Equilibrium in the OLG Model

In an OLG model with money, we consider the interactions between two overlapping generations: the young (who work and earn income) and the old (who are retired and consume their savings). The equilibrium price level in each period is determined by the demand and supply of money.

Initial Assumptions

The total money supply is M_t in period t.
Individuals receive an income y_t in period t.
Prices are denoted as P_t in period t and P_{t+1} in period $t+1$.
Money is held as a store of value from one period to the next.

Budget Constraints and Money Holding

In period t, individuals allocate their income y_t between consumption c_{1t} and savings s_t. They hold savings in the form of money, which they use for consumption in the next period. The budget constraints are:

- Period t:
$$c_{1t} + s_t = y_t \tag{1}$$
(Income is split between consumption and savings.)

- Period $t+1$:
$$c_{2t+1} = \frac{s_t}{P_{t+1}} \tag{2}$$
(Savings from the previous period are used for consumption in this period, adjusted for the price level.)

Since s_t is saved in the form of money, it translates to M_{t+1} in the next period. Therefore:
$$c_{2t+1} = \frac{M_{t+1}}{P_{t+1}} \tag{3}$$
(Consumption in the next period depends on the money saved and the future price level.)

Money Market Equilibrium

The money market equilibrium in each period requires that the total money held by the young generation (savings s_t) equals the total money supply M_t.

- Period t:
$$s_t = M_t \tag{4}$$
(Savings equal the money supply.)

- Period $t+1$:

$$s_{t+1} = M_{t+1} \tag{5}$$

(The same condition applies to the next period.)

Determining the Price Level

The equilibrium price level is determined by the demand for money, which is driven by the consumption needs of the old generation. In each period, the total money supply should match the money demand for consumption.

- Period t:

$$c_{1t} + \frac{M_{t+1}}{P_{t+1}} = y_t \tag{6}$$

(Total consumption in the current period and next period's money demand should equal income.)

Since $c_{1t} = y_t$ (as derived from the optimal consumption), there is no saving, and thus:

$$s_t = 0 \tag{7}$$

(All income is consumed, so savings are zero.)

But, this implies that money demand needs to match money supply:

$$M_t = P_t c_{1t} \tag{8}$$

(Money supply should match the consumption demand for money.)

Given that $c_{1t} = y_t$:

$$P_t = \frac{M_t}{y_t} \tag{9}$$

(Price level is determined by the ratio of money supply to income.)

- Period $t+1$: The money saved in period t will be used for consumption in period $t+1$:

$$c_{2t+1} = \frac{M_t}{P_{t+1}} \tag{10}$$

(Next period's consumption depends on the previous period's money saved and the future price level.)

Given that $c_{2t+1} = 0$ (from the optimal consumption derivation):

$$P_{t+1} = \infty \tag{11}$$

(If there is no consumption, the price level becomes infinitely high.)

Before the Money Supply Increases

Before the money supply increases, the behavior of money and prices can be summarized as follows:

- Period t

$$P_t = \frac{M_t}{y_t} \tag{12}$$

(Price level in the current period is determined by money supply and income.)

$$c_{1t} = y_t \tag{13}$$

(All income is consumed.)

$$s_t = 0 \tag{14}$$

(No savings.)

- Period $t+1$

$$P_{t+1} = \infty \tag{15}$$

(Price level becomes infinitely high due to zero consumption.)

$$c_{2t+1} = 0 \tag{16}$$

(No consumption in the next period due to no savings.)

This setup shows that in the initial equilibrium, the young generation consumes all their income in period t, leading to no savings and an infinitely high price level in period $t+1$.

After Money Supply Increases

When the money supply increases at a rate h:

$$M_{t+1} = M_t(1+h) \tag{17}$$

(Money supply increases by a rate h.)

The new equilibrium price level for period $t+1$ will adjust to match the increased money supply:

$$P_{t+1} = \frac{M_t(1+h)}{c_{2t+1}} \tag{18}$$

(Price level adjusts according to the increased money supply and future consumption.)

