Time Series Analysis

Module objectives

- Introduce time series
- Components of time series
- Deseasonalising a time series

What is a time series?

- Essentially, Time Series is a sequence of numerical data obtained at regular time intervals.

- Occurs in many areas: economics, finance, environment, medicine

- The aims of time series analysis are
  - to describe and summarize time series data,
  - fit models, and make forecasts

Why are time series data different from other data?

- Data are not independent
  - Much of the statistical theory relies on the data being independent and identically distributed
- Large samples sizes are good, but long time series are not always the best
  - Series often change with time, so bigger isn’t always better

What Are Users Looking for in an Economic Time Series?

- Important features of economic indicator series include
  - Direction
  - Turning points
  - In addition, we want to see if the series is increasing/decreasing more slowly/faster than it was before

When should time series analysis best be used?

- We do not assume the existence of deterministic model governing the behaviour of the system considered.
- Instances where deterministic factors are not readily available and the accuracy of the estimate can be compromised on the need..(be careful!)
- We will only consider univariate time series

Forecasting Horizons

- Long Term
  - 5+ years into the future
  - R&D, plant location, product planning
  - Principally judgement-based
- Medium Term
  - 1 season to 2 years
  - Aggregate planning, capacity planning, sales forecasts
  - Mixture of quantitative methods and judgement
- Short Term
  - 1 day to 1 year, less than 1 season
  - Demand forecasting, staffing levels, purchasing, inventory levels
  - Quantitative methods

Examples of Time series data

- Number of babies born in each hour
- Daily closing price of a stock.
- The monthly trade balance of Japan for each year.
- GDP of the country, measured each year.

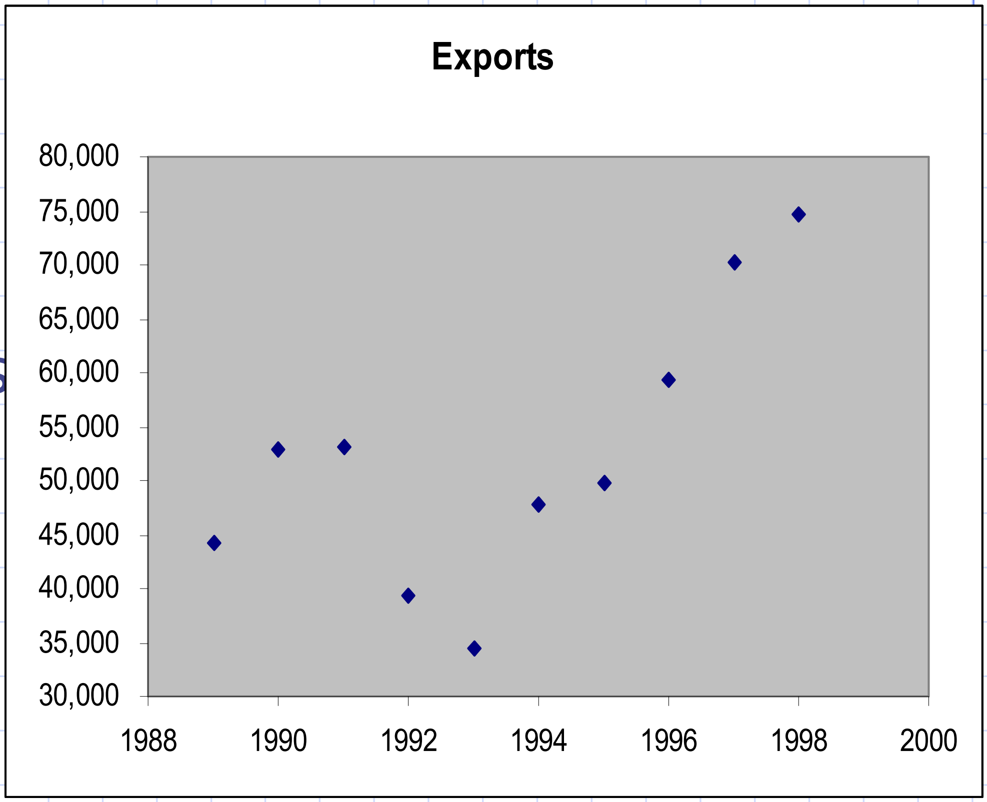
Time Series example

- How the data (x) and time (t) is recorded and presented

Exports, 1989-1998		
t	Year	x=Value
1	1989	44,320
2	1990	52,865
3	1991	53,092
4	1992	39,424
5	1993	34,444
6	1994	47,870
7	1995	49,805
8	1996	59,404
9	1997	70,214
10	1998	74,626

