N M L6

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* Newton's forward difference interpolation

If the curve $y=f(x)$ passes through the points (x_i, y_i), $i = 1, 2, \ldots m$ such that $x_{i+1} - x_i = h$.

y at $x = X$ is given by

$$\boxed{y(x) = y_1 + \frac{(x-x_1)}{h} \Delta y_1 + \frac{(x-x_1)(x-x_2)}{2!\, h^2} \Delta^2 y_1 + \cdots + \frac{(x-x_1)(x-x_2)\cdots(x-x_{m-1})}{(m-1)!\, h^{m-1}} \Delta^{m-1} y_1}$$

To find poly use this formula.

If $\frac{x-x_1}{h} = u$ in above eqn

$$\therefore \frac{x-x_2}{h} = \frac{x-(x_1+h)}{h} = \frac{x-x_1}{h} - \frac{h}{h} = u-1$$

Similarly $\frac{x-x_3}{h} = u-2$ & so on $\frac{x-x_{m-1}}{h} = u-m+2$

Also $x = X$

$$\boxed{Y = y(X) = y_1 + u\Delta y_1 + \frac{u(u-1)}{2!} \Delta^2 y_1 + \cdots + \frac{u(u-1)\cdots(u-m+2)}{(m-1)!} \Delta^{m-1} y_1}$$

Use this formula when we need to find y at X

Remark

1) To find y at $x = X$
   (i) When $x = X$ is near to the start of the table use forward diff. interpolation.
   (ii) when $x = X$ is near to the end of the table use Backward diff. interpolation.

2) Heighest order forward diff. of y_1 is $m-1$ when we have m ordinates.
   No forward diff. for y_m.

3) Heighest order backward diff. of y_m is $m-1$ when we have m ordinates.
   No backward diff. for y_1

*Examples

1) Find the polynomial passing through the points and estimate the value of y for $x = 1.5$. Also find the slope of the curve at $x = 1.5$.

x	0	2	4	6	8
y	5	29	125	341	725

→ Difference table

y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$y_1 = 5$	24	72	48	0
$y_2 = 29$	96	120	48	—
$y_3 = 125$	216	168	—	—
$y_4 = 341$	384	—	—	—
$y_5 = 725$	—	—	—	—

Newton's forward difference interpolation formula

$$y(x) = y_1 + \frac{x - x_1}{h}\Delta y_1 + \frac{(x-x_1)(x-x_2)}{2!\ h^2}\Delta^2 y_1 + \frac{(x-x_1)(x-x_2)(x-x_3)}{3!\ h^3}\Delta^3 y_1$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{4!\ h^4}\Delta^4 y_1$$

Here, $x_1 = 0$, $h = 2$

from diff. table $y_1 = 5$, $\Delta y_1 = 24$, $\Delta^2 y_1 = 72$, $\Delta^3 y_1 = 48$, $\Delta^4 y_1 = 0$

$$\therefore y(x) = 5 + \frac{x-0}{2}\times 24 + \frac{(x-0)(x-2)}{2!\ 2^2}\times 72 + \frac{(x-0)(x-2)(x-4)}{3!\ 2^3}\times 48$$

$$+ \frac{(x-0)(x-2)(x-4)(x-6)}{4!\ 2^4}\times 0$$

$$= 5 + 12x + 9x^2 - 18x + x^3 - 6x^2 + 8x + 0$$

$$\boxed{y(x) = x^3 + 3x^2 + 2x + 5}$$ — polynomial.

$$\therefore \boxed{y(1.5) = 18.125}$$ — y at $x=1.5$

$$\text{slope} = \frac{dy}{dx} = 3x^2 + 6x + 2$$

$$\therefore \boxed{y'(1.5) = 17.75}$$ — slope of the curve at $x=1.5$

2) By using Newton's forward diff. interpolation formula find the value of y at $x=0.5$ from the following table of x and y values

x	0	1	2	3	4
y	1	5	25	100	250

→ Diff. table

y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	4	16	39	−19
5	20	55	20	—
25	75	75	—	—
100	150	—	—	—
250	—	—	—	—

$$y(x) = y_1 + u\Delta y_1 + \frac{u(u-1)}{2!}\Delta^2 y_1 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_1 +$$

$$\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_1$$

Where, $u = \frac{x - x_1}{h} = \frac{0.5 - 0}{1}$ — given

$\therefore u = 0.5$

and $y_1 = 1$, $\Delta y_1 = 4$, $\Delta^2 y_1 = 16$, $\Delta^3 y_1 = 39$, $\Delta^4 y_1 = -19$

$$\therefore y(0.5) = 1 + 0.5 \times 4 + \frac{0.5(0.5-1)}{2} \times 16 +$$

$$\frac{0.5(0.5-1)(0.5-2)}{6} \times 39 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24} \times (-19)$$

$$\therefore \boxed{y(0.5) = 4.1797}$$

3) From the following table estimate the number of students who obtained marks between 40 & 45.

marks	30-40	40-50	50-60	60-70	70-80
no. of students	31	42	51	35	31

→ Consider the given tabular values as

→ Consider the given tabular values as marks upto the value

marks less than (x)	40	50	60	70	80
no. of students (Y)	31	73	124	159	190

Then find no. of students obtained marks less than 45

$Y(45) = ?$

no. of students who obtained marks betn 40 & 45

$= Y(45) - Y(40)$