4 Aggregation simulation of independent processes

The general idea is making use of fixed intrinsic parameters such as γ and ρ_A to generate 10 independent process as 10 "states". The loss function will be the sum of loss of the 10 "states"− aggregate the loss function $d_s^{(i)}$ to construct new total profile loss, then optimize the values for $gamma$, rho_A. Here intrinsic parameters and mobility data are fixed across all "states":

Aggregation simulation 1

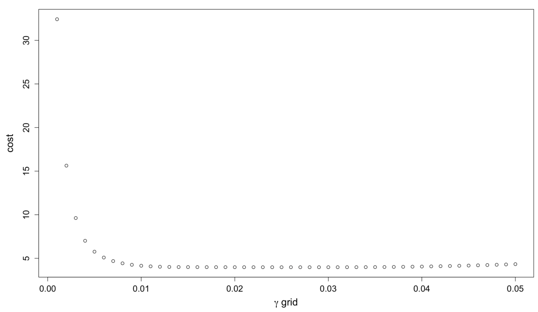
- Time (Days) = 100
- $\gamma = 0.03$
- $\rho_A = 0.06$
- $\alpha = 0.2$
- $\rho_H(t) = (rep(0.03, 20), rep(0.04, 40), rep(0.05, 40))$
- $\phi(t)$ is a linear function of Time with a small positive slope (reflecting increasing testing efficiency):

$$\phi(t) = 0.001 + 1 : Time * 0.00005$$

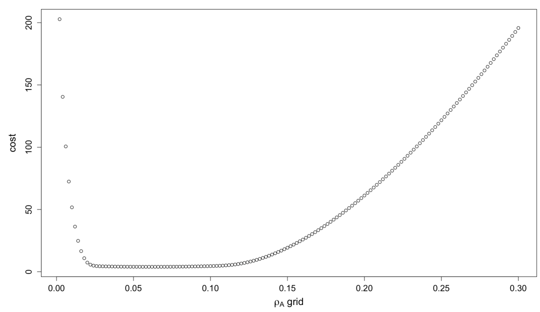
- $\kappa(t)$ = *lowess* version of (rep(1.2,15),rep(0.8,10),rep(0.4,20),rep(0.6,15),rep(0.7,15),rep(0.9,15),rep(0.7,10))
- $\delta(t)$ =(rep(0.007,50),rep(0.005,70),rep(0.003,60),rep(0.004,20))
- number of Time block = 9; window length = 30;

grid search for γ and ρ_A are:

$$\hat{\gamma} = 0.027, \hat{\rho}_A = 0.06$$



(a)



(b)

Figure 21: grid of γ and ρ_A against the costs; minimum is achieved at $\hat{\gamma} = 0.027$ and $\hat{\rho}_A = 0.06$ respectively

estimation for "state" 7

estimation for "state" 4

estimation for $\phi(t), \rho_H(t), \alpha\sqrt{\kappa(t)}$

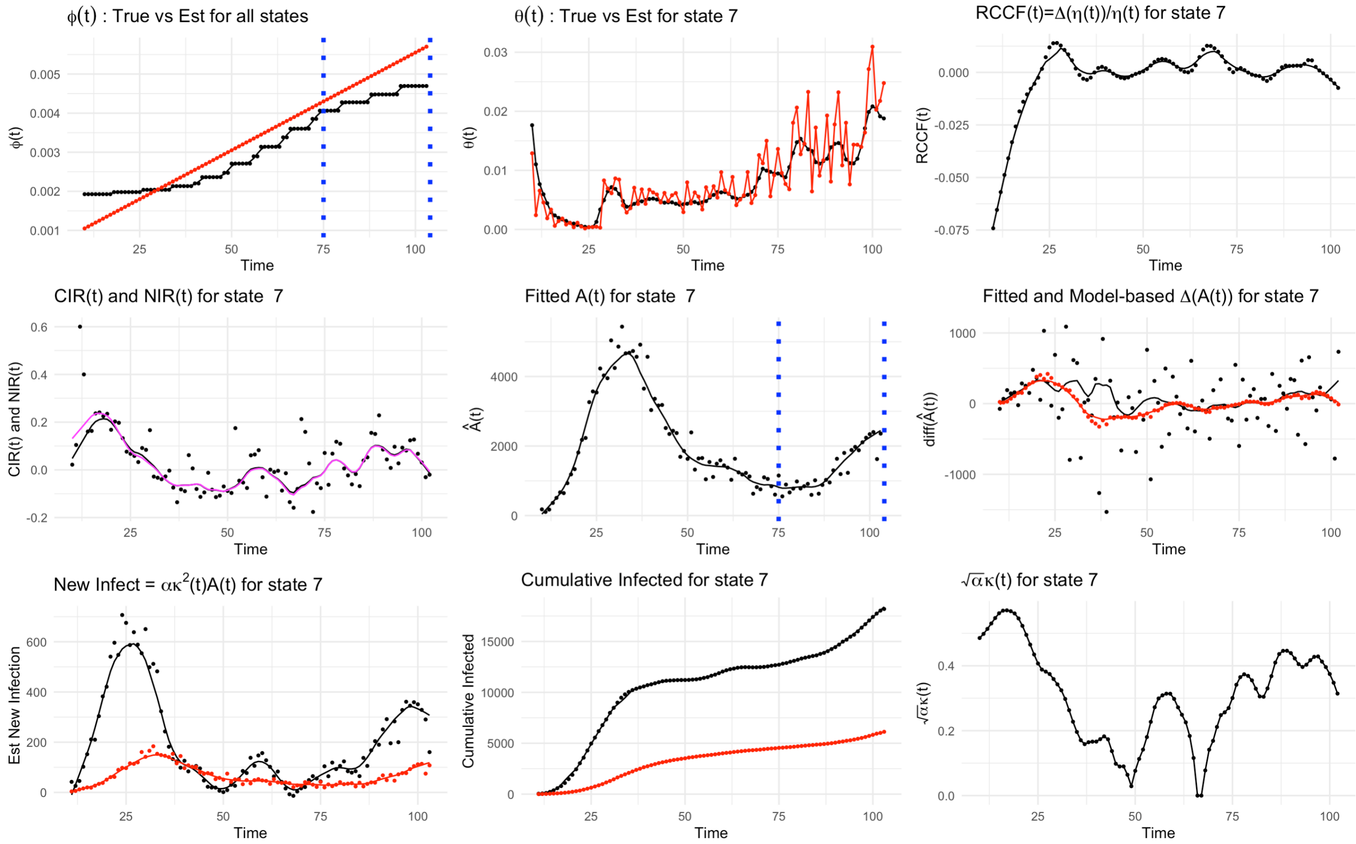


Figure 22: plots of estimations for "states" 7 (CIR(t) vs NIR(t); Est vs TRUE)

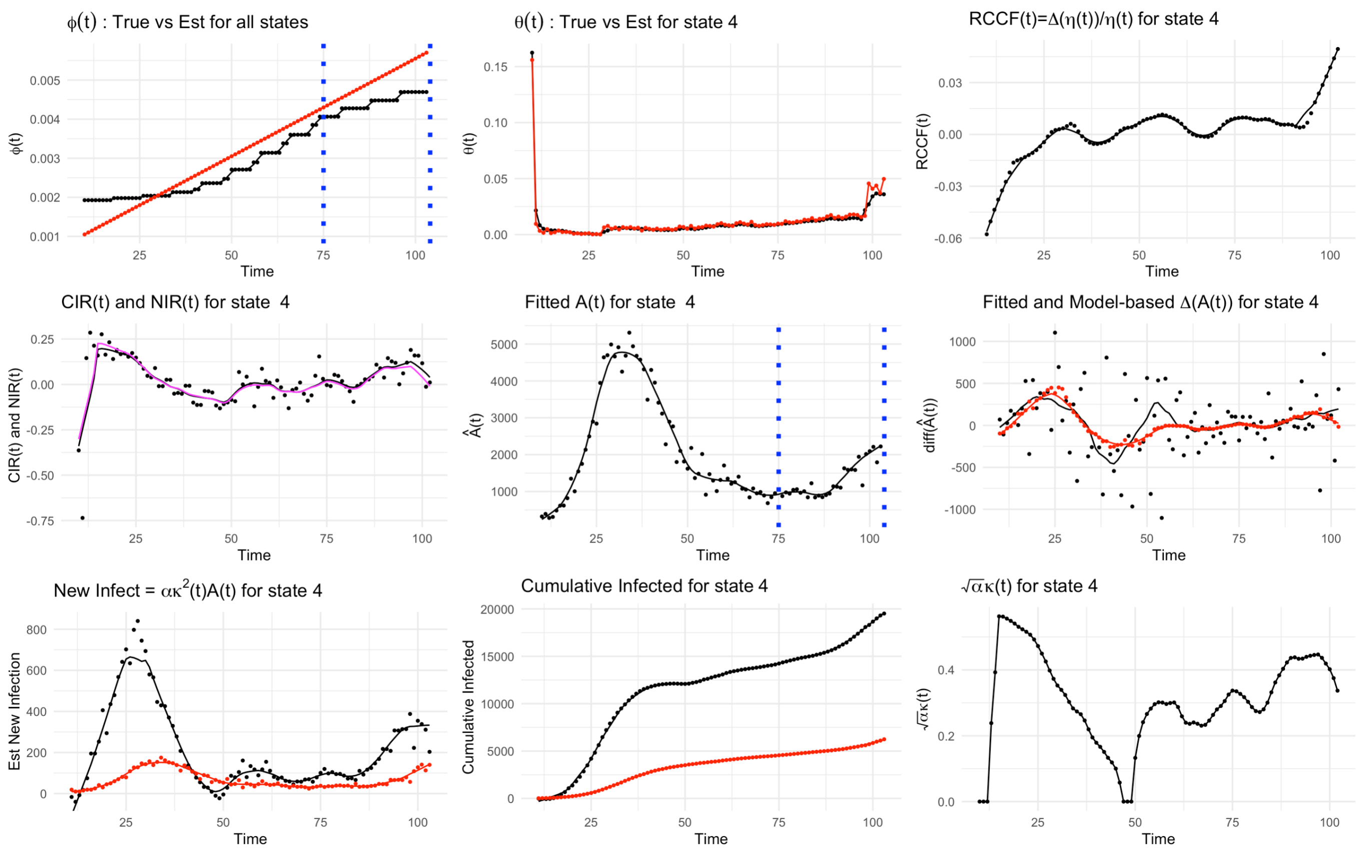


Figure 23: plots of estimations for "states" 4 (CIR(t) vs NIR(t); Est vs TRUE)

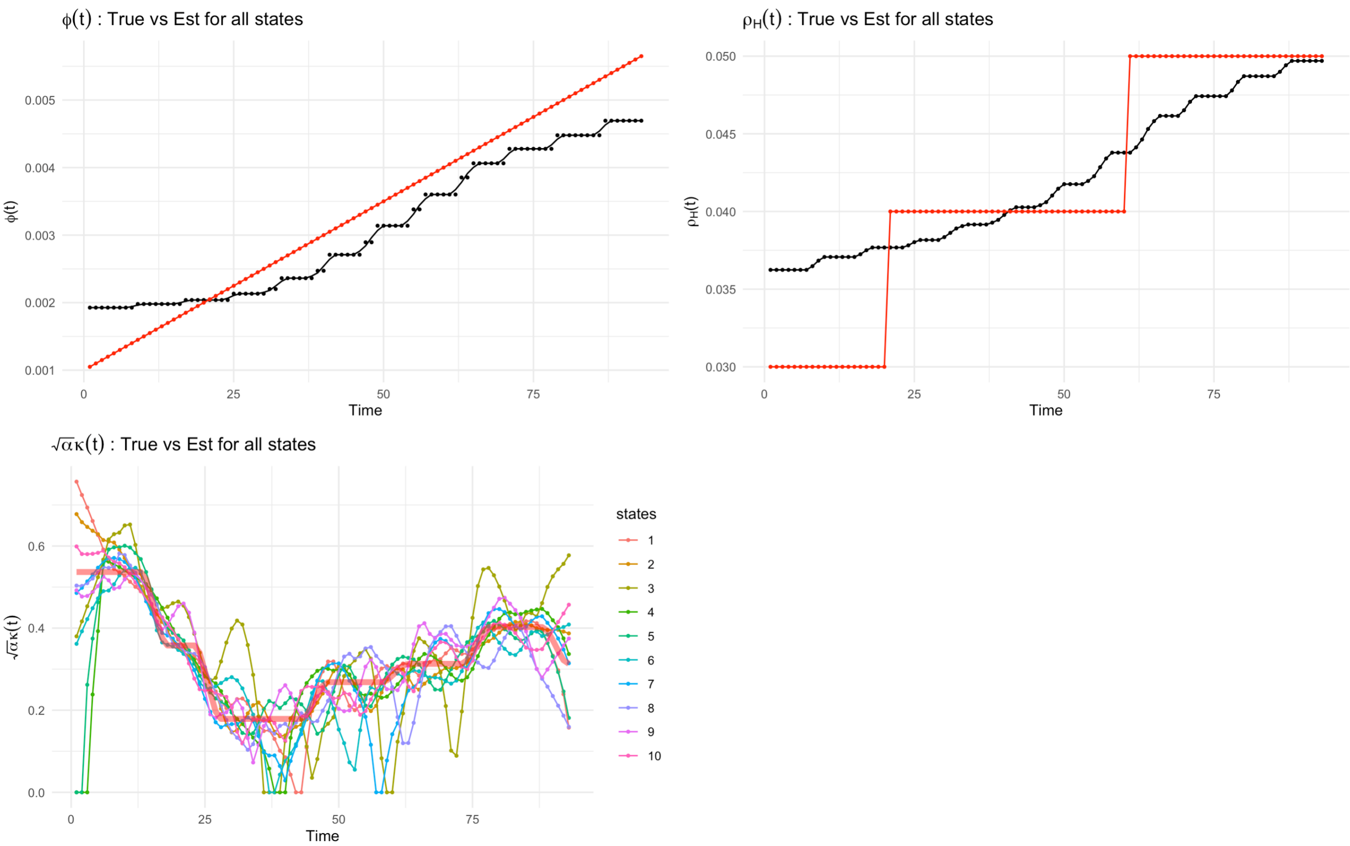


Figure 24: plots of estimations for $\phi(t), \rho_H(t), \sqrt{\alpha}\kappa(t)$ (Est vs TRUE)

Sampling distributions (CI)

Under the same true parameter setting, we simulate for 1050 times. Each time generate 10 independent processes.