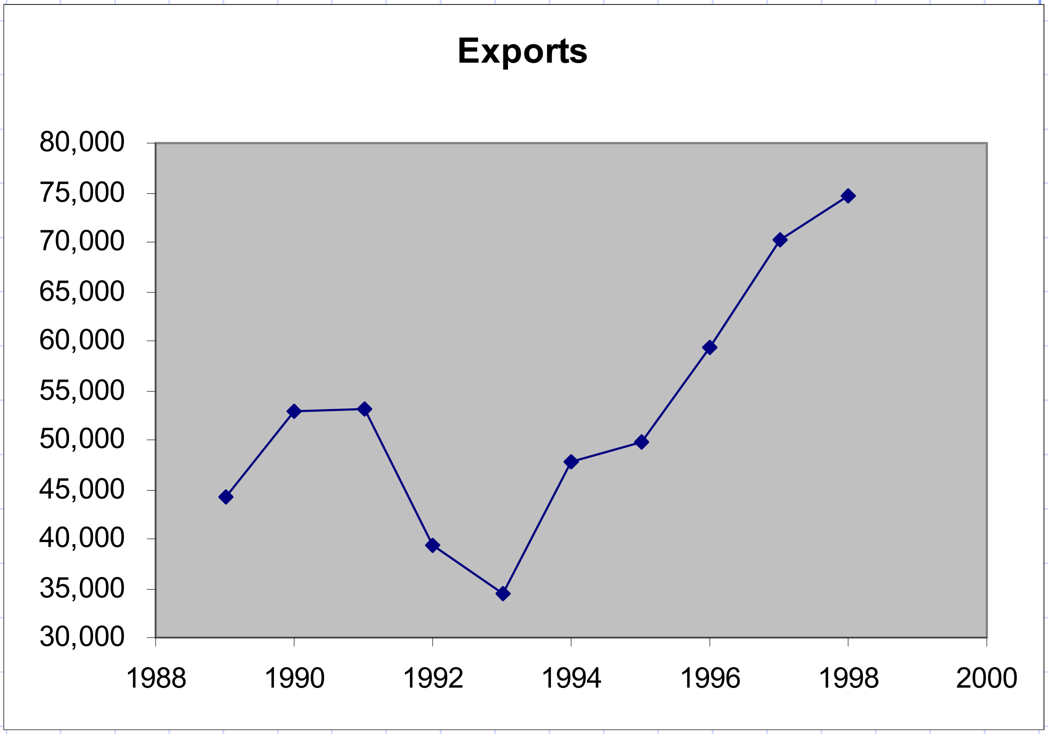
Time Series

- Coordinates (t,x) is established in the 2 axis
- (1, **44,320)**
- (2, **52,865)**
- (3, **53,092)**
- etc..



Time Series

- A graphical representation of time series.
- We use x as a function of t: x= f(t)
- Data points connected by a curve



Importance of time series analysis

- Understand the past.

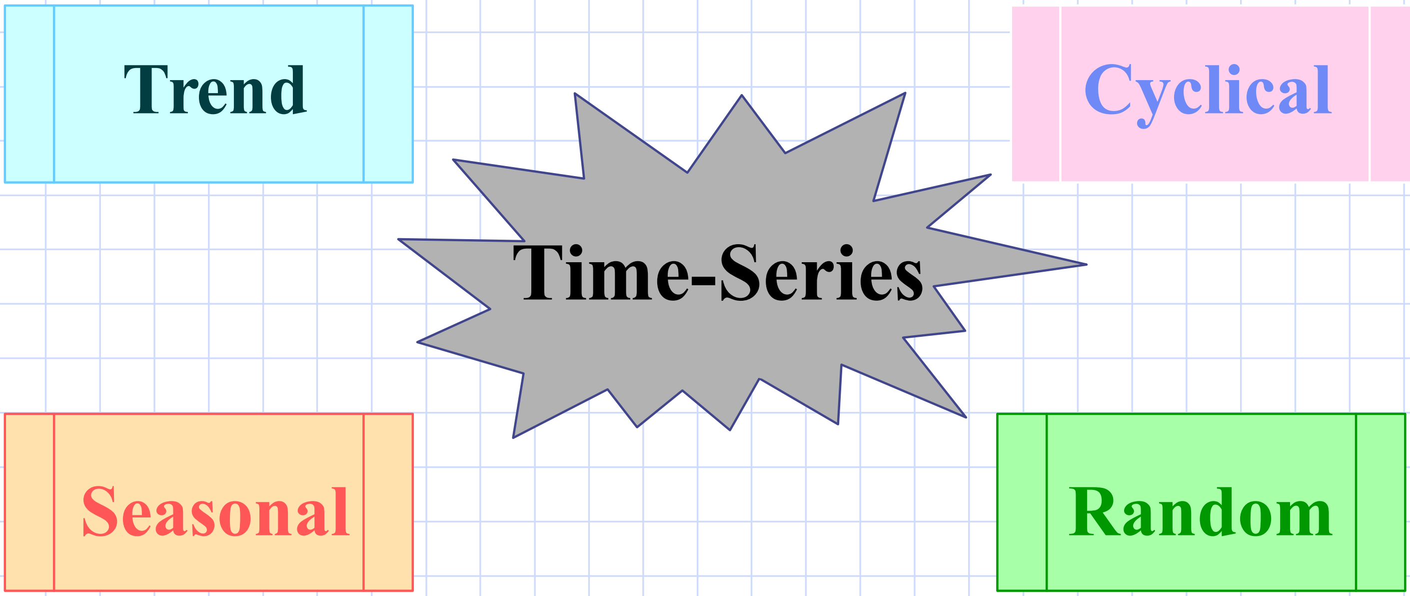
  What happened over the last years, months?

- Forecast the future.

  Government wants to know future of unemployment rate, percentage increase in cost of living etc.

  For companies to predict the demand for their product etc.

Time-Series Components



Components of Time Series

- Trend (T_t)
- Seasonal variation (S_t)
- Cyclical variation (C_t)
- Random variation (R_t) or irregular

Components of Time Series

Trend (T_t)

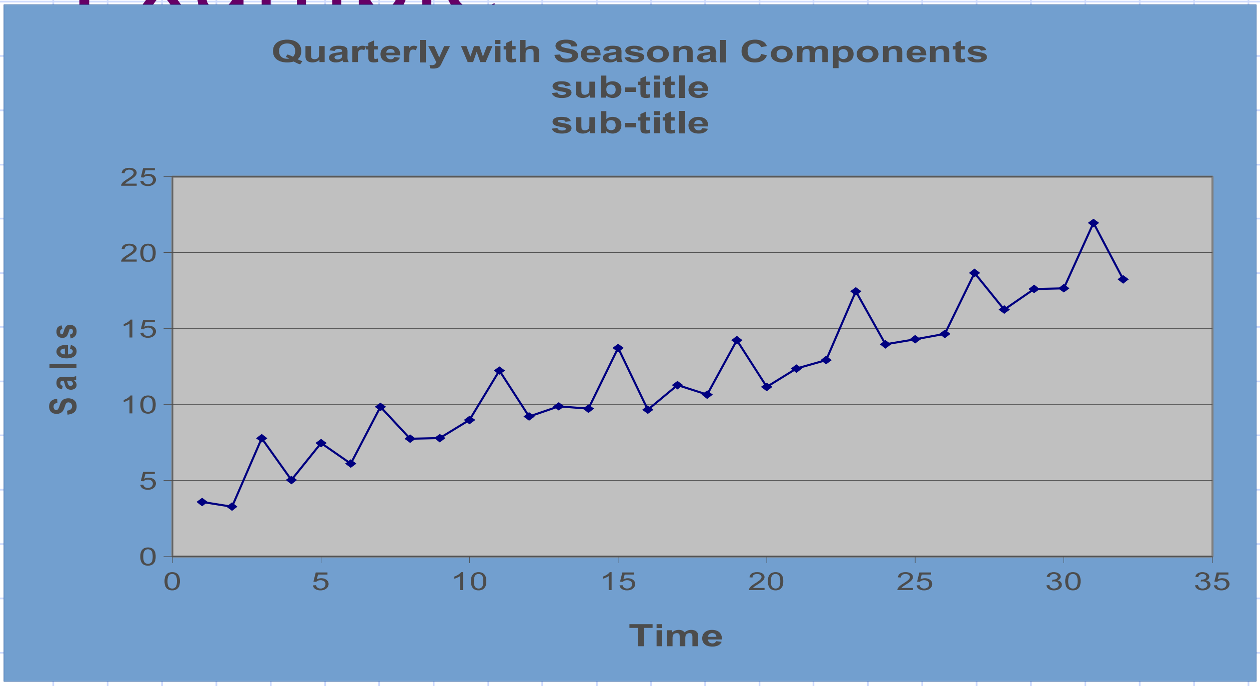
- Trend: the long-term patterns or movements in the data.
- Overall or persistent, long-term upward or downward pattern of movement.
- The trend of a time series is not always linear.

