N M L7

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* Newton's Backward Difference Interpolation:

If the curve $y=f(x)$ passes through the points (x_i, y_i), $i=1,2,3,\ldots,m$ and $x_{i+1}-x_i=h$

$$y(x)=y_m+\boxed{\frac{x-x_m}{h}}\nabla y_m+\frac{(x-x_m)(x-x_{m-1})}{2!\,h^2}\nabla^2 y_m+\cdots$$
$$+\frac{(x-x_m)(x-x_{m-1})\cdots(x-x_2)}{(m-1)!\,h^{m-1}}\nabla^{m-1}y_m$$

Let $\frac{x-x_m}{h}=v$

$$\therefore \frac{x-x_{m-1}}{h}=\frac{x-(x_m-h)}{h}=v+1$$

$$\therefore y(x)=y_m+v\nabla y_m+\frac{v(v+1)}{2!}\nabla^2 y_m+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_m+\cdots$$

* Examples

1) If $f(1.15)=1.0723$, $f(1.2)=1.0954$, $f(1.25)=1.118$ and $f(1.3)=1.1401$ then find $f(1.28)$.

→ Given

x	$x_1=1.15$	$x_2=1.2$	$x_3=1.25$	$x_4=1.3$
y	$y_1=1.0723$	$y_2=1.0954$	$y_3=1.118$	$y_4=1.1401$

We have to find $f(1.28)$ i.e. y at $x = 1.28$
here 1.28 is closed to end value of x & values of x are equally spaced

$\therefore$ we use Newton's backward diff. interpl^n formula

$$y(x) = y_4 + v\,\nabla y_4 + \frac{v(v+1)}{2!}\nabla^2 y_4 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_4 \quad \text{—} (1)$$

where $v = \frac{x - x_4}{h} = \frac{1.28 - 1.3}{0.05} = -0.4$ — (2)

Difference table

y	∇y	$\nabla^2 y$	$\nabla^3 y$
$y_1 = 1.0723$	0.0231	-0.0005	0 ($\nabla^3 y_4$)
$y_2 = 1.0954$	0.0226	-0.0005 ($\nabla^2 y_4$)	—
$y_3 = 1.118$	0.0221 (∇y_4)	—	—
$y_4 = 1.1401$ (y_4)	—	—	—

from eq^n (1), (2) & diff. table

$$y(1.28) = 1.1401 + (-0.4)\times(0.0221) + \frac{(-0.4)(-0.4+1)}{2}\times(-0.0005)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)}{} (0)$$

$$+\frac{(-0.4)(-0.4+1)(-0.4+2)^{2}}{6}\times(0)$$

$$\boxed{y(1.28) = 1.1313}$$

2) From the following data, estimate the number of persons earning wages 110 rupees

wages (x in rupees)	40	60	80	100	120
No. of persons (y in thousand)	250	120	100	70	50

→ Difference table

y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
250	−130	110	−120	140 $\nabla^4 y_5$
120	−20	−10	20	—
100	−30	10	— $\nabla^3 y_5$	—
70	−20	— $\nabla^2 y_5$	—	—
50 y_5	— ∇y_5	—	—	—

$$\therefore\ y(x) = y_5 + v\,\nabla y_5 + \cdots + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_5$$

where $v = \frac{110 - 120}{20} = -0.5$

$$\therefore y(110) = 50 + (-0.5) \times (-20) + \frac{-0.5(-0.5+1)}{2} \times 10 + \frac{-0.5(-0.5+1)(-0.5+2)}{6} \times 20 + \frac{-0.5(-0.5+1)(-0.5+2)(-0.5+3)}{24} \times 140$$

$$\therefore y(110) = 50 + 10 - \frac{2.5}{2} - \frac{0.5 \times 0.5 \times 1.5}{6} 20 - \frac{0.5 \times 0.5 \times 1.5 \times 2.5}{24} \times 140$$

$$y(110) = 52.03125 \cong 52$$

3) Find Newton's interpolating polynomial for the following data:

x	0.1	0.2	0.3	0.4	0.5
y	1.4	1.56	1.76	2	2.28

Also find the value of y at $x = 0.23$ and $\frac{dy}{dx}$ at $x = 0.33$