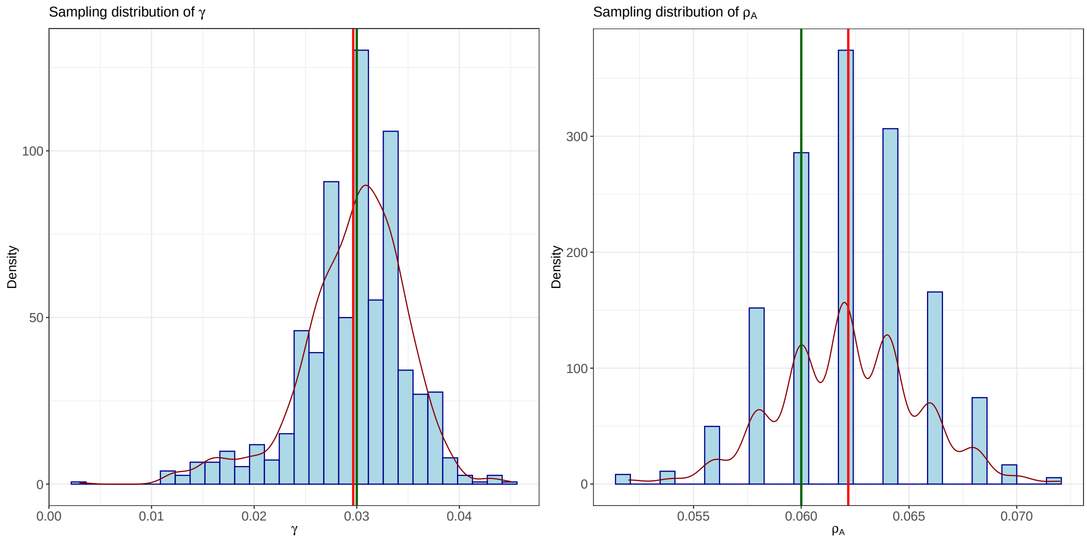


Figure 25: Sampling distribution of γ and ρ_A; (the mean is in red; the true is in darkgreen)

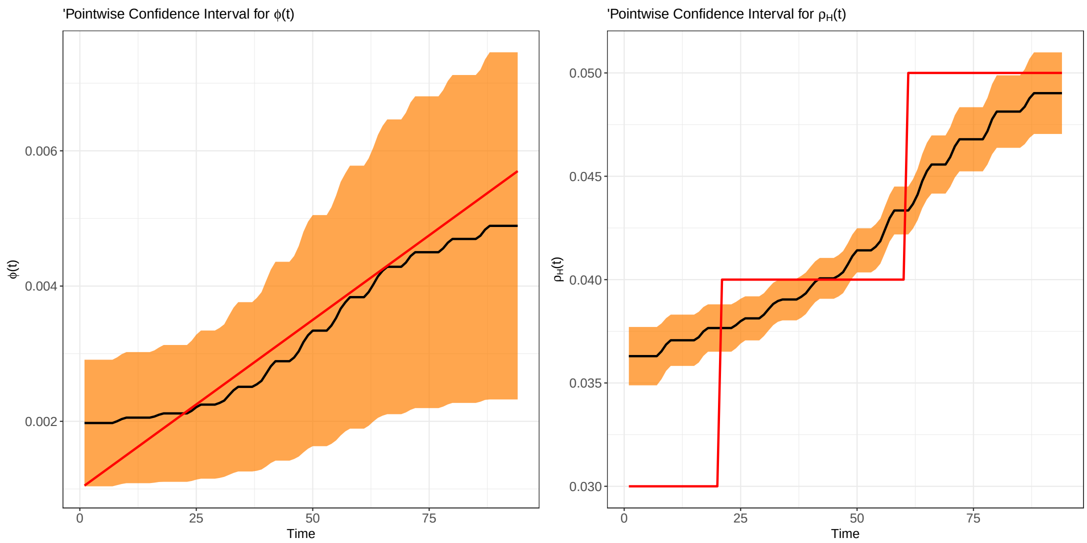


Figure 26: Sampling distribution of $\phi(t)$ and ρ_H; mean is in black and true is in red

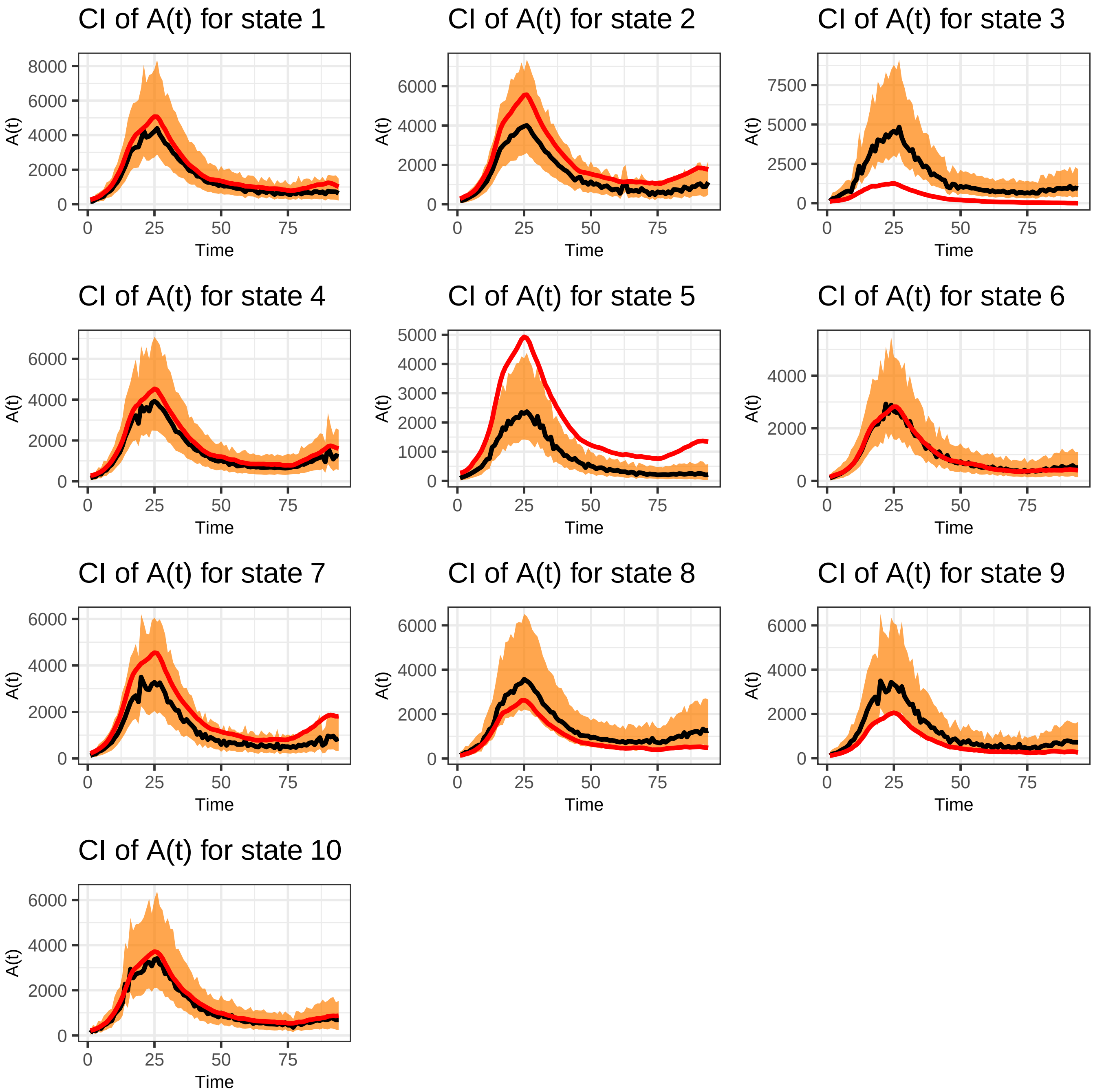


Figure 27: Pointwise Confidence Interval for A_t; mean is in black and true is in red

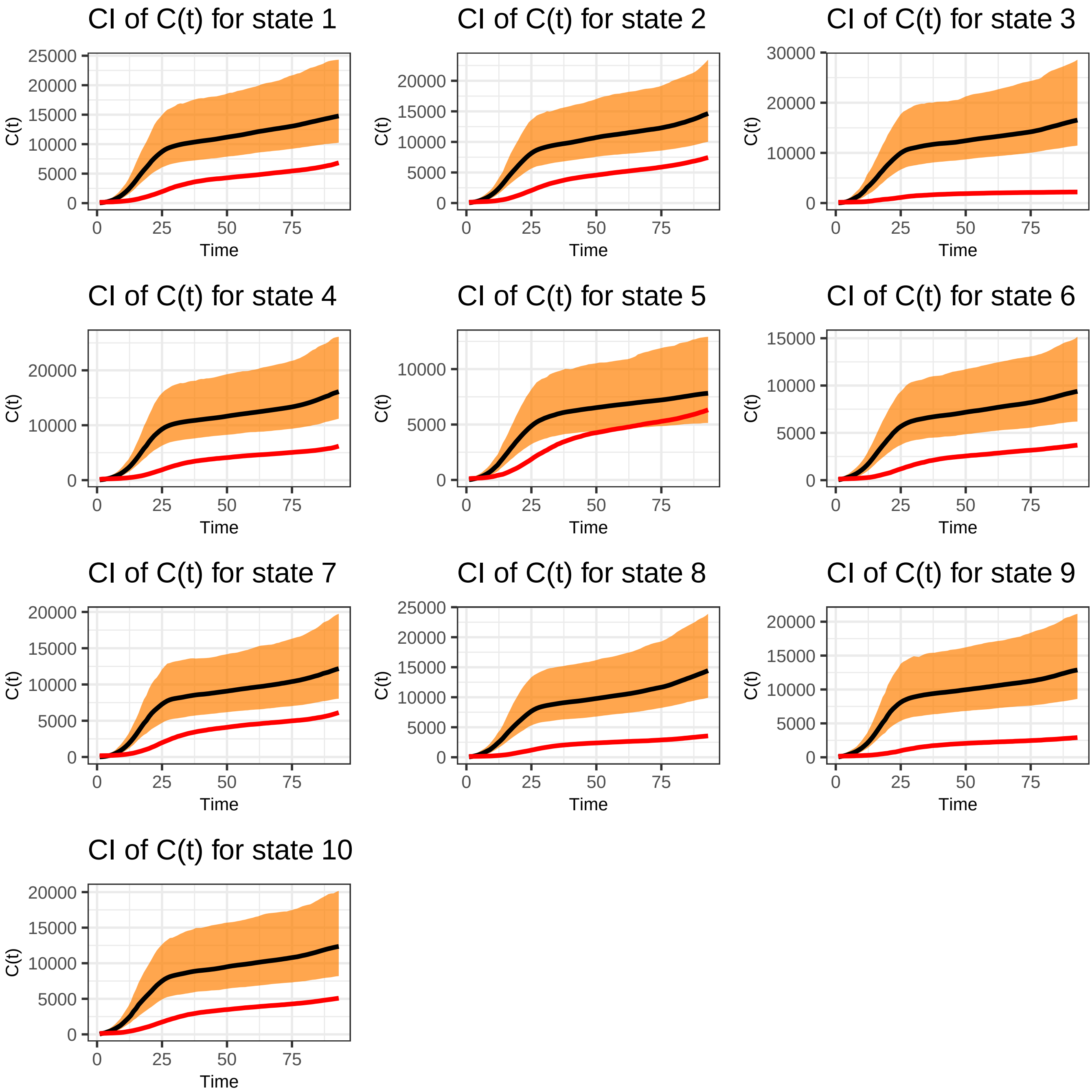


Figure 28: Pointwise Confidence Interval for C_t; mean is in black and true is in red