Components of Time Series

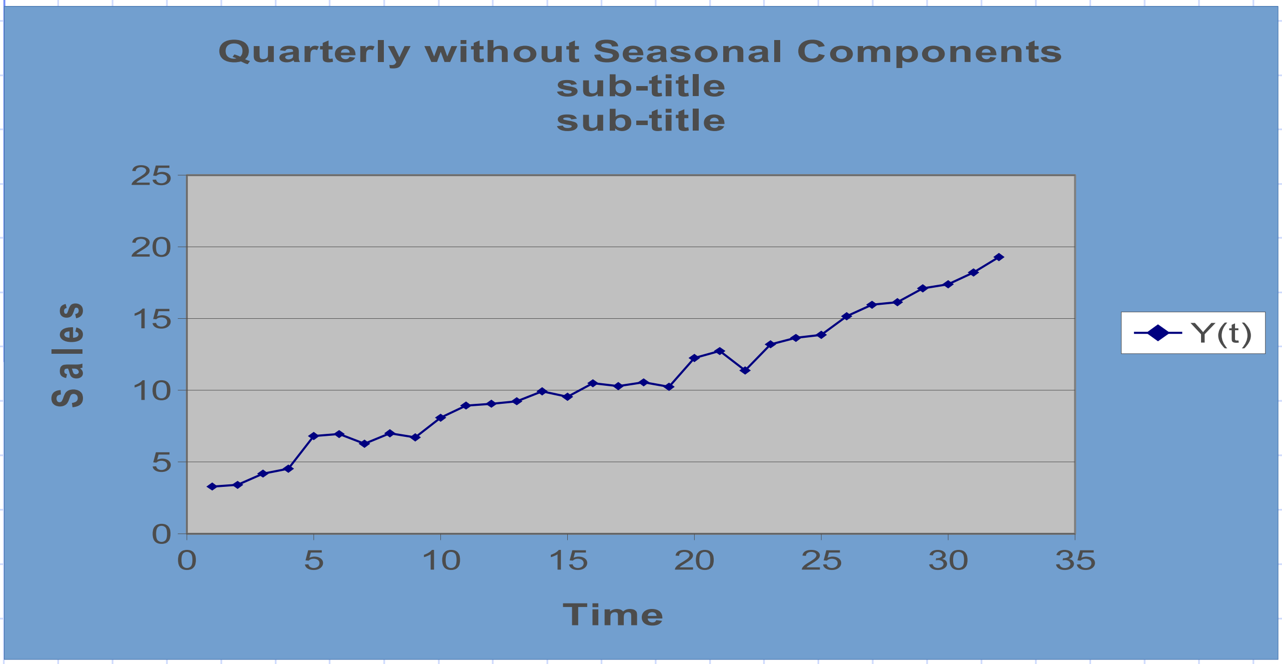
Seasonal variation (S_t)

- Regular periodic fluctuations that occur within year.
- Examples:
- Consumption of heating oil, which is high in winter, and low in other seasons of year.
- Gasoline consumption, which is high in summer when most people go on vacation.

Example


Quarterly with Seasonal Components
sub-title
sub-title
Sales
25
20
15
10
5
0
0
5
10
15
20
25
30
35
Time

Seasonal Components Removed



Why Do Users Want Seasonally Adjusted Data?

Seasonal movements can make features difficult or impossible to see

Causes of Seasonal Effects

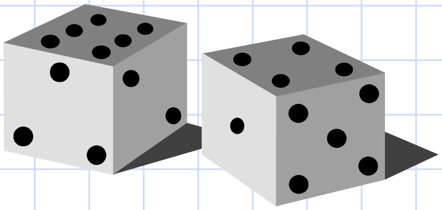
- Possible causes are
  - Natural factors
  - Administrative or legal measures
  - Social/cultural/religious traditions (e.g., fixed holidays, timing of vacations)

Components of Time Series
Cyclical variation (C_t)

- Cyclical variations are similar to seasonal variations. Cycles are often irregular both in height of peak and duration.
- Examples:
- Long-term product demand cycles.
- Cycles in the monetary and financial sectors. (Important for economists!)

Irregular Component

- Unpredictable, random, “residual” fluctuations
- Due to random variations of
  - Nature
  - Accidents or unusual events
- “Noise” in the time series



