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Calculus Geogebra Project

INTRODUCTION.

To complete my project, I did some analytical work before using Geogebra.

Analytical Work: I recognised that all the variables, y, x, and Θ, depend on time. Hence, t is the primary variable.

Rewriting y, x, and Θ as functions of t.

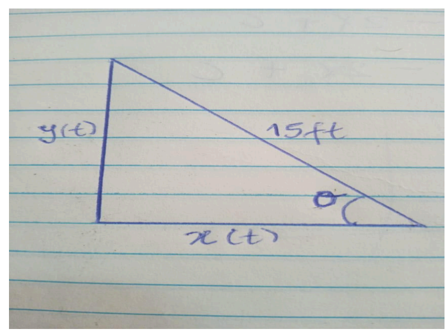
If the rate of change of x is 4ft/sec => $x(t) = 4t$ (x is the distance moved with speed 4ft/sec at any time, t).

$y^2 + x^2 = 15^2$ => $y = (15^2 - x^2)^{0.5}$ (From Pythagoras theorem)

$y(t) = \sqrt{15^2 - (4t)^2}$

$sin\Theta = \frac{y}{15}$ => $\Theta = arcsin(\frac{y}{15})$ (From the Trigonometry theorem)

$\Theta(t) = arcsin(\frac{\sqrt{15^2-(4t)^2}}{15})$



INTERPRETATION OF THE FUNCTIONS AND VISUALIZATION IN GEOGEBRA. (time=t)

Questions (i): Since the 2D plane in GeoGebra takes values in the form (a,b), I used the *curve function* in the input bar to draw the wall and the ground: For wall [0,y(t)] and the ground [x(t),0], (I could also use the point function to plot it, but you will need to find the derivative later). This will automatically give a maximum value of 15 to the ground and wall because they depend on time. I inserted images to represent the wall and ground. I used the *point function* to create points A=ground (time) on the ground and B=wall (time) on the wall: At time 0, the distance between A and B is 15ft. Since both vary with time, the distance between them will be the same at any time. Using the *segment function*, I created a segment to join the A and B, which made the ladder.

Questions (ii) and (iii): The max value of x is 15, and the max value of t is 3.75 sec (from $x(t) = 4t$).

I created another slider called Control, from 1 to 5. I used the *number function* to generate a time value with the *condition function IF*: time = If[Control $\stackrel{?}{=}$ 1, 0, Control $\stackrel{?}{=}$ 2, 6 / 4, Control $\stackrel{?}{=}$ 3, 8 / 4, Control $\stackrel{?}{=}$ 4, 10 / 4, Control $\stackrel{?}{=}$ 5, 13 / 4, 15 / 4]. Animating the Control slider automatically changes time to its value at *x* = 6, 8, 10, 13, and 15. I added buttons to control the slider using the button function in the menu bar.

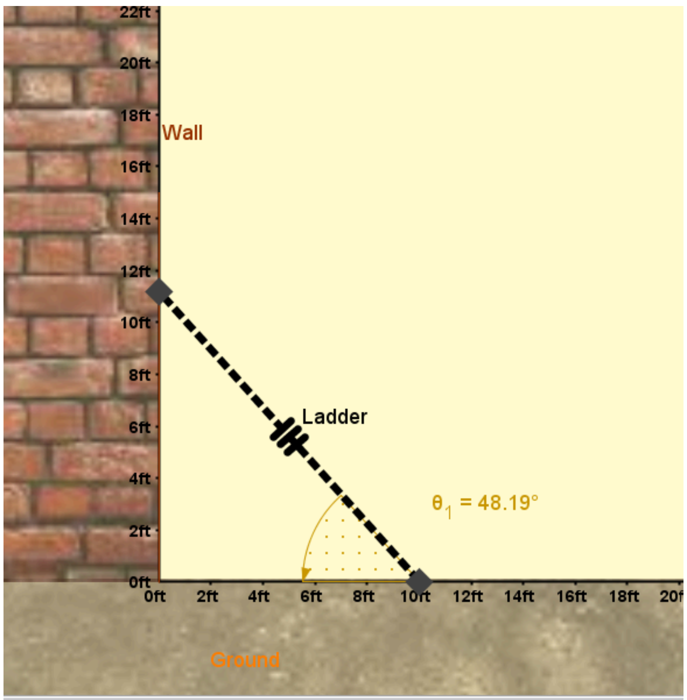
I used the *derivative function* in the input bar to differentiate the wall curve, wall'(time). The y value of the curves wall(time) and wall'(time) gives the value of y and $\frac{dy}{dt}$ respectfully, and I recorded it on a table using the *spreadsheet view* in the view option of the menu bar. I created a point C(0,0). Using the *angle function*, I connected points A, B, and C to form the angle. I made a curve, Angle(0,ϴ(t)) and used the *derivative function* to differentiate it, Angle'(time). The x value of the curves Angle(time) and Angle'(time) gives the value of ϴ and $\frac{d\Theta}{dt}$ respectfully, and I recorded it on a table using the *spreadsheet view*.