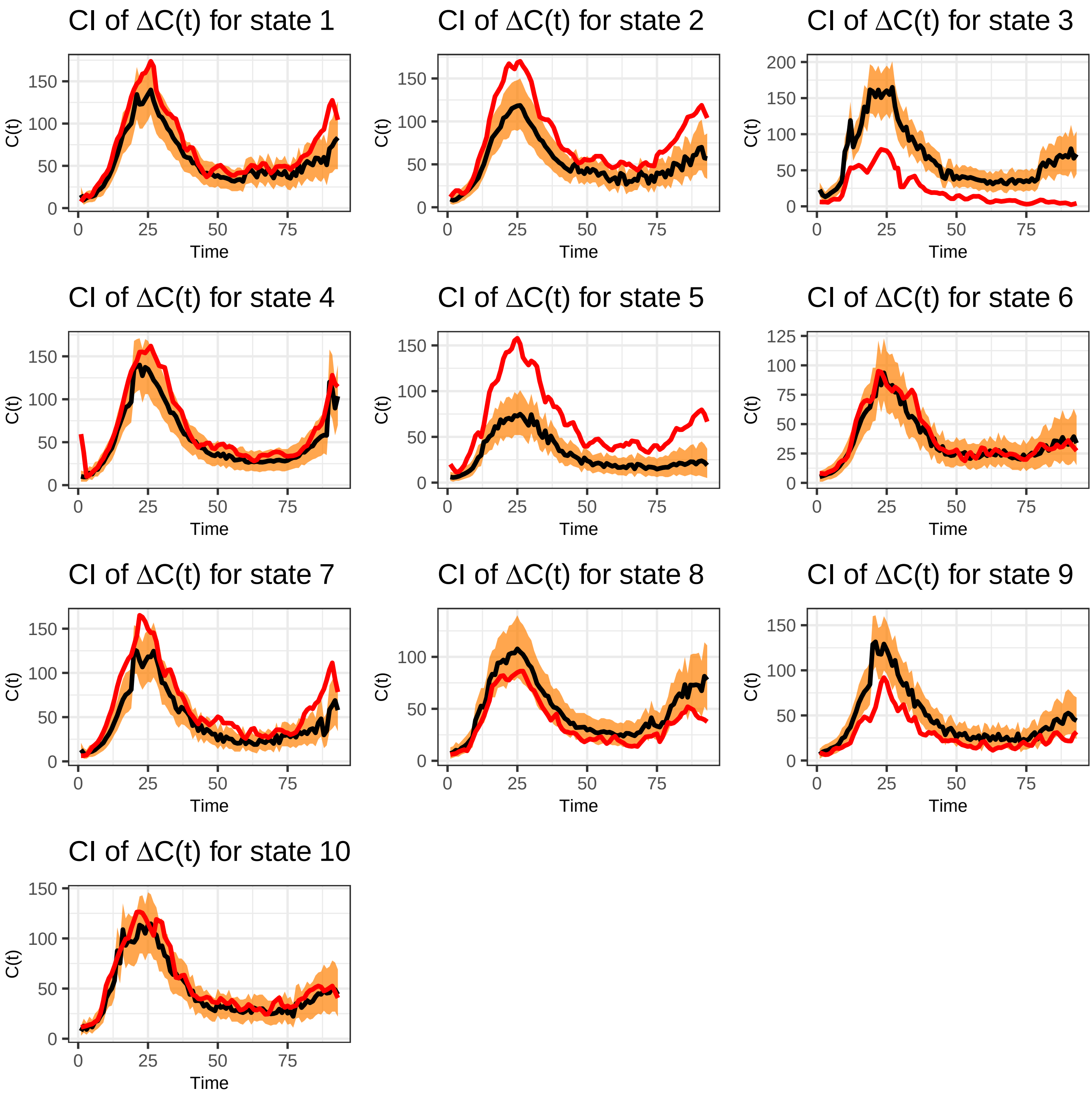


Figure 29: Pointwise Confidence Interval for ΔC_t (true smoothed ΔC_t and $(\hat{\theta}(t)+\hat{\gamma})\hat{A}_t$); mean is in black and true is in red

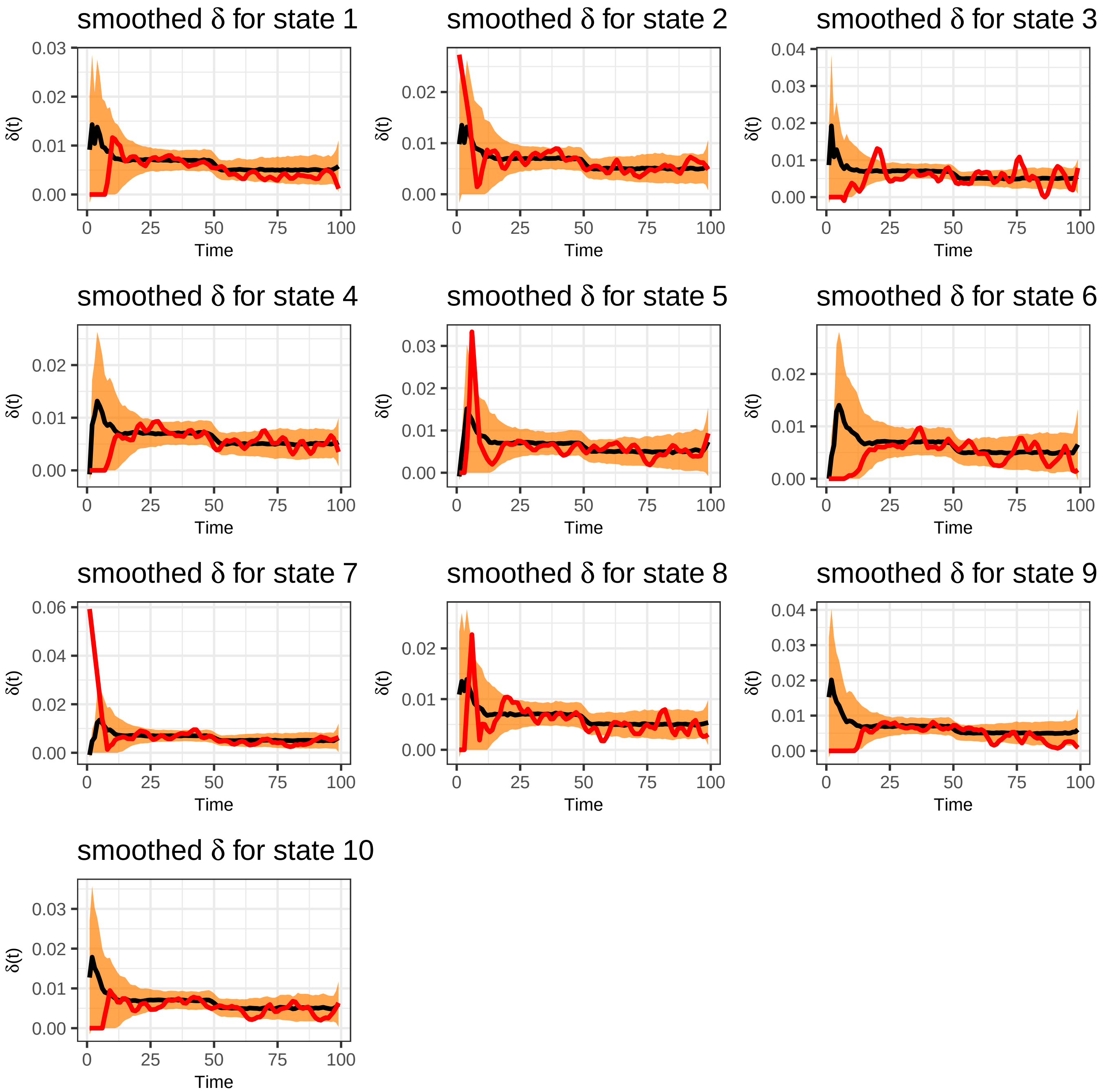


Figure 30: Pointwise Confidence Interval for smoothed $\delta(t)$; mean is in black and true is in red

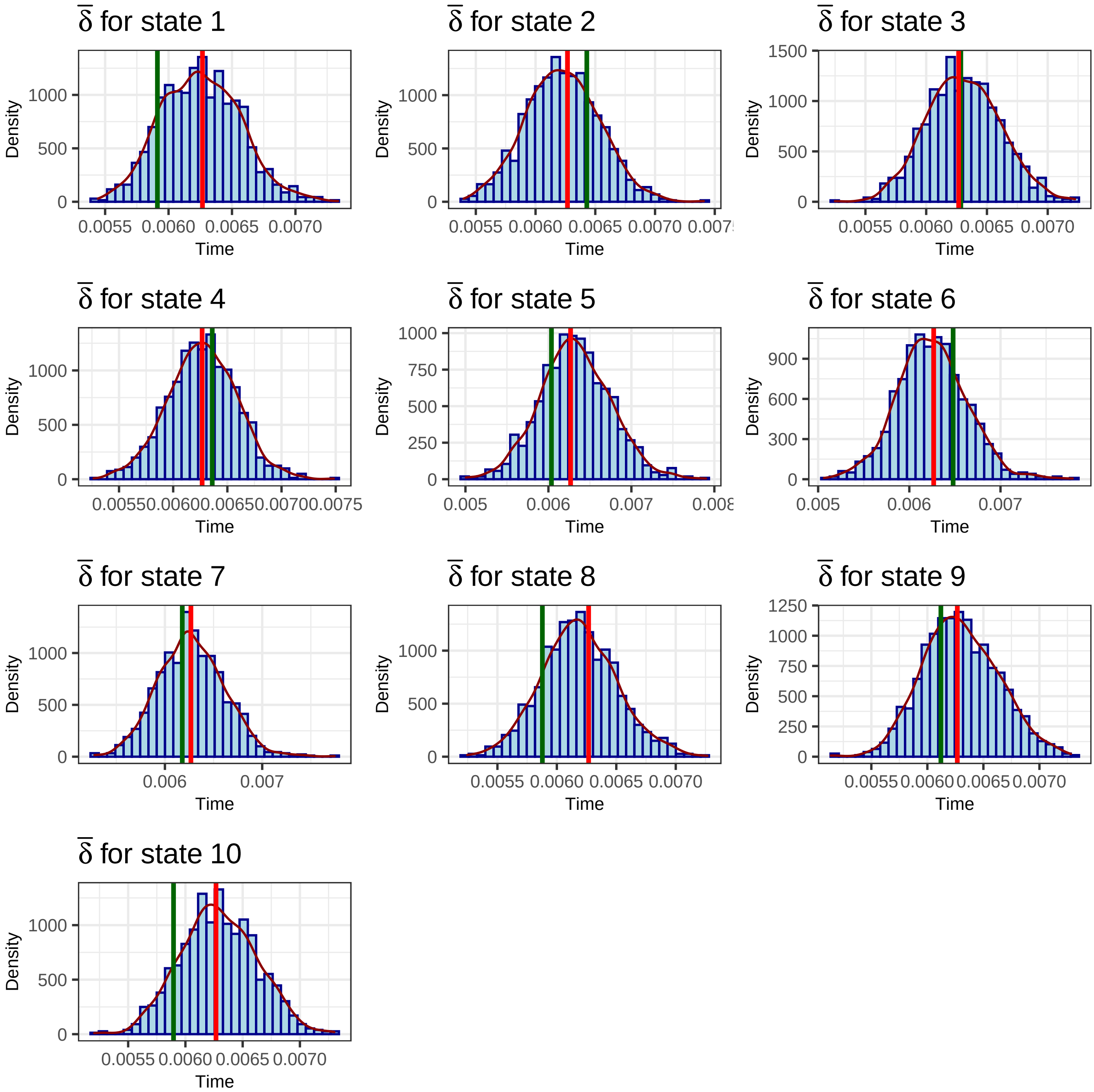


Figure 31: Sampling distribution of $\delta(t)$ (estimated by $D \sim \bar{\delta} H$); mean is in red and true is in darkgreen

