## II) Simpson's $\frac{1}{3}^{rd}$ Rule :

If the curve $y=f(x)$ passes through the points. $(x_i, y_i)$ , $i=1,2,\dots,m$ (such that $m$ is an odd integer) and $x_{i+1}-x_i=h$

Since $m$ is odd , $n=m-1$ should be an even integer.

$$\boxed{I = \int_{x_1}^{x_m} y\,dx = \frac{h}{3}\left[y_1+y_m+4(y_2+y_4+y_6+\cdots)+2(y_3+y_5+y_7+\cdots)\right]}$$

### * Examples

1) Find area bounded by the curve $y=f(x)$, x-axis between $x=0$ and $x=1$. If the curve passes through the following points:

| $x$ | 0 | 1/8 | 2/8 | 3/8 | 4/8 | 5/8 | 6/8 | 7/8 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0 | 1.0001 | 1.1001 | 1.0098 | 1.0307 | 1.0735 | 1.1473 | 1.2514 | 1.4142 |
| | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ | $y_8$ | $y_9$ |

→ Area bdd by $y=f(x)$, x-axis & $x=0$ , $x=1$ $= \int_0^1 y\,dx = ?$

Here we have, $h=\frac{1}{8}$ , $m=9$ (odd) , $n=8$ (even)

∴ By using Simpson's $1/3^{rd}$ rule

$$\int_0^1 y\,dx = \frac{h}{3}\left[y_1+y_9+4(y_2+y_4+y_6+y_8)+2(y_3+y_5+y_7)\right]$$

$$= \frac{1/8}{3}\Big[0+1.4142+4(1.0001+1.0098+1.0735+1.2594)+$$

$$2(1.1001+1.0307+1.1473)\Big]$$

$= \frac{\text{[illegible]}}{3}\Big[$ [illegible] $+$

$2\left(1.1001 + 1.0307 + 1.1473\right)\Big]$

$$= \frac{1}{24}\left[1.4142 + 4\left(4.3428\right) + 2\left(3.2781\right)\right]$$

$$= \frac{1}{24}\left[1.4142 + 17.3712 + 6.5562\right]$$

$$\boxed{\int_0^1 y\, dx = 1.0559 \text{ units.}}$$

2) Evaluate using Simpson's $\frac{1}{3}^{rd}$ rule : $\int_0^1 \frac{1}{1+x}\, dx$ by dividing the interval into six subintervals.

→ Given:

$y = f(x) = 1/(1+x)$ — integrand fun.

$x_1 = 0$ — lower limit

$x_m = 1$ — upper limit

$n = 6$ — no. of intervals.

$\therefore m = n+1 = 7$ — no. of ordinates

$h = \frac{x_m - x_1}{n} = \frac{1-0}{6} = 1/6$ — step size.

Tabulated data points

| $x$ | 0 | 1/6 | 2/6 | 3/6 | 4/6 | 5/6 | 1 |
|---|---|---|---|---|---|---|---|
| $y$ | 1<br>$y_1$ | 6/7<br>$y_2$ | 6/8<br>$y_3$ | 6/9<br>$y_4$ | 6/10<br>$y_5$ | 6/11<br>$y_6$ | 1/2<br>$y_7$ |

$\therefore$ Using Simpson's $\frac{1}{3}$ rule

$$\int_0^1 \frac{dx}{1+x} = \frac{h}{3}\left[y_1 + y_7 + 4(y_2+y_4+y_6) + 2(y_3+y_5)\right]$$

$$\boxed{\int_0^1 \frac{dx}{1+x} = 0.6931}$$ — check.

III) Simpson's $\frac{3}{8}$th Rule :

In this rule, we approximate $\int_{x_1}^{x_m} y\,dx$ by considering $n$ (multiples of three) intervals.

| $x$ | $x_1$ | $x_2$ | ..... | $x_m$ |
|---|---|---|---|---|
| $y$ | $y_1$ | $y_2$ | — — | $y_m$ |

, $x_{i+1} - x_i = h$ , $n$ is multiple of 3 $(n = m-1)$

$$\boxed{I = \int_{x_1}^{x_m} y\,dx = \frac{3h}{8}\left[y_1 + y_m + 3(y_2+y_3+y_5+y_6+\cdots\cdots) + 2(y_4+y_7+y_{10}+\cdots)\right]}$$

* Examples :

1) Evaluate using Simpson's 3/8 rule : $\int_0^1 \frac{1}{1+x}\,dx$ , by dividing interval into 6 equal parts.

