* Topics

1. Integration: Trapezoidal, Simpson's $1/3^{rd}$, Simpson's $3/8^{th}$ Rules.
2. Interpolation: Lagrange's, Newtons Interpolation.
   - Newton's forward interp^n
   - Newton's Backward interpol^n.
3. System of linear equation: Gauss elimination method ✓
   L-U decomposition method.

* Integration.

$$I = \int_{a=x_1}^{b=x_m} y\,dx = ?$$

If the curve passes through the points

x	x_1	x_2	$\cdots$	x_m
y	y_1	y_2	$\cdots$	y_m

& $x_{i+1} - x_i = h$

I) Trapezoidal Rule:

$$I = \int_{x_1}^{x_m} y\,dx = \frac{h}{2}\left[y_1 + y_m + 2(y_2 + y_3 + \cdots + y_{m-1})\right]$$

* Examples

1) Find the area bounded by the curve $y=f(x)$, X-axis between $x=1$, $x=7$. If the curve passes through the points

x	1	2	3	4	5	6	7
y	81	75	80	83	78	70	60

→ Area bdd by $y=f(x)$, x-axis between $x=1$, $x=7$ $= \int_1^7 y\,dx = ?$

Given:

x	$x_1=1$	$x_2=2$	$x_3=3$	$x_4=4$	$x_5=5$	$x_6=6$	$x_7=7$
y	$y_1=81$	$y_2=75$	$y_3=80$	$y_4=83$	$y_5=78$	$y_6=70$	$y_7=60$

∴ from values of x: $h=1$

∴ By using Trapezoidal rule

$$\int_1^7 y\,dx = \frac{h}{2}\left[y_1 + y_7 + 2(y_2 + y_3 + y_4 + y_5 + y_6)\right]$$

$$= \frac{1}{2}\left[81 + 60 + 2(75 + 80 + 83 + 78 + 70)\right]$$

$$= \frac{1}{2}\left[141 + 2(386)\right]$$

$$= \frac{1}{2}\left[141 + 772\right]$$

$$= \frac{1}{2}\left[\quad 913\right]$$

$$= 456.5$$

$\therefore$ Area bdd by $y=f(x)$, x-axis bet^n $x=1$ and $x=7$ is 456.5 units

2) Evaluate using Trapezoidal rule: $\int_0^{0.6} e^{-x^2}$ by taking 7 ordinates.

→ Given :

$f(x) = e^{-x^2}$

lower limit $(x_1) = 0$

upper limit $(x_m) = 0.6$

no. of points $(m) = 7$

$\therefore$ no. of intervals$(n) = m-1 = 6$

step size $(h) = \frac{x_m - x_1}{n} = \frac{x_7 - x_1}{n} = \frac{0.6-0}{6} = 0.1$

$\therefore$ Tabulated values of x and y :

				x_4	x_5	x_6	
x	$x_1 = 0$	$x_2 = x_1 + h = 0.1$	$x_3 = x_1 + 2h = 0.2$	0.3	0.4	0.5	$x_7 = 0.6$
y	$y_1 = e^{-0^2} = 1$	$y_2 = e^{-(0.1)^2} = 0.99$	$y_3 = e^{-(0.2)^2} = 0.9608$	$y_4 = e^{-(0.3)^2} = 0.9139$	$y_5 = 0.8521$	0.7788	0.6977

$\therefore$ By using Traperoidal rule

$$\int_0^{0.6} e^{-x^2} dx = \frac{h}{2}\left[y_1 + y_7 + 2(y_2+y_3+y_4+y_5+y_6)\right]$$

$$= \frac{0.1}{2}\left[1 + 0.6977 + 2(0.99 + 0.9608 + 0.9139 + 0.8521 + 0.7788)\right]$$

$$\boxed{\int_0^{0.6} e^{-x^2} dx = 0.534365} \cong 0.5344$$

3) Evaluate: $\int_0^4 y\, dx$ if $y=f(x)$ passes through

x	0	1	2	3	4
y	20	40	45	30	25

4) Evaluate : $\int_0^5 \frac{dx}{4x+5}$, dividing the range into 10 equal parts

→ Given : $f(x) = \dfrac{1}{4x+5}$, no. of intervals $= n = 10$

lower limit $= \underline{x_1} = \underline{0}$ & upper limit $= \underline{x_m} = \underline{5}$

Since $n = 10$, $m =$ no. of points $= n+1 = 11$

Step size $= h = \dfrac{x_m - x_1}{n} = \dfrac{x_{11} - x_1}{n} = \dfrac{5-0}{10} = \dfrac{1}{2} = \underline{0.5}$

x :	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y :	1/5	1/7	1/9	1/11	1/13	1/15	1/17	1/19	1/21	1/23	1/25

$$\therefore \quad I = \int_0^5 \frac{1}{4x+5}\,dx = \frac{h}{2}\left[y_1 + y_{11} + 2\left(y_2 + y_3 + \cdots + y_{10}\right)\right]$$

$$= \frac{0.5}{2}\left[\frac{1}{5} + \frac{1}{25} + 2\left(\frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \frac{1}{19} + \frac{1}{21} + \frac{1}{23}\right)\right]$$

$$\boxed{\int_0^5 \frac{dx}{4x+5} = \quad}$$