Aggregation simulation 2 - Different parameter settings

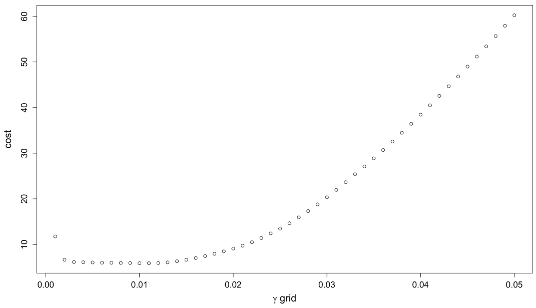
- Time (Days) = 100
- $\gamma = 0.01$
- $\rho_A = 0.04$
- $\alpha = 0.2$
- $\rho_H(t) = (rep(0.02, 20), rep(0.03, 40), rep(0.04, 40))$
- $\phi(t)$ is a linear function of Time with a small positive slope (reflecting increasing testing efficiency):

$$\phi(t) = 0.002 + 1 : Time * 0.00005$$

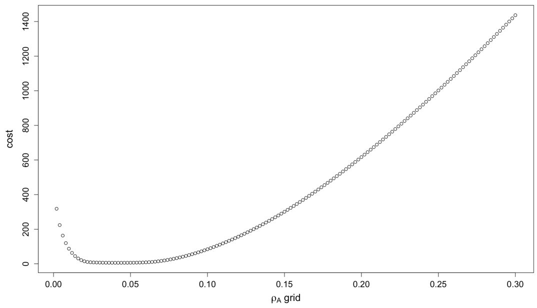
- $\kappa(t)$ = *lowess* version of (rep(1.2,15),rep(0.8,10),rep(0.4,20),rep(0.6,15),rep(0.7,15),rep(0.9,15),rep(0.7,10))
- $\delta(t)$ =(rep(0.005,50),rep(0.008,50))
- number of Time block = 9; window length = 30;

grid search for γ and ρ_A are:

$$\hat{\gamma} = 0.011, \hat{\rho}_A = 0.042$$



(a)



(b)

Figure 36: grid of γ and ρ_A against the costs; minimum is achieved at $\hat{\gamma} = 0.011$ and $\hat{\rho}_A = 0.042$ respectively

estimation for "state" 5

estimation for "state" 6

estimation for $\phi(t), \rho_H(t), \alpha\sqrt{\kappa(t)}$

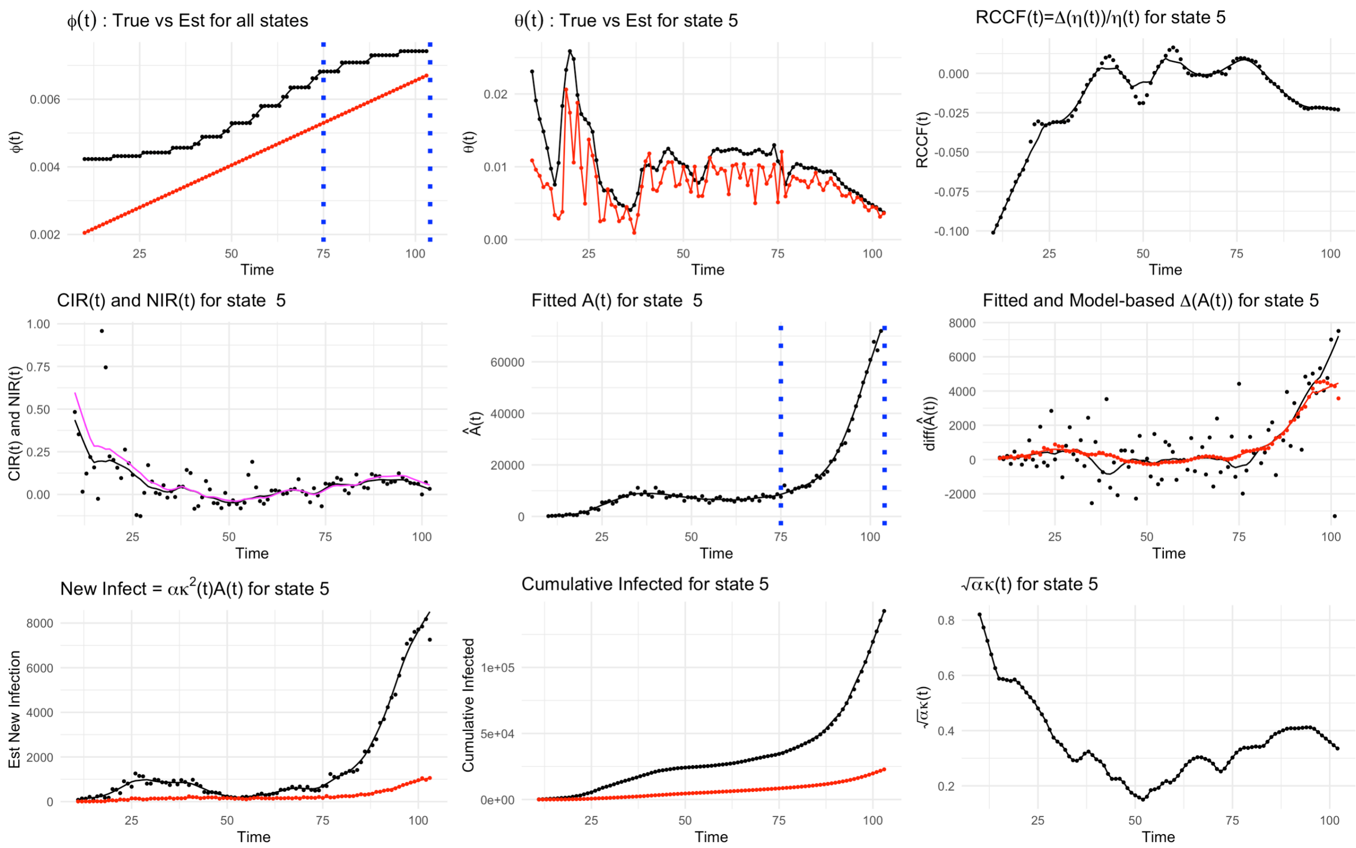


Figure 37: plots of estimations for "states" 5 (CIR(t) vs NIR(t); Est vs TRUE)

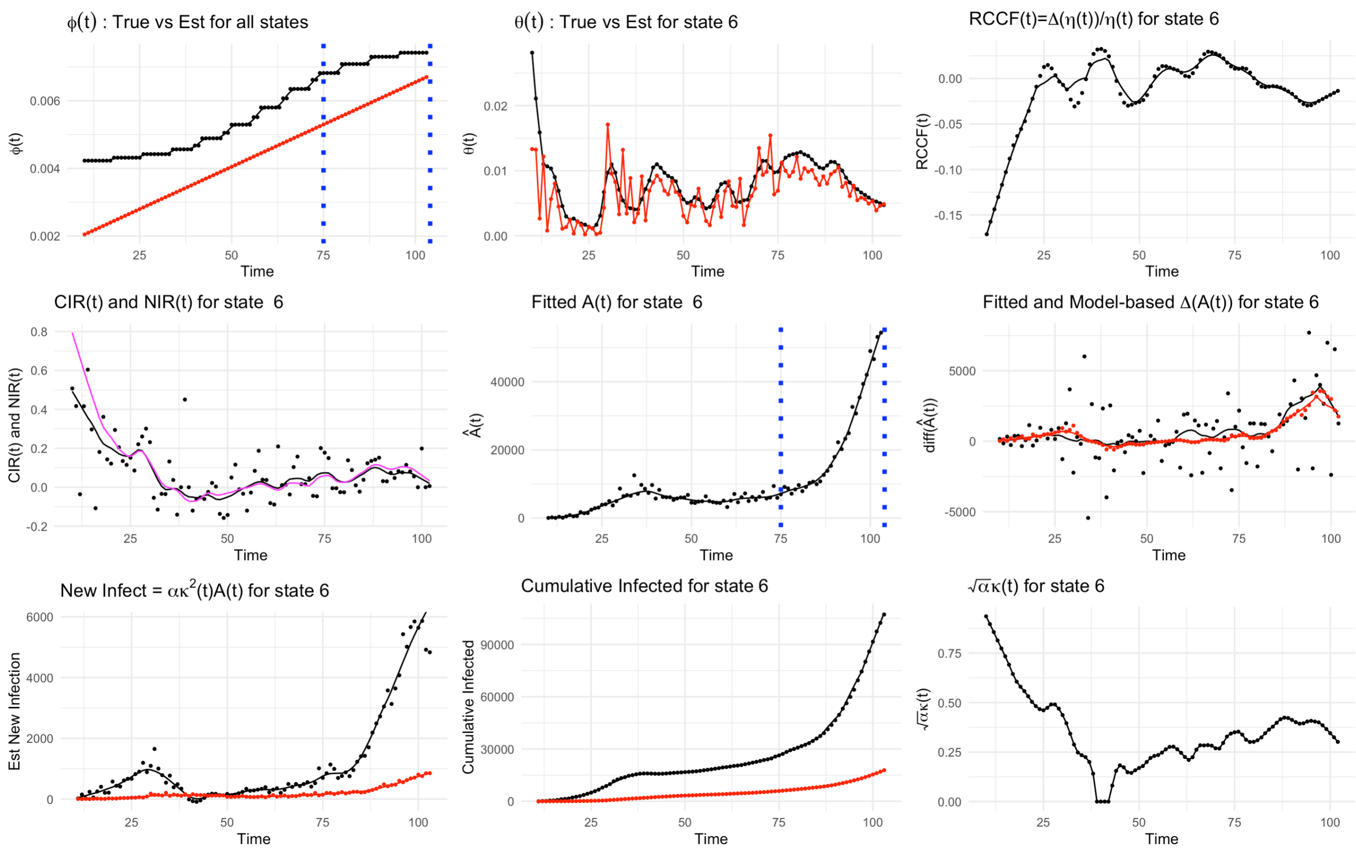


Figure 38: plots of estimations for "states" 6 (CIR(t) vs NIR(t); Est vs TRUE)

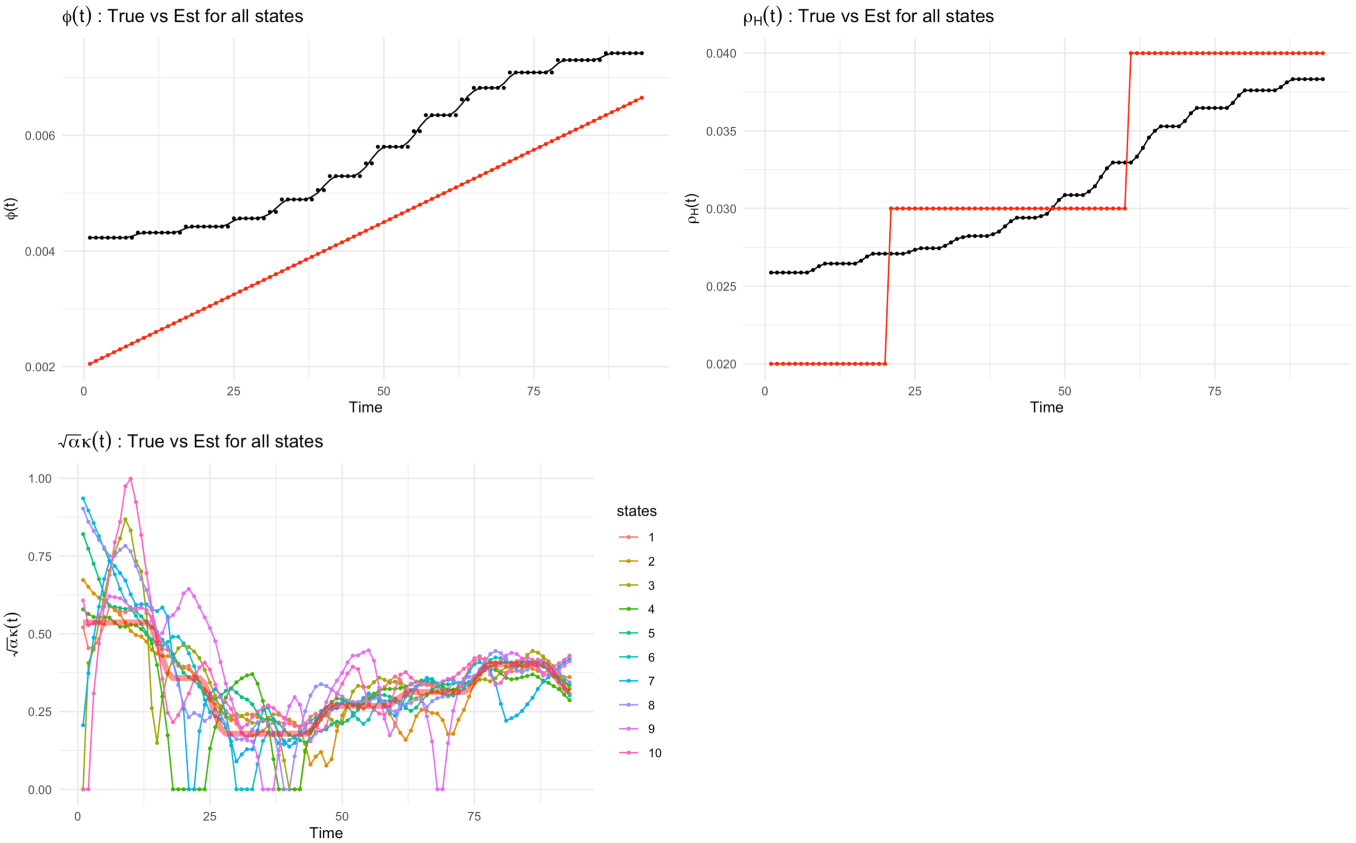


Figure 39: plots of estimations for $\phi(t), \rho_H(t), \sqrt{\alpha}\kappa(t)$ (Est vs TRUE)

Sampling distributions (CI)

Under the same true parameter setting, we simulate for 1050 times. Each time generate 10 independent processes.