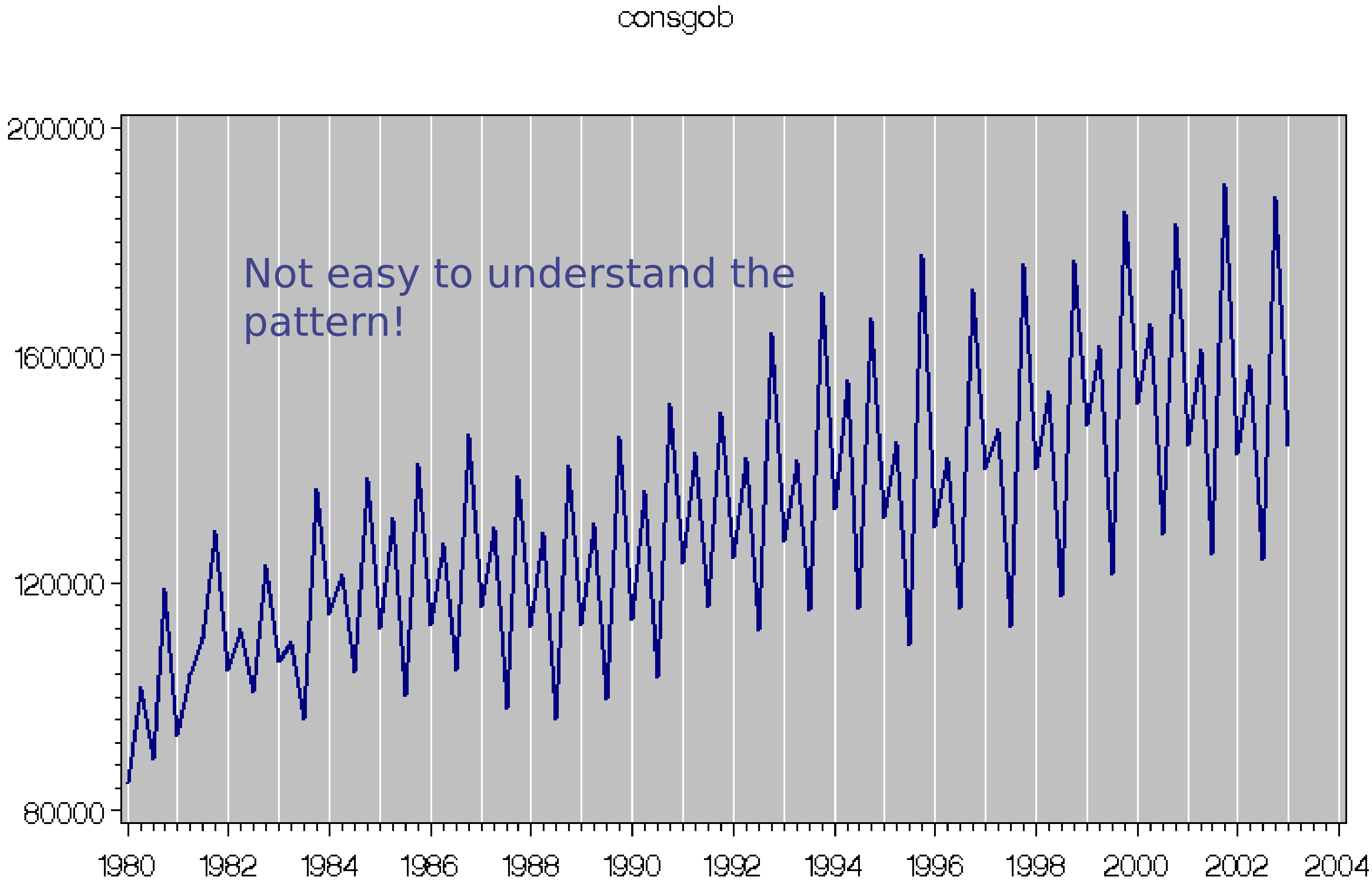
Causes of Irregular Effects

- Possible causes
  - Unseasonable weather/natural disasters
  - Strikes
  - Sampling error
  - Nonsampling error

Classical Decomposition

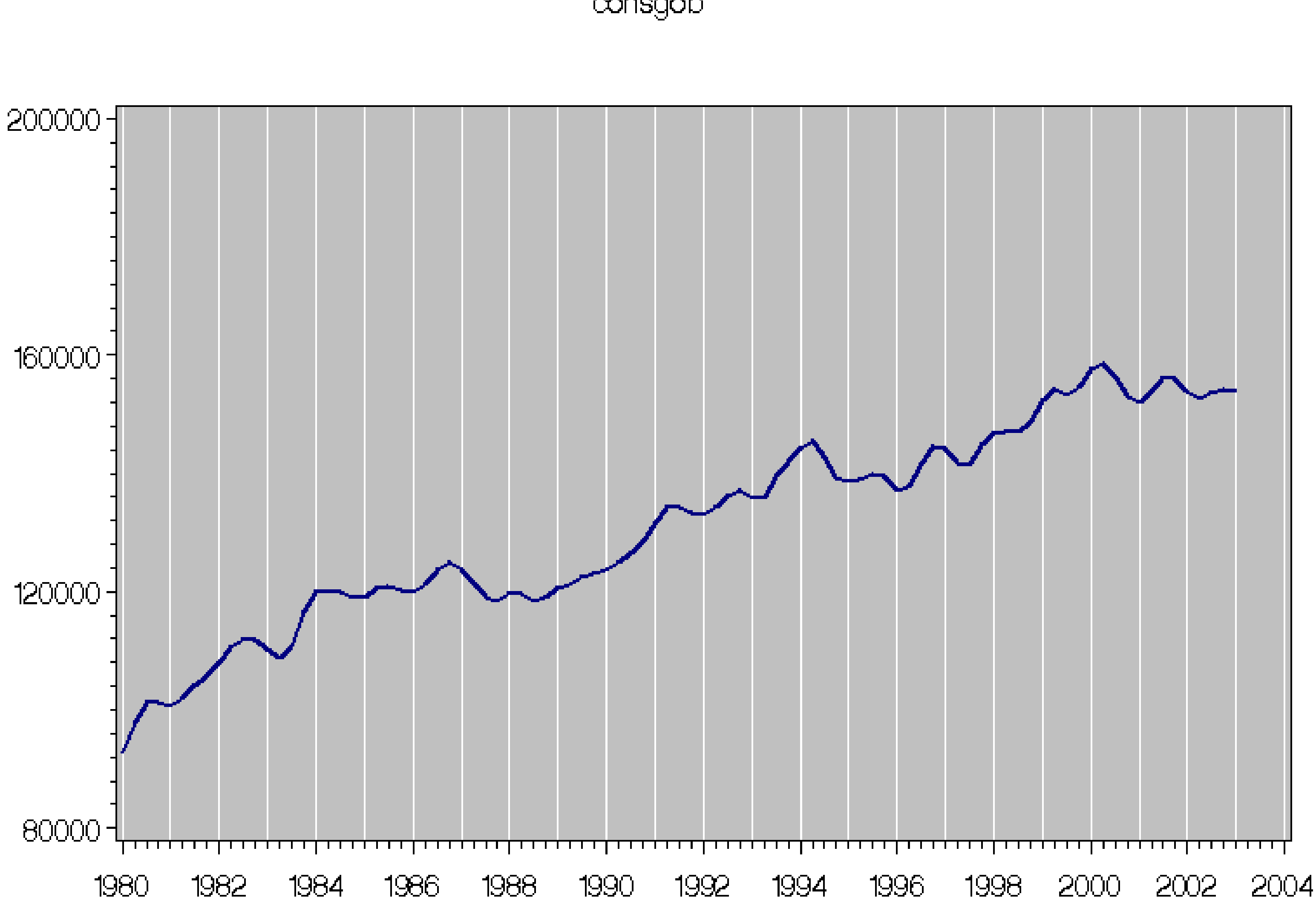
- One method of describing a time series
- Decompose the series into various components
  - Trend – long term movements in the level of the series
  - Seasonal effects – cyclical fluctuations reasonably stable in terms of annual timing (including moving holidays and working day effects)
  - Cycles – cyclical fluctuations longer than a year
  - Irregular – other random or short-term unpredictable fluctuations