CONCLUSION

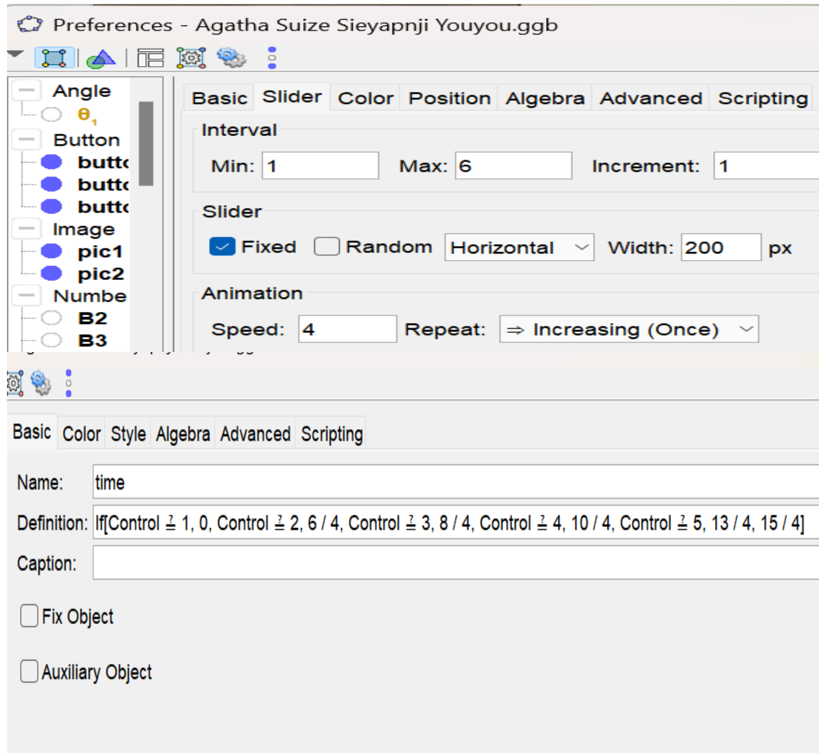
Questions (vi) and (vii)

- Calculating $\frac{d\Theta}{dt}$ and $\frac{dy}{dt}$ for each value of *x* analytically and using GeoGebra to approximately 2 decimal places give the same results.
- When x = 15, the height of the top of the ladder above the ground is zero, and the angle too is zero. Hence, $\frac{d\Theta}{dt}$ and $\frac{dy}{dt}$ is negative infinity at x = 15. Negative infinity is not a specific number, but rather, it indicates that the values are decreasing indefinitely in the negative direction

APPENDIX



Questions (i)



Questions (ii), and (iii)

Spreadsheet

	B	C	D	E
1	Y	Y'	θ	θ'
2	15	0	1.57	?
3	13.75	-1.75	1.16	-0.29
4	12.69	-2.52	1.01	-0.32
5	11.18	-3.58	0.84	-0.36
6	7.48	-6.95	0.52	-0.53
7	0	-∞	0	-∞

$y^2 + x^2 = 15^2$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

$\boxed{\frac{dy}{dt} = \frac{x}{\sqrt{15^2-x^2}}(4)}$

$\cos\theta = \frac{x}{15}$

$-\sin\theta\frac{d\theta}{dt} = \frac{1}{15}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-(4)}{15\sin\theta}$

$\frac{d\theta}{dt} = \left(-4\sqrt{15^2-x^2}\right)$

x/ft	6	8	10	13	15
$\frac{dy}{dt}$	-1.75	-2.52	-3.58	-6.95	-∞
$\frac{d\theta}{dt}$	-0.29	-0.32	-0.36	-0.53	-∞

Questions (vi) and (vii)