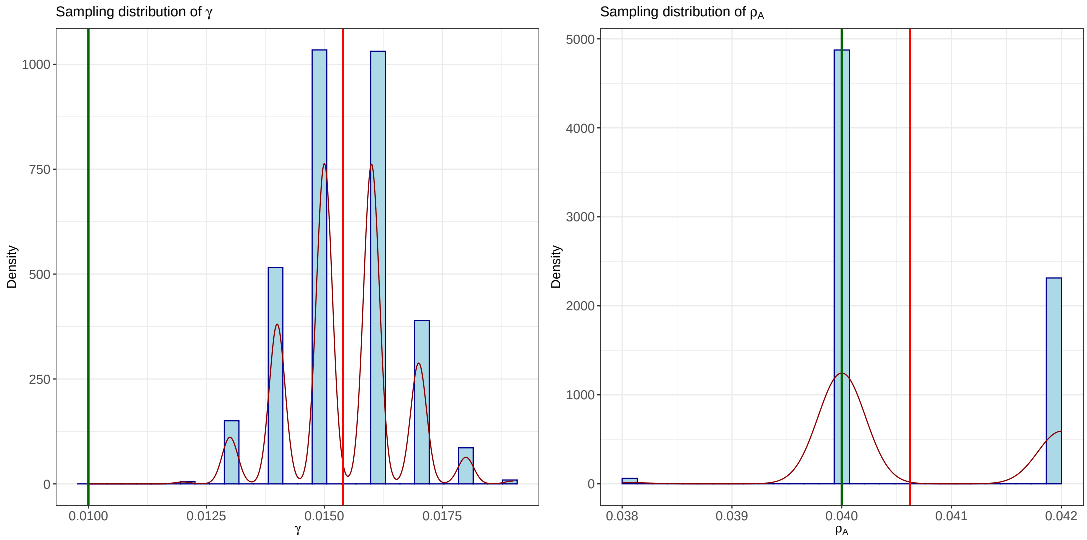


Figure 40: Sim2: Sampling distribution of γ and ρ_A; (the mean is in red; the true is in darkgreen)

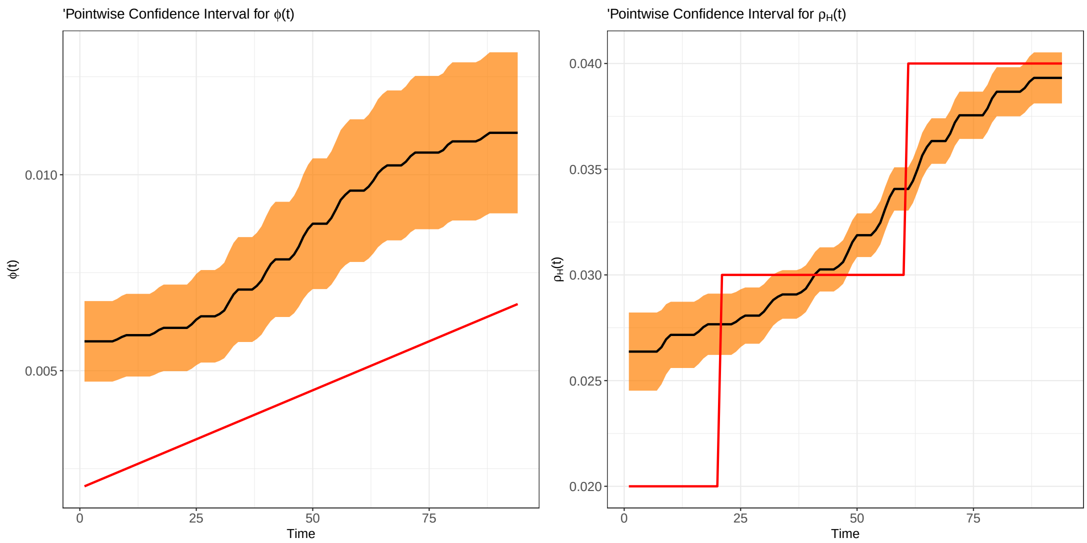


Figure 41: Sim2: Sampling distribution of $\phi(t)$ and ρ_H; mean is in black and true is in red

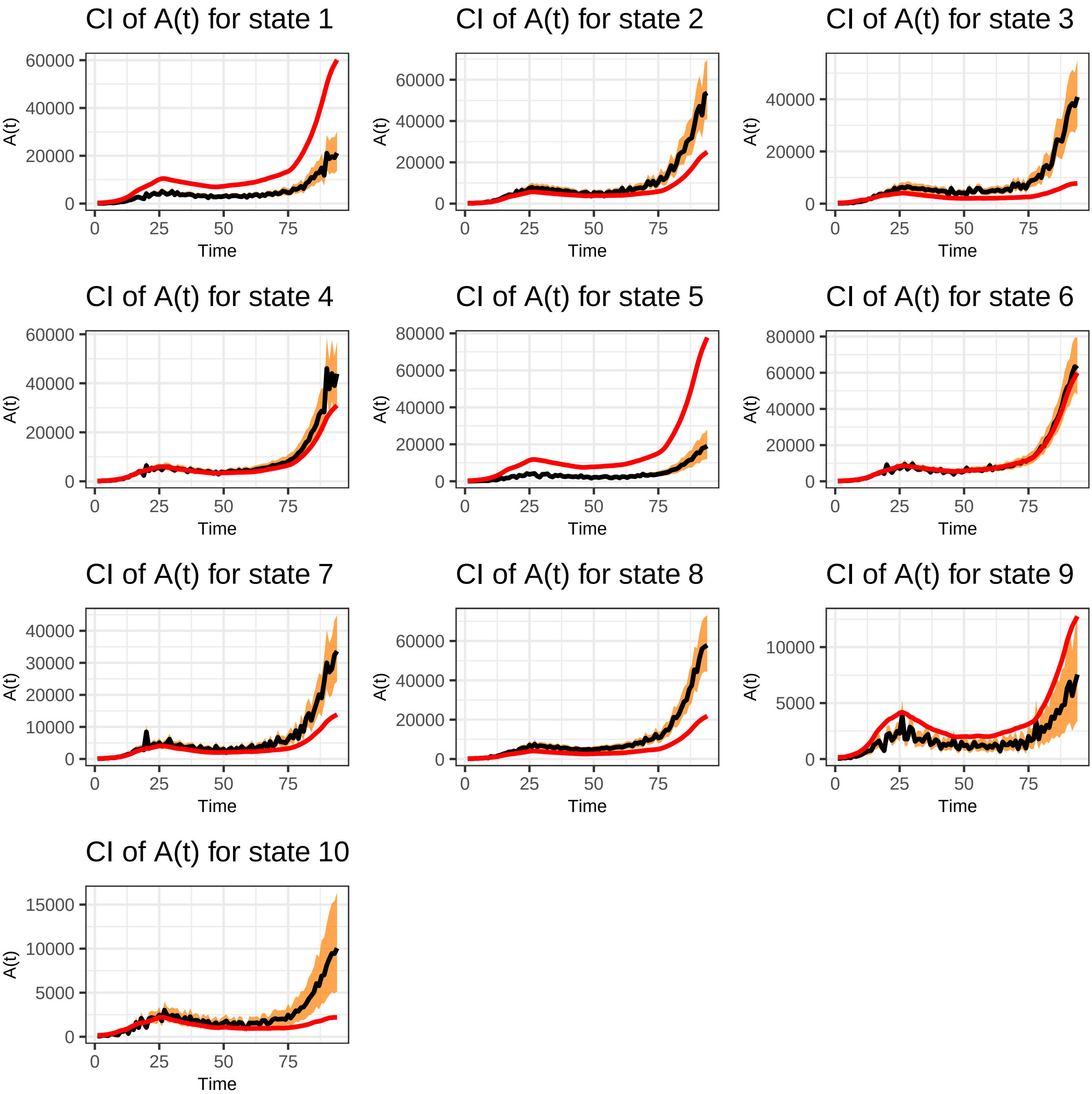


Figure 42: Sim2: Pointwise Confidence Interval for A_t; mean is in black and true is in red

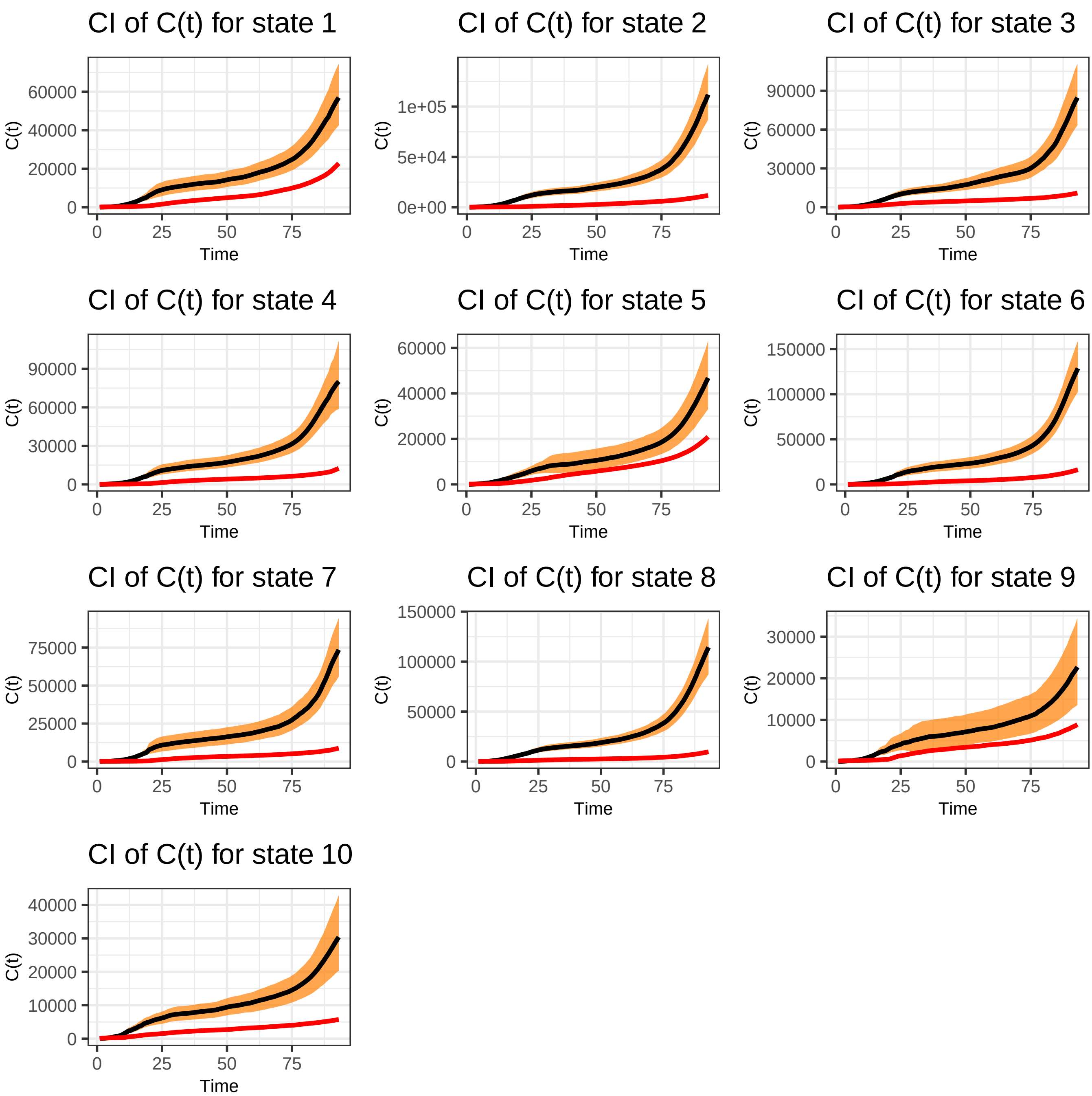


Figure 43: Sim2: Pointwise Confidence Interval for C_t; mean is in black and true is in red