Original Series



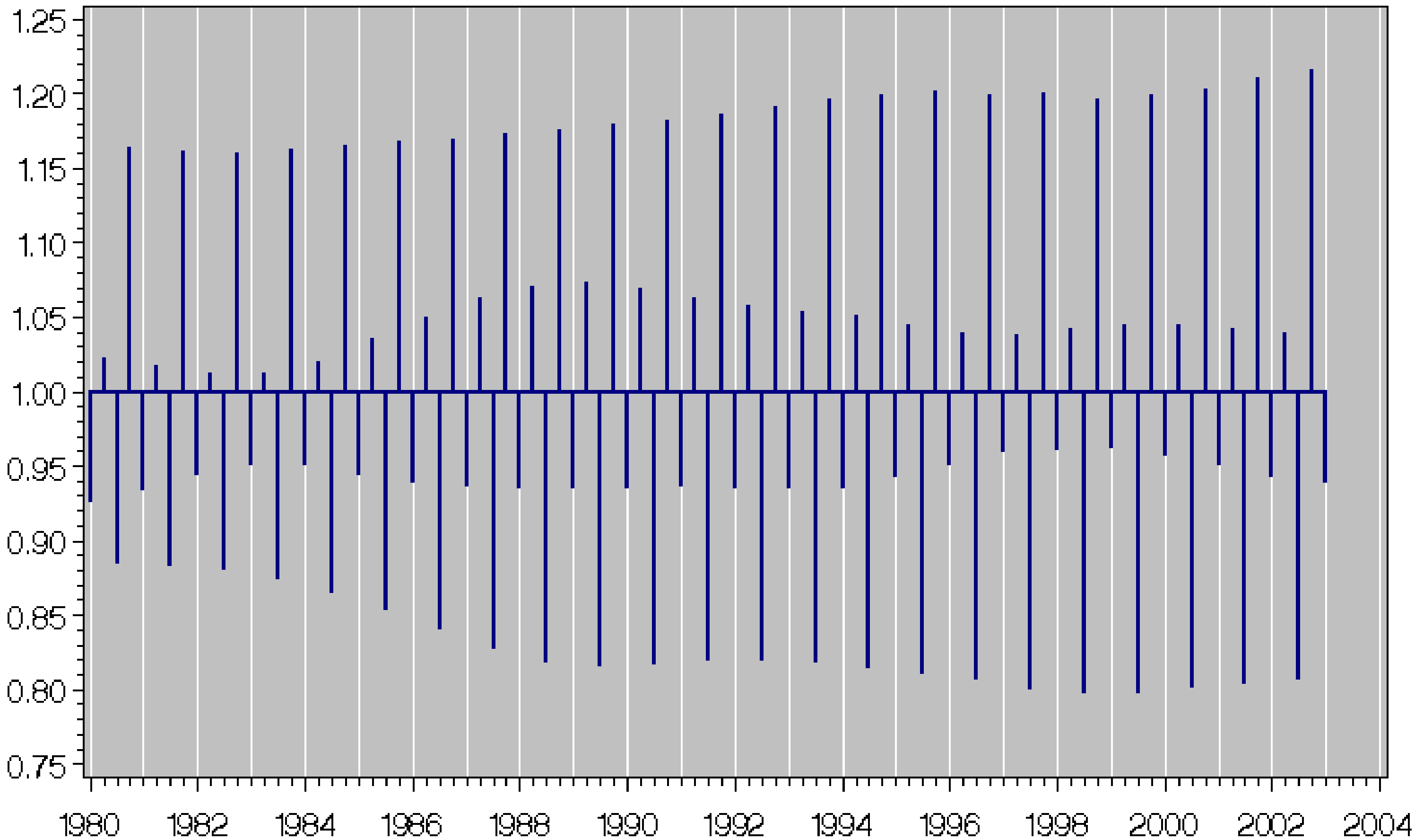
Our aim

- is to understand and identify different variations so that we can easily predict the future variations separately and combine together
- Look how the above complicated series could be understood as follows separately


consgob
200000
160000
120000
80000
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004

