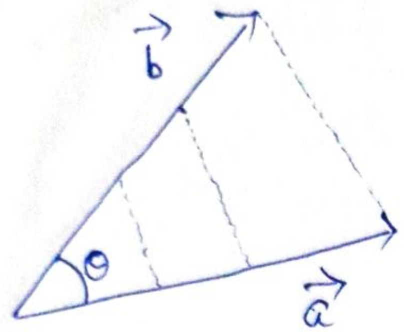

cosine similarity

$\cos\theta$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Euclidian Similarity

$$\frac{1}{1+(\text{Euclidian distance})^2}$$

$$= \frac{1}{1+\left((a_x-b_x)^2+(a_y-b_y)^2\right)}$$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{b} = b_x\hat{i} + b_y\hat{j}$

when normalized:

$|\vec{a}| = 1,\ |\vec{b}| = 1$

$\Rightarrow \sqrt{a_x^2 + a_y^2} = 1$

$\Rightarrow \boxed{a_x^2 + a_y^2 = 1}$

similarly $\boxed{b_x^2 + b_y^2 = 1}$

$\rightarrow \cos\theta = \vec{a}\cdot\vec{b}$

$\Rightarrow \cos\theta = (a_x\hat{i} + a_y\hat{j})\cdot(b_x\hat{i} + b_y\hat{j})$

$\Rightarrow \cos\theta = a_x\hat{i}\times b_x\hat{i} + a_y\hat{j}\times b_x\hat{i} + a_x\hat{i}\times b_y\hat{j} + a_y\hat{j}\times b_y\hat{j}$

$\Rightarrow \cos\theta = a_x b_x + a_y b_y \quad (\because \hat{i}\times\hat{j} = 0,\ \hat{i}\times\hat{i} = 1,\ \hat{j}\times\hat{j} = 1)$

Euclidian distance^2:

$(a_x-b_x)^2 + (a_y-b_y)^2 = a_x^2 + b_x^2 - 2a_xb_x + a_y^2 + b_y^2 - 2a_yb_y$

$\Rightarrow 1 + 1 - 2(a_xb_x + a_yb_y)$

$= 2 - 2\cos\theta$

$\therefore \boxed{(a_x-b_x)^2 + (a_y-b_y)^2 = 2(1-\cos\theta)} \Rightarrow \boxed{\text{Euclidian Distance}^2 = 2\times \text{cosine distance}}$

$$\text{score} = \frac{1}{1+\left((a_x-b_x)^2+(a_y-b_y)^2\right)} = \frac{1}{1+2(1-\cos\theta)} = \frac{1}{3-2\cos\theta}$$

$$\boxed{\text{Euclidian score} = \frac{1}{3 - 2\,\text{cosine score}}}$$

— when vectors are normalized