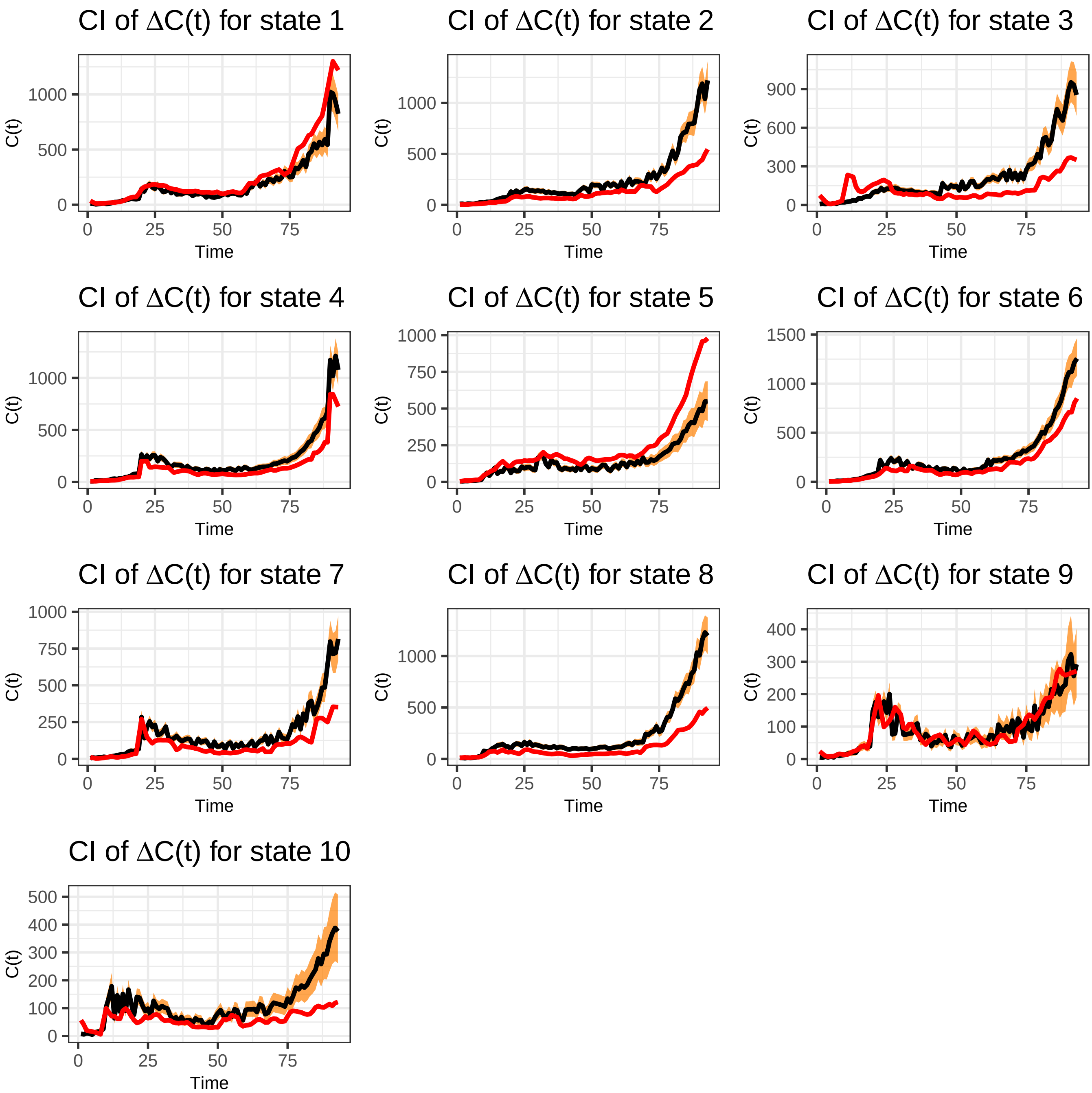


Figure 44: Pointwise Confidence Interval for ΔC_t (true smoothed ΔC_t and $(\hat{\theta}(t)+\hat{\gamma})\hat{A}_t$); mean is in black and true is in red

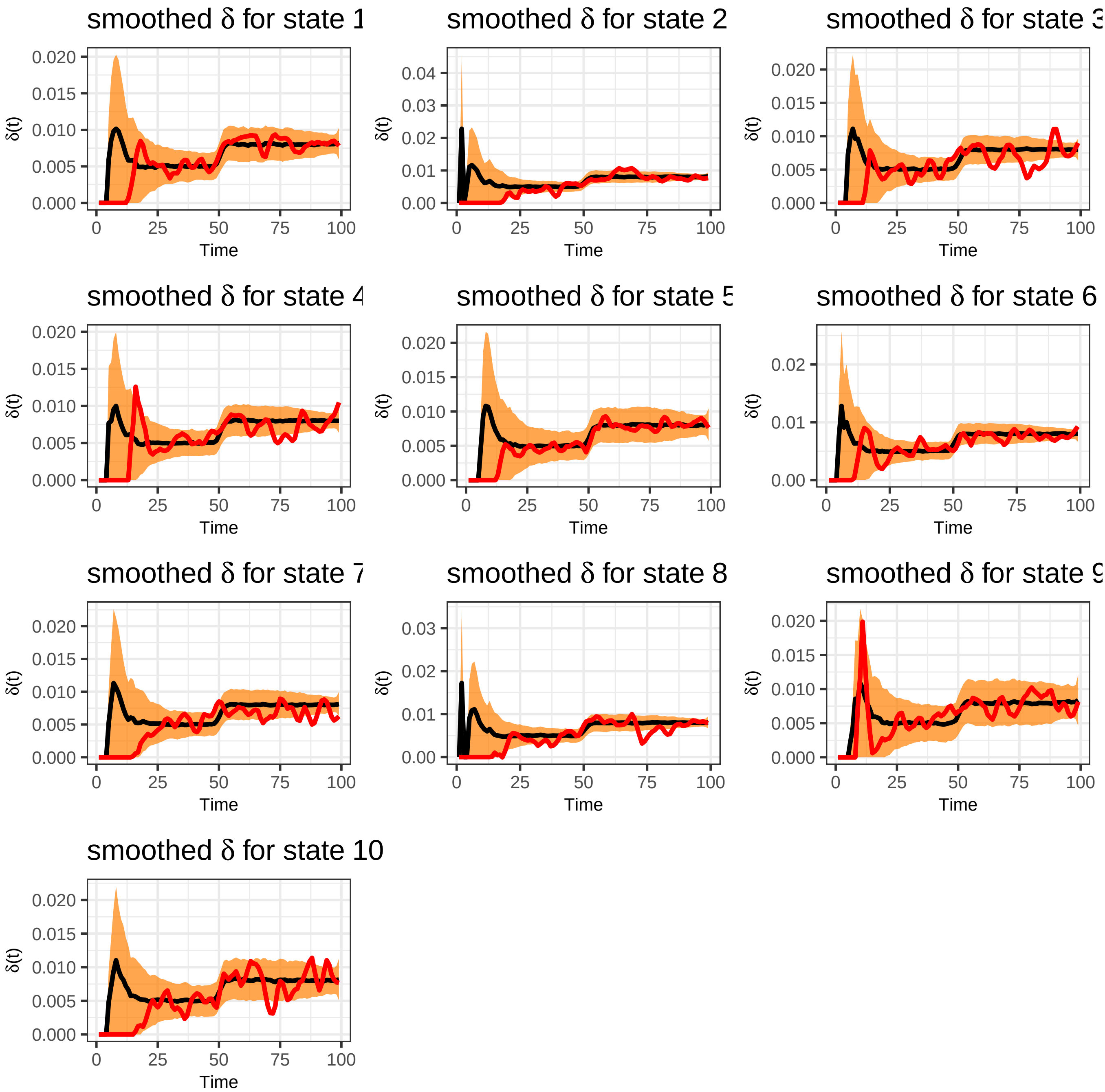


Figure 45: Sim2: Pointwise Confidence Interval for smoothed $\delta(t)$; mean is in black and true is in red

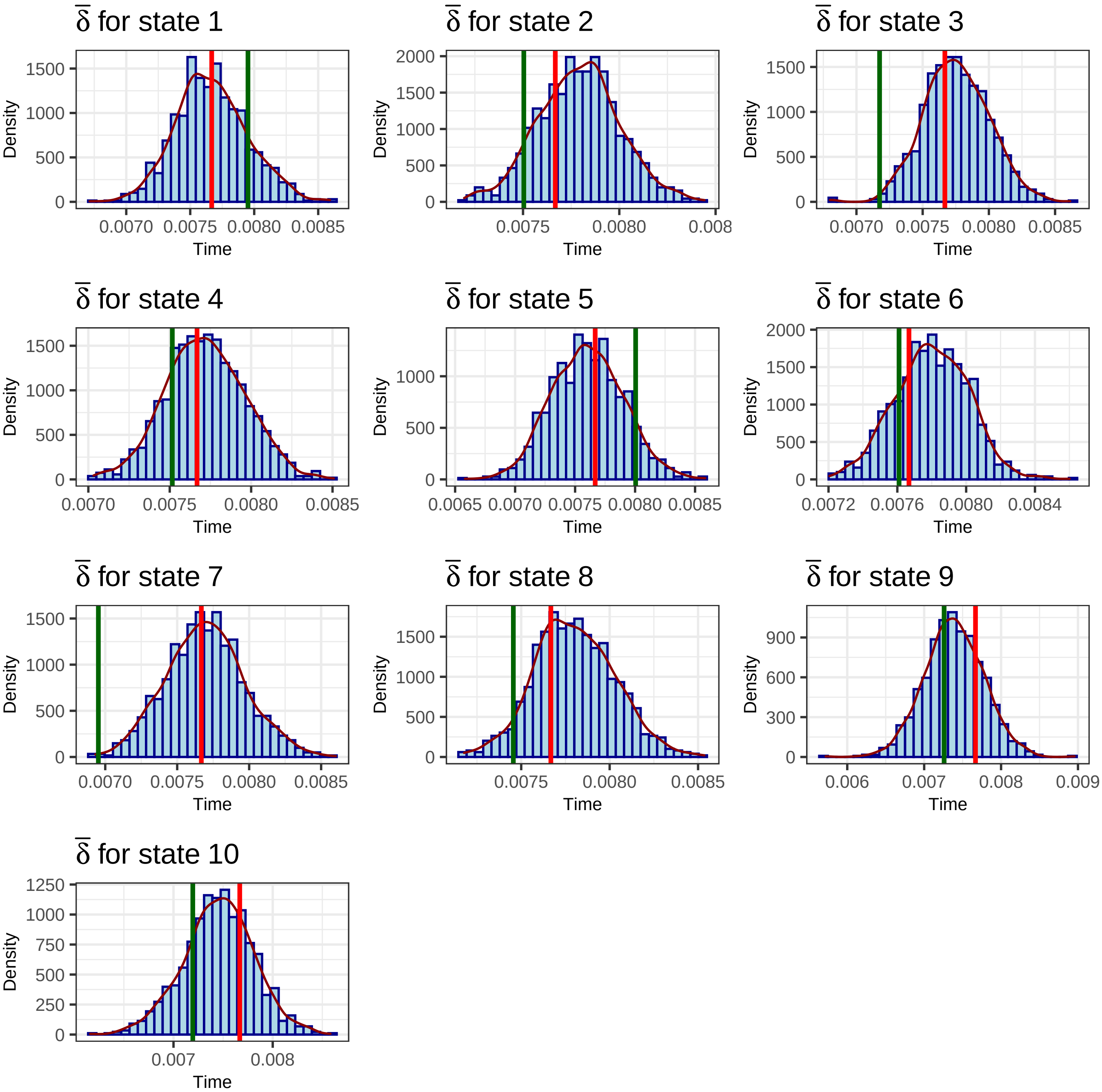


Figure 46: Sim2: Sampling distribution of $\delta(t)$ (estimated by $D \sim \bar{\delta}H$); mean is in red and true is in darkgreen