Seasonal Factors

consgob



Irregular

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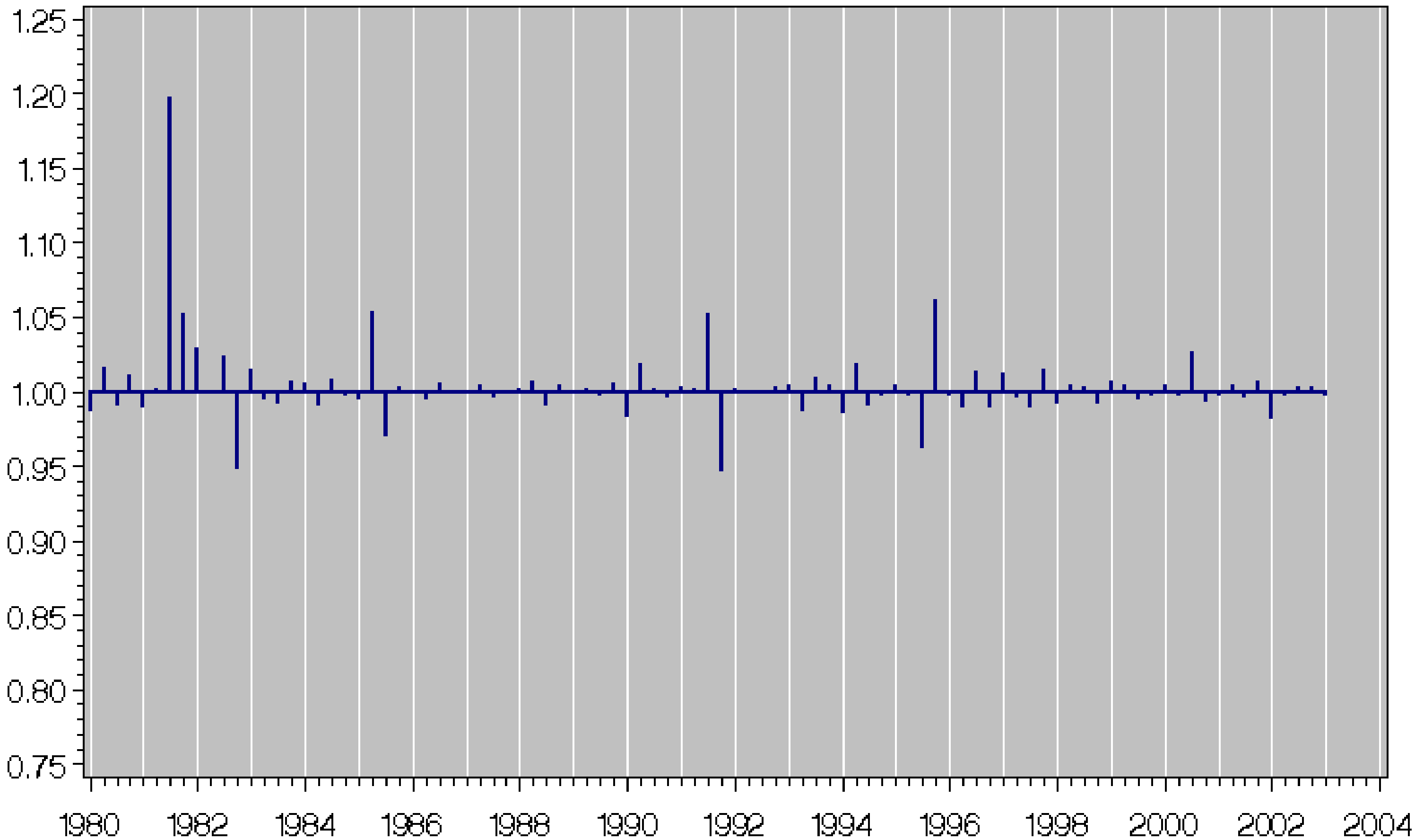
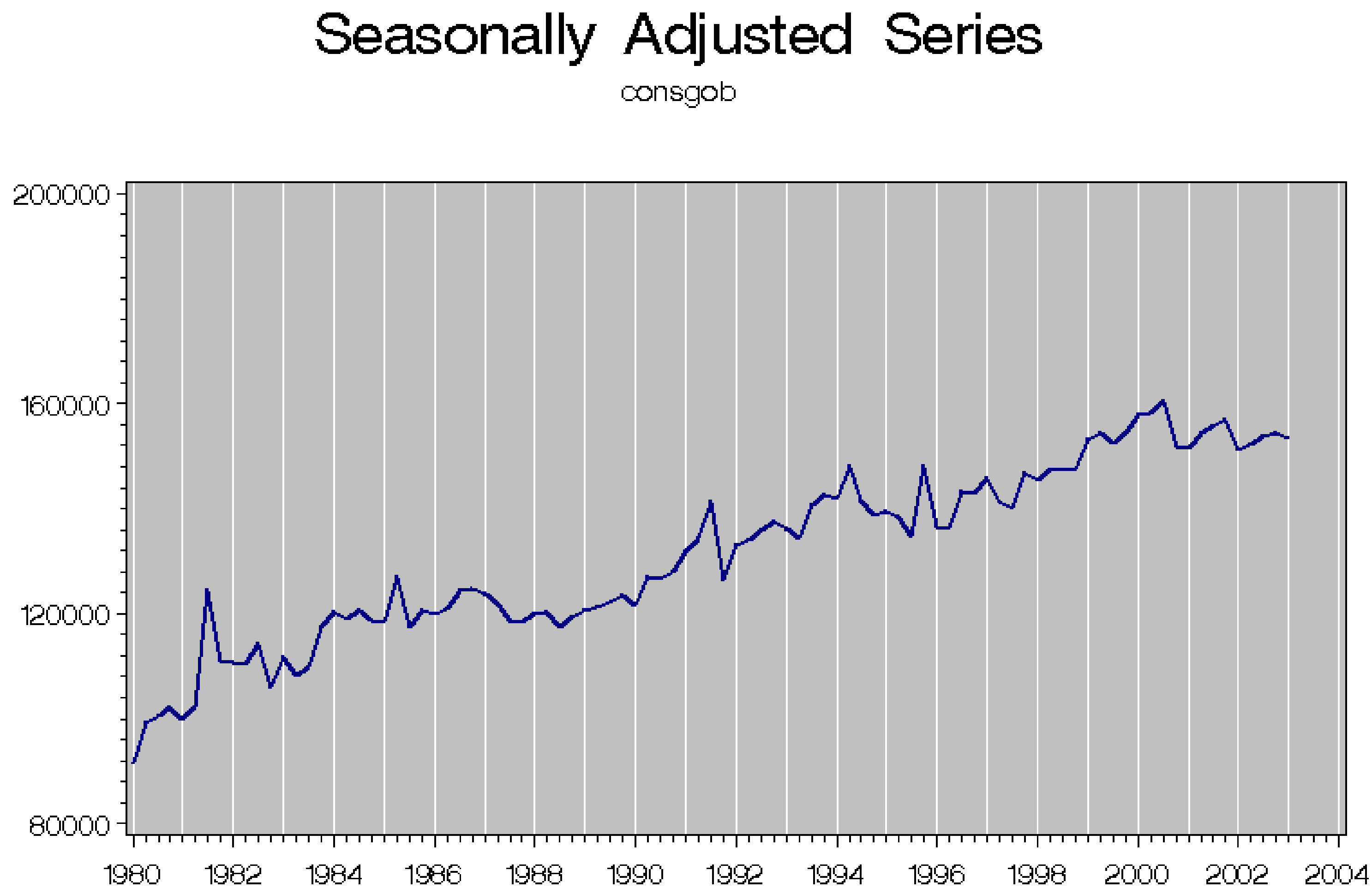
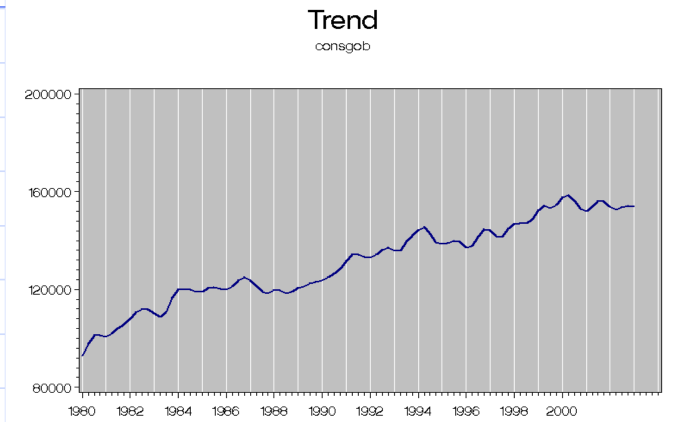
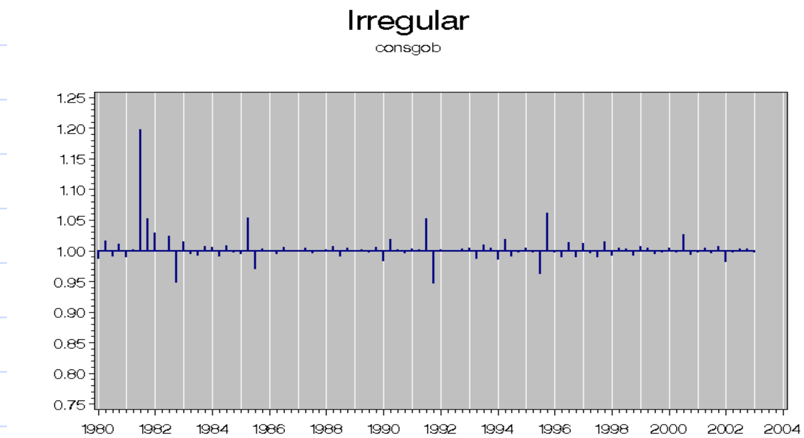