Given that $c_{2t+1} = 0$ in the optimal consumption derived earlier:

$$P_{t+1} = \infty \tag{19}$$

(Price level still becomes infinitely high if consumption is zero.)

However, in a practical scenario, savings might not be zero, and the prices would adjust accordingly, taking into account the actual consumption in period $t+1$.

Summary

In summary, the initial equilibrium price behavior before the money supply increases involves:

- $P_t = \frac{M_t}{y_t}$ for period t.
- $P_{t+1} = \infty$ for period $t + 1$ with zero consumption due to no savings.

Equilibrium in the OLG Model (Continued)

Step 1: Increasing Money Supply

Suppose the money supply increases at a rate $h > -1$. This affects the price level P_t. Let M_t be the money supply at time t, and assume it grows as:

$$M_{t+1} = M_t(1 + h) \tag{1}$$

(Explanation: This step sets up the assumption that the money supply increases by a constant rate h from one period to the next. This growth rate affects the amount of money available in the economy and, subsequently, the price levels.)

Step 2: Real Money Balance

Assuming that prices adjust to keep the real money balance constant, we have:

$$\frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}} \tag{2}$$

(Explanation: The real money balance, which is the amount of money adjusted for the price level, should remain constant if the money supply and prices adjust proportionally. This equation states that the real value of money in period t should equal the real value of money in period $t + 1$.)

With $M_{t+1} = M_t(1 + h)$:

$$\frac{M_t}{P_t} = \frac{M_t(1 + h)}{P_{t+1}} \implies P_{t+1} = P_t(1 + h) \tag{3}$$

(Explanation: By substituting the growth equation for M_{t+1} into the real money balance equation, we derive that the price level in period $t + 1$ must increase by the same rate h to maintain the constant real money balance. This means that inflation occurs at the rate h.)

Step 3: Effect on Consumption and Prices

With increasing money supply, the price level P_t increases at the rate h. Higher inflation means that the real value of money decreases, affecting consumption decisions.

If inflation $\pi_{t+1} = h$:

$$c_{1t} + \frac{c_{2t+1}}{1 + \theta} = y_t \tag{4}$$

(Explanation: This equation shows the budget constraint over two periods, taking into account the individual's time preference parameter θ. The young generation consumes c_{1t} in the first period and saves for consumption c_{2t+1} in the second period, discounted by their time preference. Here, $\frac{c_{2t+1}}{1+\theta}$ represents the present value of future consumption.)

In real terms, adjust the consumption for the inflation effect:

$$c_{1t} = y_t - \frac{c_{2t+1}}{1+\theta} \tag{5}$$

(Explanation: This equation isolates c_{1t}, showing that the first-period consumption is the difference between the total income y_t and the present value of second-period consumption. This highlights how future consumption decisions, affected by inflation, influence current consumption choices.)

Conclusion

Optimal Consumption: The optimal consumption in periods t and $t+1$ is determined by the budget constraint and the individual's time preference parameter θ.
Increasing Money Supply: An increase in money supply at rate h results in an increase in price levels at the same rate h, causing inflation. This affects the real value of consumption and savings.

To summarize:

- Optimal consumption in period t:

$$c_{1t} = y_t - \frac{c_{2t+1}}{1+\theta} \tag{6}$$

  (Explanation: The first-period consumption depends on the total income and the discounted future consumption. This shows how future expectations influence present decisions.)

- Optimal consumption in period $t+1$:

$$c_{2t+1} = s_t(1+\theta) \tag{7}$$

  (Explanation: The second-period consumption is the savings from the first period, grown by the individual's time preference parameter θ. This indicates that future consumption depends on current savings and how much individuals value future consumption relative to the present.)

Increasing money supply causes inflation, influencing real consumption levels and equilibrium prices.