^{(e)} = \frac{1}{2}\int_{\Delta e} |\nabla U|^2 ds = \frac{1}{2}\sum_i \sum_j U_i U_j \int_{\Delta e} \nabla a_i \cdot \nabla a_j \, ds = \frac{1}{2} U^T S^{(e)} U$$

where $S_{ij}^{(e)} = \int_e \nabla a_i \cdot \nabla a_j \, ds = \nabla a_i \cdot \nabla a_j \cdot A$

$\nabla a_1 = \langle y_2 - y_3, x_3 - x_2 \rangle \cdot \frac{1}{2A}$ $\quad x_1 = 0$ $\quad y_1 = 0.02$

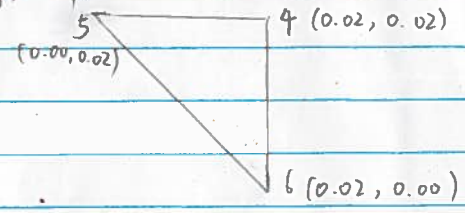
$\nabla a_2 = \langle y_3 - y_1, x_1 - x_3 \rangle \cdot \frac{1}{2A}$ $\quad x_2 = 0$ $\quad y_2 = 0$

$\nabla a_3 = \langle y_1 - y_2, x_2 - x_1 \rangle \cdot \frac{1}{2A}$ $\quad x_3 = 0.02$ $\quad y_3 = 0$

Plug in the values

$$S^{(1)} = \begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 1 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

Similarly, for



$A = 0.0002\ m^2$

$\nabla a_4 = \langle y_5 - y_6, x_6 - x_5 \rangle \cdot \frac{1}{2A}$ $\quad x_4 = 0.02$ $\quad y_4 = 0.02$

$\nabla a_5 = \langle y_6 - y_4, x_4 - x_6 \rangle \cdot \frac{1}{2A}$ $\quad x_5 = 0$ $\quad y_5 = 0.02$

$\nabla a_6 = \langle y_4 - y_5, x_5 - x_4 \rangle \cdot \frac{1}{2A}$ $\quad x_6 = 0.02$ $\quad y_6 = 0$

$$S^{(2)} = \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \end{bmatrix}$$