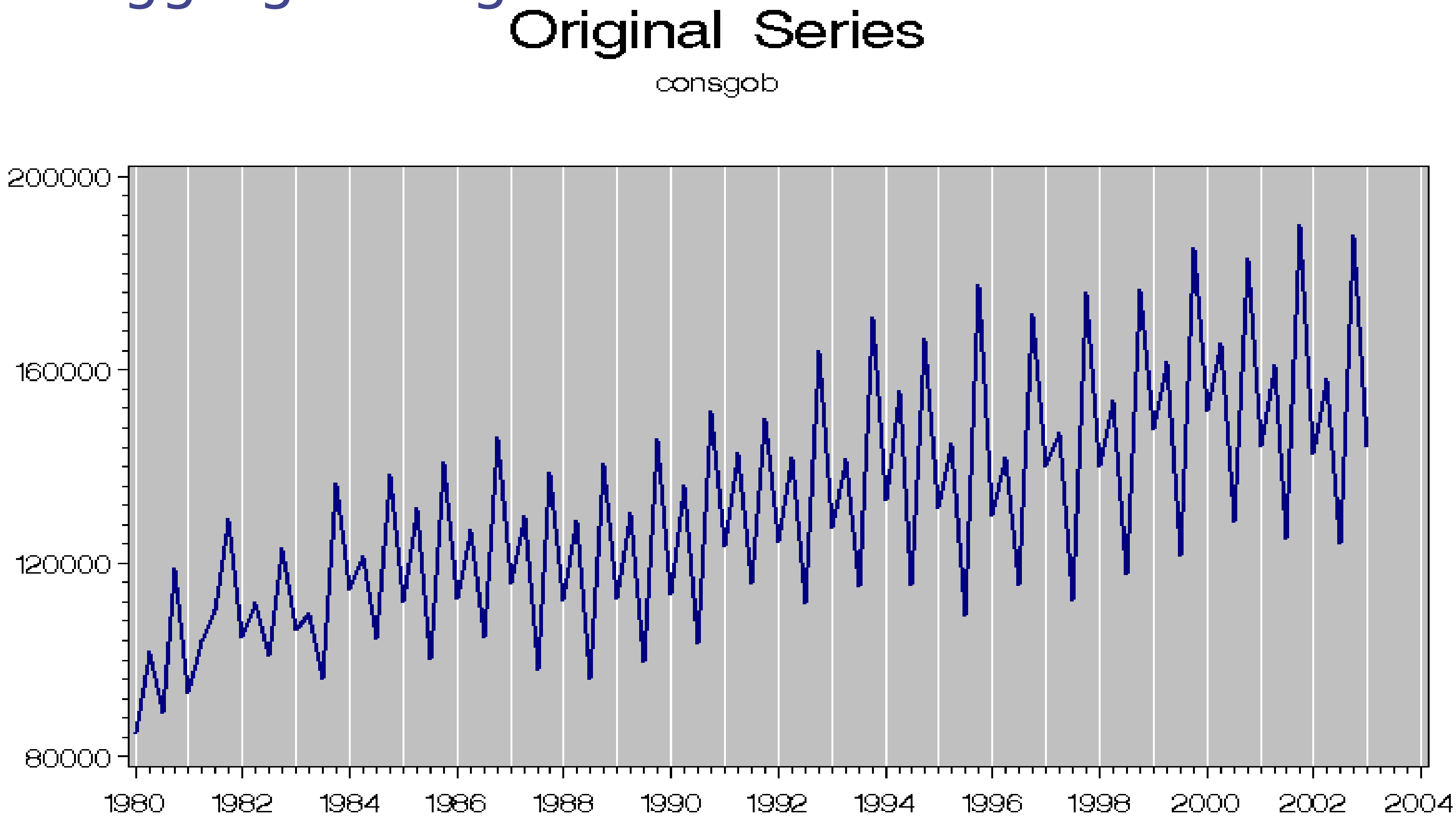

Seasonally Adjusted Series
consgob
200000
160000
120000
80000
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004

Few variations separately





Can you imagine how all components aggregate together to form this?