Aggregation simulation 3 - Different parameter settings

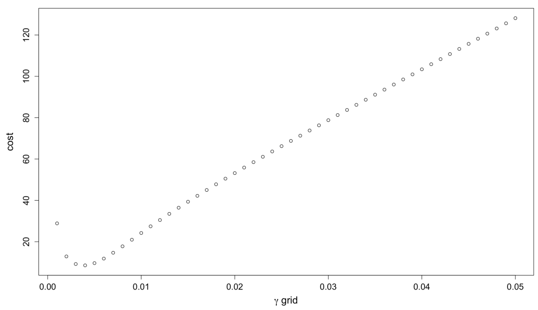
- Time (Days) = 100
- $\gamma = 0.01$
- $\rho_A = 0.02$
- $\alpha = 0.18$
- $\rho_H(t) = (rep(0.02, 20), rep(0.05, 50), rep(0.04, 30))$
- $\phi(t)$ is a linear function of Time with a small positive slope (reflecting increasing testing efficiency):

$$\phi(t) = 0.0001 + 1 : Time * 0.00005$$

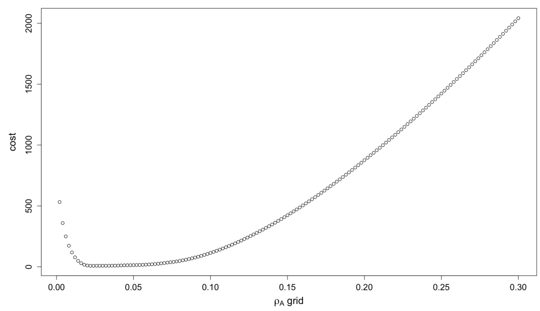
- $\kappa(t)$ = *lowess* version of (rep(1,15),rep(0.8,20),rep(0.7,10),rep(0.6,10),rep(0.7,15),rep(0.9,15),rep(1,15))
- $\delta(t)$ =(rep(0.005,30),rep(0.009,40),rep(0.007,30))
- number of Time block = 9; window length = 30;

grid search for γ and ρ_A are:

$$\hat{\gamma} = 0.004, \hat{\rho}_A = 0.024$$



(a)



(b)

Figure 51: grid of γ and ρ_A against the costs; minimum is achieved at $\hat{\gamma} = 0.004$ and $\hat{\rho}_A = 0.024$ respectively

estimation for "state" 2

estimation for "state" 9

estimation for $\phi(t), \rho_H(t), \alpha\sqrt{\kappa(t)}$

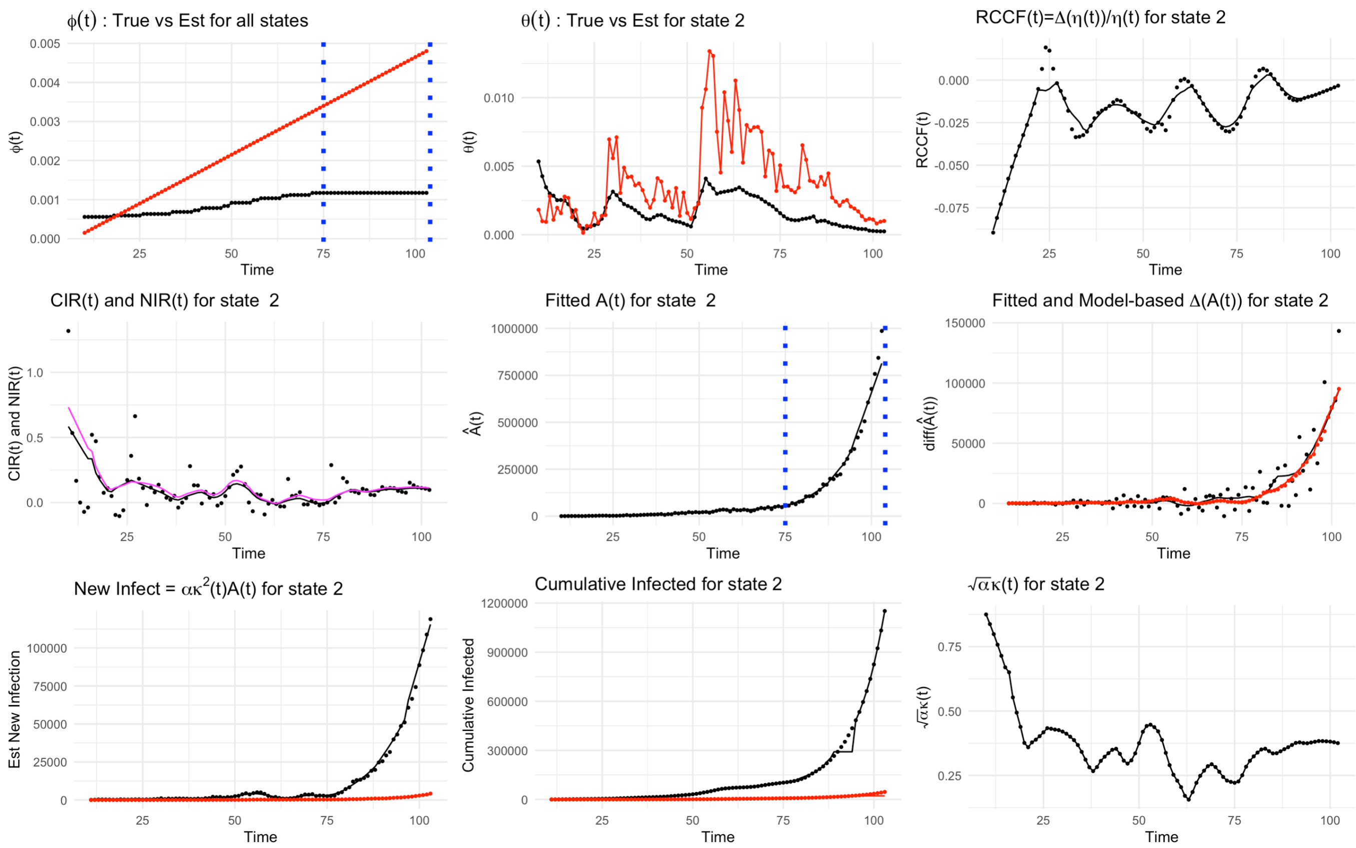


Figure 52: plots of estimations for "states" 2 (CIR(t) vs NIR(t); Est vs TRUE)

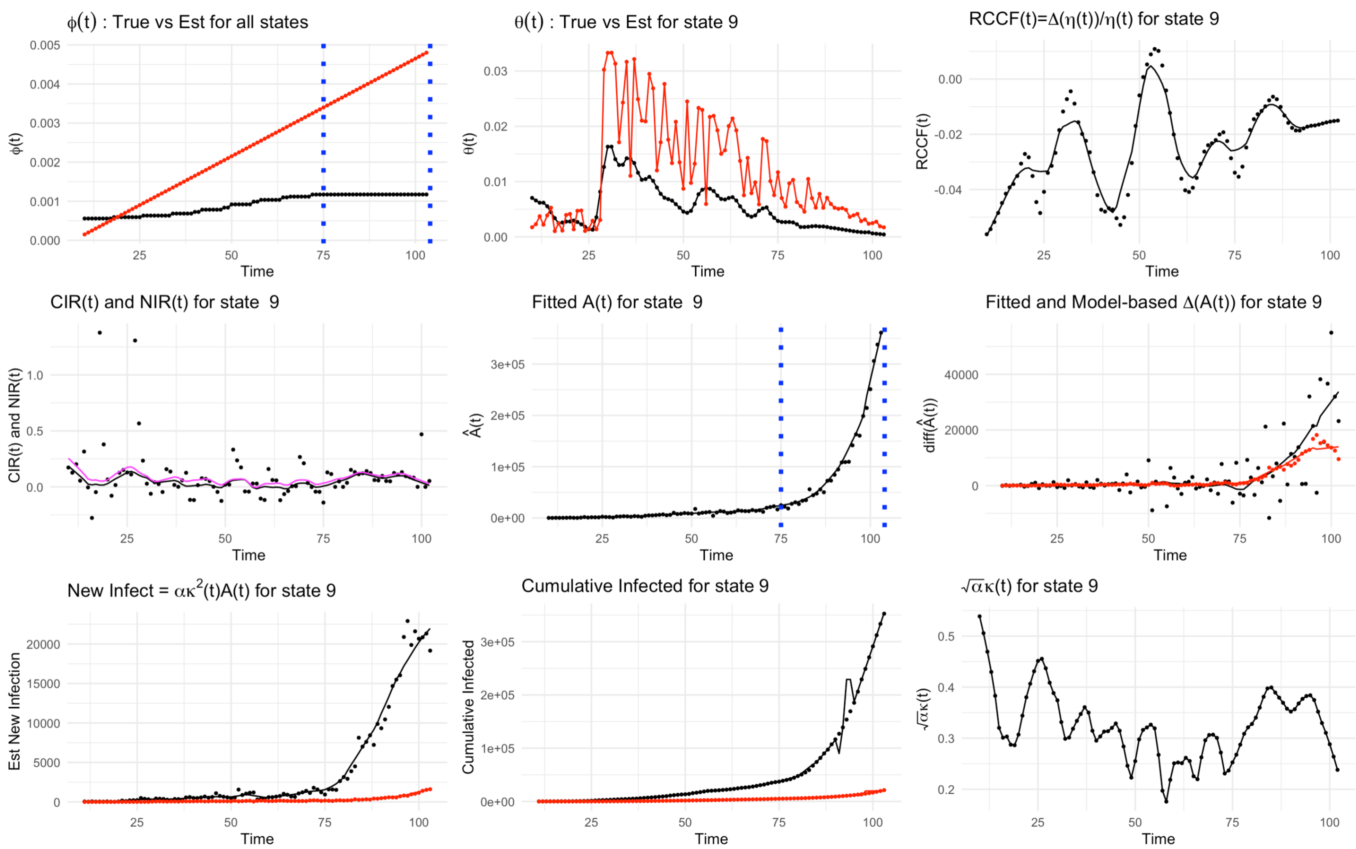


Figure 53: plots of estimations for "states" 9 (CIR(t) vs NIR(t); Est vs TRUE)

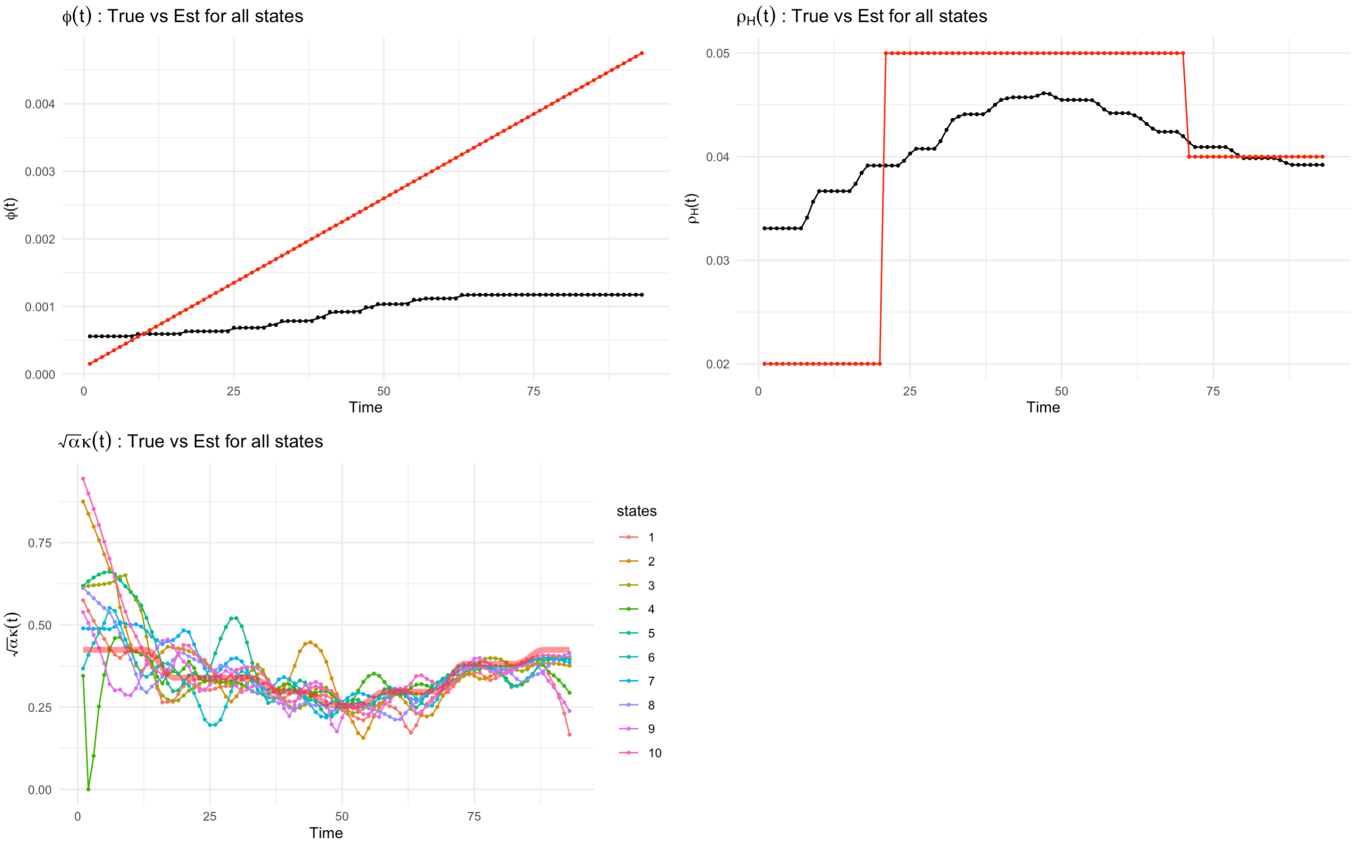


Figure 54: plots of estimations for $\phi(t), \rho_H(t), \sqrt{\alpha}\kappa(t)$ (Est vs TRUE)

Sampling distributions (CI)

Under the same true parameter setting, we simulate for 1050 times. Each time generate 10 independent processes.