→ data points.
(Same as Q.2 in Simpson's $\frac{1}{3}$ rule)

By using Simpson's $\frac{3}{8}$ rule

$$\int_0^1 \frac{1}{1+x}\,dx = \frac{3h}{8}\left[y_1 + y_7 + 3(y_2+y_3+y_5+y_6) + 2(y_4)\right]$$

$$\int_0^1 \frac{1}{1+x}\,dx = \frac{3h}{8}\left[y_1+y_7+3(y_2+y_3+y_5+y_6)+2(y_4)\right]$$

$$\boxed{\int_0^1 \frac{1}{1+x}\,dx = 0.6931}$$ — check.

2) Find $\int_1^7 y\,dx$ if the curve passes through

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $y$ | 81 | 75 | 80 | 83 | 78 | 70 | 60 |
| | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ |

→ $$\int_1^7 y\,dx = \frac{3h}{8}\left[y_1+y_7+3(y_2+y_3+y_5+y_6)+2(y_4)\right]$$

$$\boxed{\int_1^7 y\,dx = 456}$$ — check

* Examples: Trapezoidal / Simpson's 1/3 / Simpson's 3/8 rules.

Q. Evaluate $\int_{x_1}^{x_m} y\,dx$ for the following cases

1) If the curve passes through

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| $y$ | 7 | 11 | 14 | 18 | 24 | 32 |

→ Since $m=6$, $n=5$

$\therefore$ $n=5$ is neither an even integer nor multiple of 3

( $\therefore n=5$ is neither an even integer nor multiple of 3)

By using Trapezoidal rule

$$\int_0^{25} y\,dx = \frac{h}{2}\left[y_1+y_6+2(y_2+y_3+y_4+y_5)\right]$$

$$\boxed{\int_0^{25} y\,dx = 432.5}$$

2) A rocket is launched from the ground. It's accl^n is registered during the first 80 sec and is given in the table below.
Find the velocity of the rocket at t=80 sec.

| t (sec) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| a (cm/sec²) | 30 | 31.63 | 33.34 | 35.47 | 37.75 | 40.33 | 43.25 | 46.69 | 50.67 |
| | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ | $y_8$ | $y_9$ |

→ Velocity of the rocket at t=80 sec $= \int_0^{80} a\,dt$

(Here $m=9 \Rightarrow n=8$ not multiple of 3 $\Rightarrow$ 3/8 rule not applicable)

$n=8$ is an even $\Rightarrow$ Use Simpson's 1/3 rule.

$\boxed{\text{Velocity at } t=80 \text{ sec is } 3086.1 \text{ cm/sec.}}$

3) Evaluate $\int^{\pi} \sin^2\theta \; d\theta$ by taking $h=\pi$ .

3) Evaulate $\int_0^{\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta$ by taking $h = \frac{\pi}{6}$.

→ Given: $f(x) = \frac{\sin^2 x}{5+4\cos x}$

$x_1 = 0$

$x_m = \pi$

$h = \pi/6$

$\therefore n = \frac{x_m - x_1}{h} = \frac{\pi - 0}{\pi/6} = 6$ it is multiple of 3

∴ By using $3/8$ th rule

| $x$ | $0$ | $\pi/6$ | $2\pi/6$ | $3\pi/6$ | $4\pi/6$ | $5\pi/6$ | $\pi$ |
|---|---|---|---|---|---|---|---|
| $y$ | $0$ | | | | | | $0$ |
| | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ |

$$\int_0^{\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta = \frac{3h}{8}\left[y_1 + y_7 + 3(y_2 + y_3 + y_5 + y_6) + 2y_4\right]$$