Multiplicative Time-Series Model for Annual Data

- Used primarily for forecasting
- Observed value in time series is the product of components

$$Y_i = T_i \times C_i \times I_i$$

where T_i = Trend value at year i

C_i = Cyclical value at year i

I_i = Irregular (random) value at year i

Multiplicative Time-Series Model with a Seasonal Component

- Used primarily for forecasting
- Allows consideration of seasonal variation

$$Y_i = T_i \times S_i \times C_i \times I_i$$

where T_i = Trend value at time i

S_i = Seasonal value at time i

C_i = Cyclical value at time i

I_i = Irregular (random) value at time i

Smoothing techniques

- Smoothing helps to see overall patterns in time series data.
- Smoothing techniques smooth or “iron” out variation to get the overall picture.
- There are several smoothing techniques of time series.

Smoothing techniques

- We will study :
- Moving average.
- Exponential smoothing

Smoothing the Annual Time Series

- Calculate moving averages to get an overall impression of the pattern of movement over time

Moving Average: averages of consecutive
time series values for a chosen period of length L

Moving Averages

- Used for smoothing
- A series of arithmetic means over time
- Result dependent upon choice of L (length of period for computing means)
- Examples:
  - For a 3 year moving average, L = 3
  - For a 5 year moving average, L = 5

Smoothing techniques: Moving Average (MA)

- Odd number of points. Points (k) – length for computing MA
- k=3

$$MA_1 = \frac{y_1 + y_2 + y_3}{3}$$

$$MA_2 = \frac{y_2 + y_3 + y_4}{3}$$

and so on.

Smoothing techniques: Moving Average (MA) k=3

Year	Series	3 Point MA
1990	5	
1991	6	6
1992	7	
1993	8	
1994	10	
1995	11	
1996	12	
1997	12	
1998	12	12.0
1999	12	12.3
2000	13	12.7
2001	13	

MA_1

MA_2

Smoothing techniques: Moving Average (MA) k=5

Year	Series	5 Point MA
1990	5	
1991	6	
1992	7	7.2
1993	8	8.4
1994	10	9.6
1995	11	10.6
1996	12	11.4
1997	12	11.8
1998	12	12.2
1999	12	12.4
2000	13	
2001	13	

Smoothing techniques: Moving Average (MA)

- We need even numbered MA s for seasonal adjustments
  - eg: 4 – quarterly data
  - 12 – monthly data

Smoothing techniques: Moving Average (MA)

- Even number of points.

  Two stages:

  1. Obtain MA, centered halfway between t and t-1.

  2. To get a trend take the average of two successive estimates. Estimate centered halfway between t and t-1.

Smoothing techniques: Moving Average (MA)

- for k=4.

  Stage 1.

$$MA_{1,1}=\frac{(y_1+y_2+y_3+y_4)}{4}$$

$$MA_{1,2}=\frac{(y_2+y_3+y_4+y_5)}{4}$$

  Stage 2.

$$\overline{MA_1}=\frac{MA_{1,1}+MA_{1,2}}{2}$$

and so on.

Smoothing techniques: Moving Average (MA)

$MA_{1,1}$ $MA_{1,2}$ $\overline{MA}_1$

Observation	Series	MA stage 1	MA stage 2: MA Centered
1	5		#NA
2	6		#NA
		6.5	
3	7		7.1
4	8		
		9.0	
5	10		
		10.3	
6	11		#NA
7	12		#NA

Measuring the seasonal effect

- To measure seasonal effect construct seasonal indices.
- Seasonal indices is a degree to which the seasons differ from one another.
- Requirement: time series should be sufficiently long to allow to observe seasonal fluctuations.

Measuring the seasonal effect

- Computation:
  - Calculating MA.
  - Set the number of periods equal to the number of types of season.
  - Use multiplicative model:

$$Y_t = T_t \cdot C_t \cdot S_t \cdot R_t$$

  - MA remove S_t and R_t

Measuring the seasonal effect

- Calculate MA_t (step 1)
- Compute the ratio (step 2):

$$\frac{Y_t}{MA_t} = \frac{T_t \cdot C_t \cdot S_t \cdot R_t}{T_t \cdot C_t} = S_t \cdot R_t$$

- For each type of season calculate the average of the ratios (step 3).
- The seasonal indices are the average ratios from ratios step 3 adjusted.

Step 1

Step 2

Step 3 (calculation see next slide)

Measuring seasonal effect					
Year	**Quarter**	**Hotel Occupancy Yt**	**Centered MA**	**Ratio Yt/MA**	**Seasonal Index Si**
1997	1	0.527			0.895
	2	0.660			1.098
	3	0.752	0.642	1.171	1.144
	4	0.534	0.658	0.811	0.864
1998	1	0.541	0.635	0.852	0.895
	2	0.694	0.632	1.098	1.098
	3	0.816	0.657	1.241	1.144
	4	0.569	0.658	0.864	0.864
1999	1	0.558	0.628	0.889	0.895
	2	0.694	0.617	1.124	1.098
	3	0.685	0.642	1.068	1.144
	4	0.564	0.650	0.867	0.864
2000	1	0.585	0.637	0.918	0.895
	2	0.666	0.650	1.023	1.098
	3	0.758	0.688	1.101	1.144
	4	0.594	0.705	0.843	0.864
2001	1	0.625	0.696	0.898	0.895
	2	0.785	0.703	1.116	1.098
	3	0.821			1.144
	4	0.630			0.864

Calculating seasonal index

	Quarterly ratios				
Year	**1**	**2**	**3**	**4**	**Total**
1997			1.171	0.811	
1998	0.852	1.098	1.241	0.864	
1999	0.889	1.124	1.068	0.867	
2000	0.918	1.023	1.101	0.843	
2001	0.898	1.116			
Average	0.889	1.090	1.137	0.858	**3.974**
Seasonal index	0.895	1.098	1.144	0.864	4.000

Example:
Seasonal index for Quarter 1 = 0.889/3.974*4.000=0.895

Thank You!!!

FORECAST THE FUTURE!!!