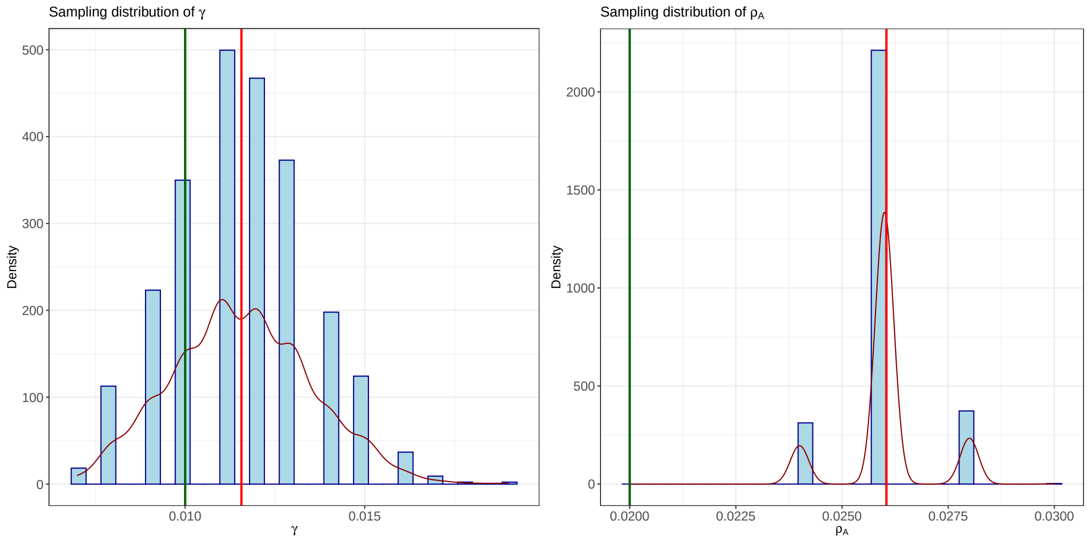


Figure 55: Sim3: Sampling distribution of γ and ρ_A; (the mean is in red; the true is in darkgreen)

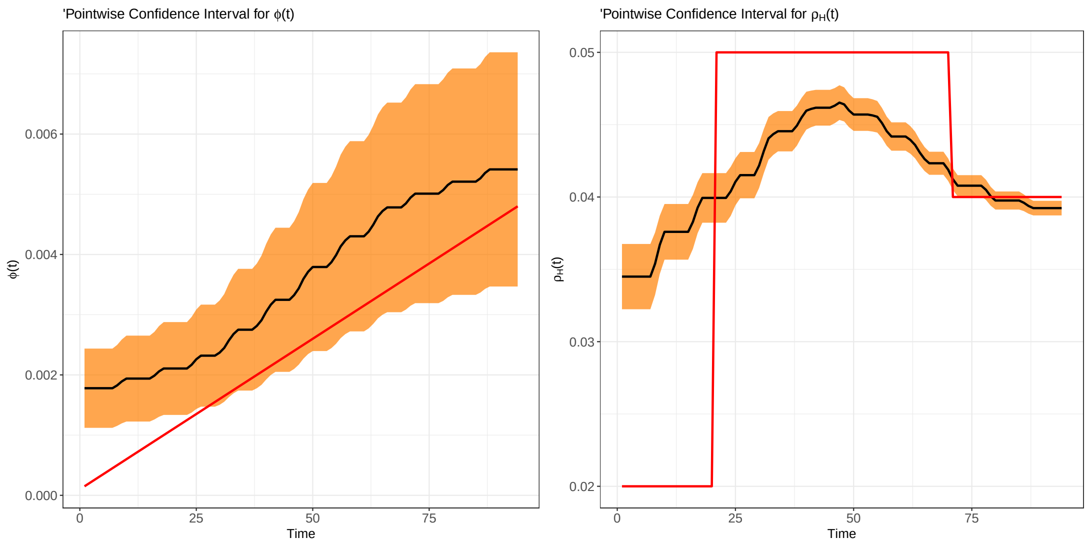


Figure 56: Sim3: Sampling distribution of $\phi(t)$ and ρ_H; mean is in black and true is in red

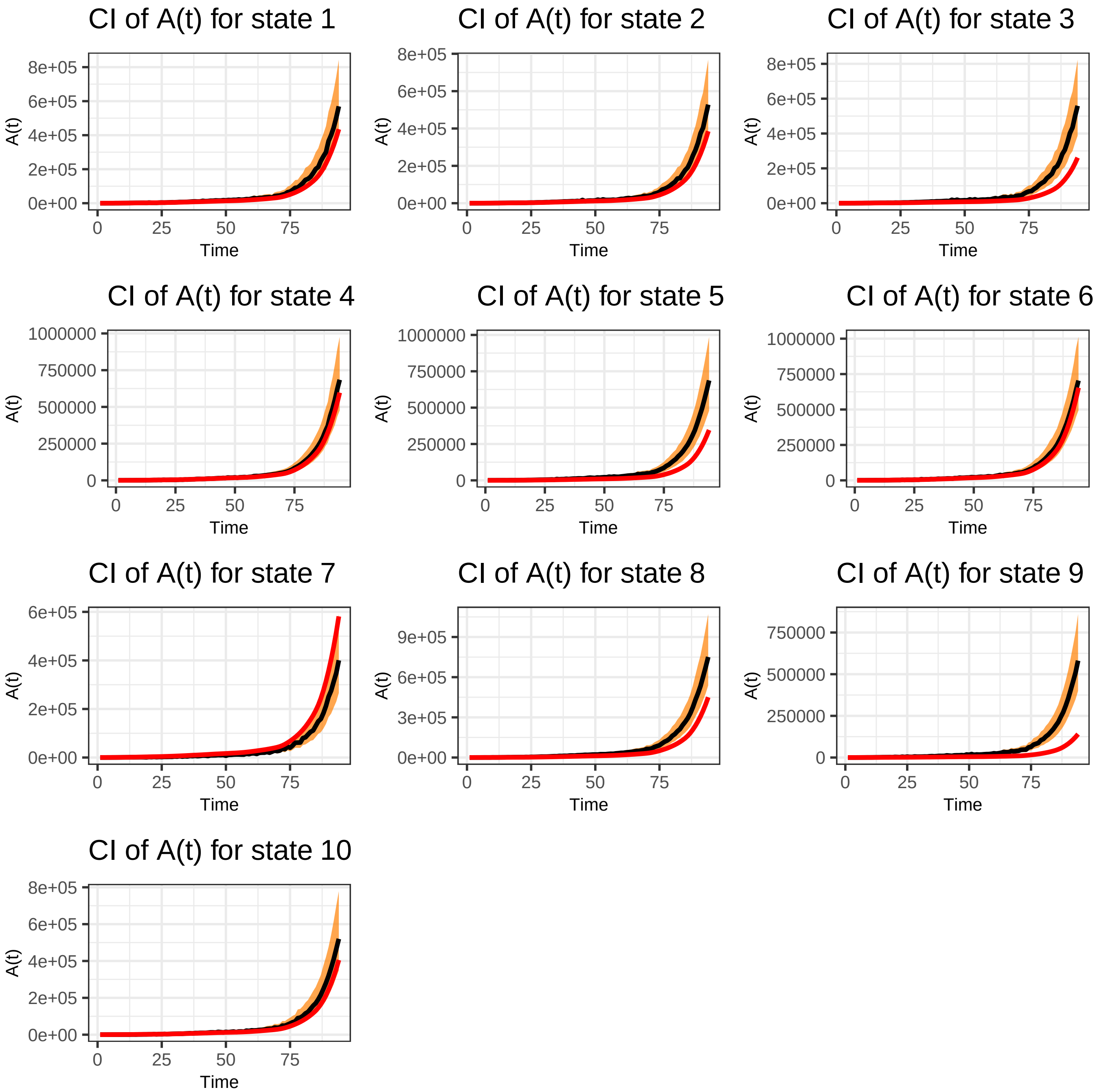


Figure 57: Sim3: Pointwise Confidence Interval for A_t; mean is in black and true is in red

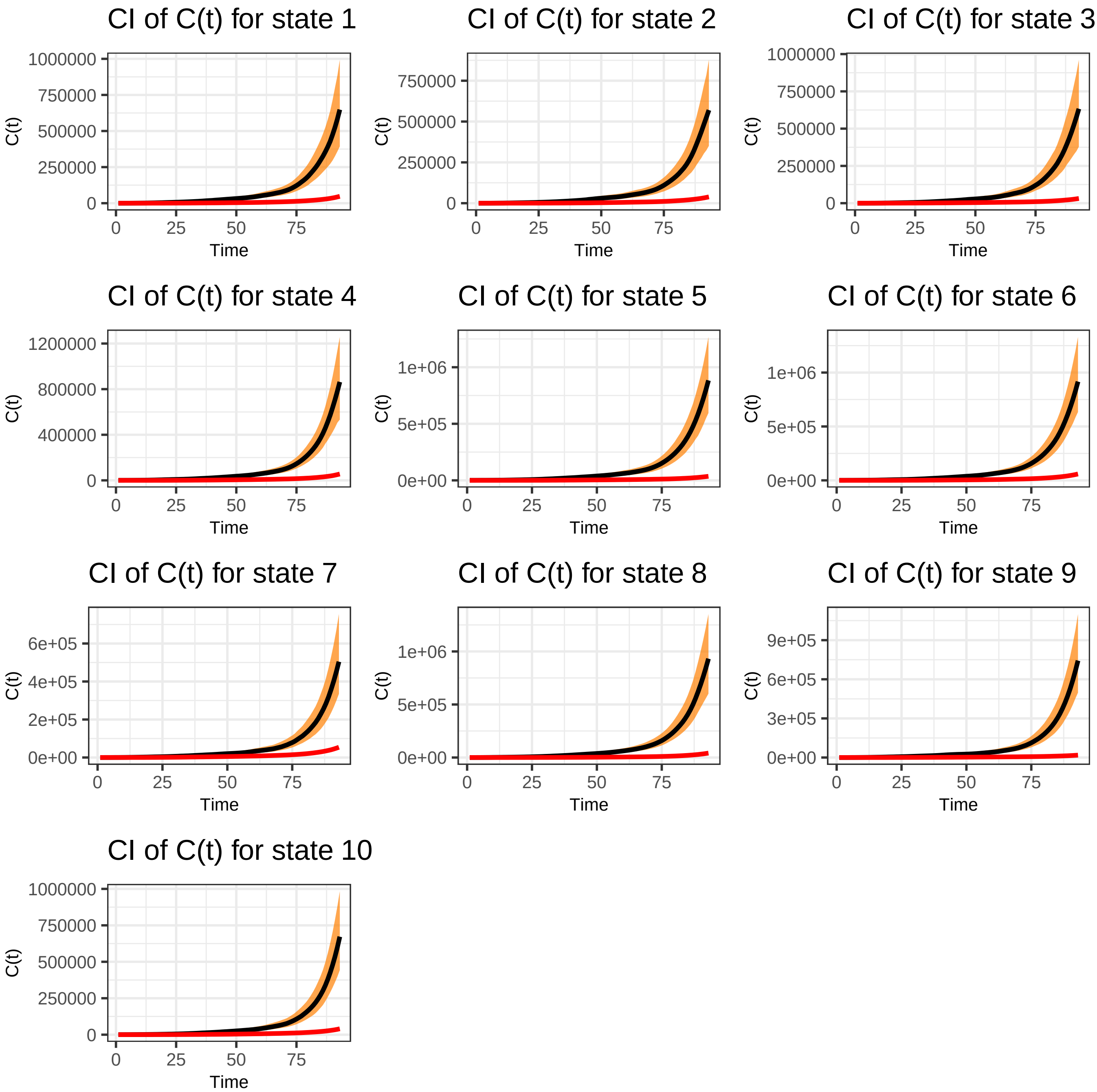


Figure 58: Sim3: Pointwise Confidence Interval for C_t; mean is in black and true is in red

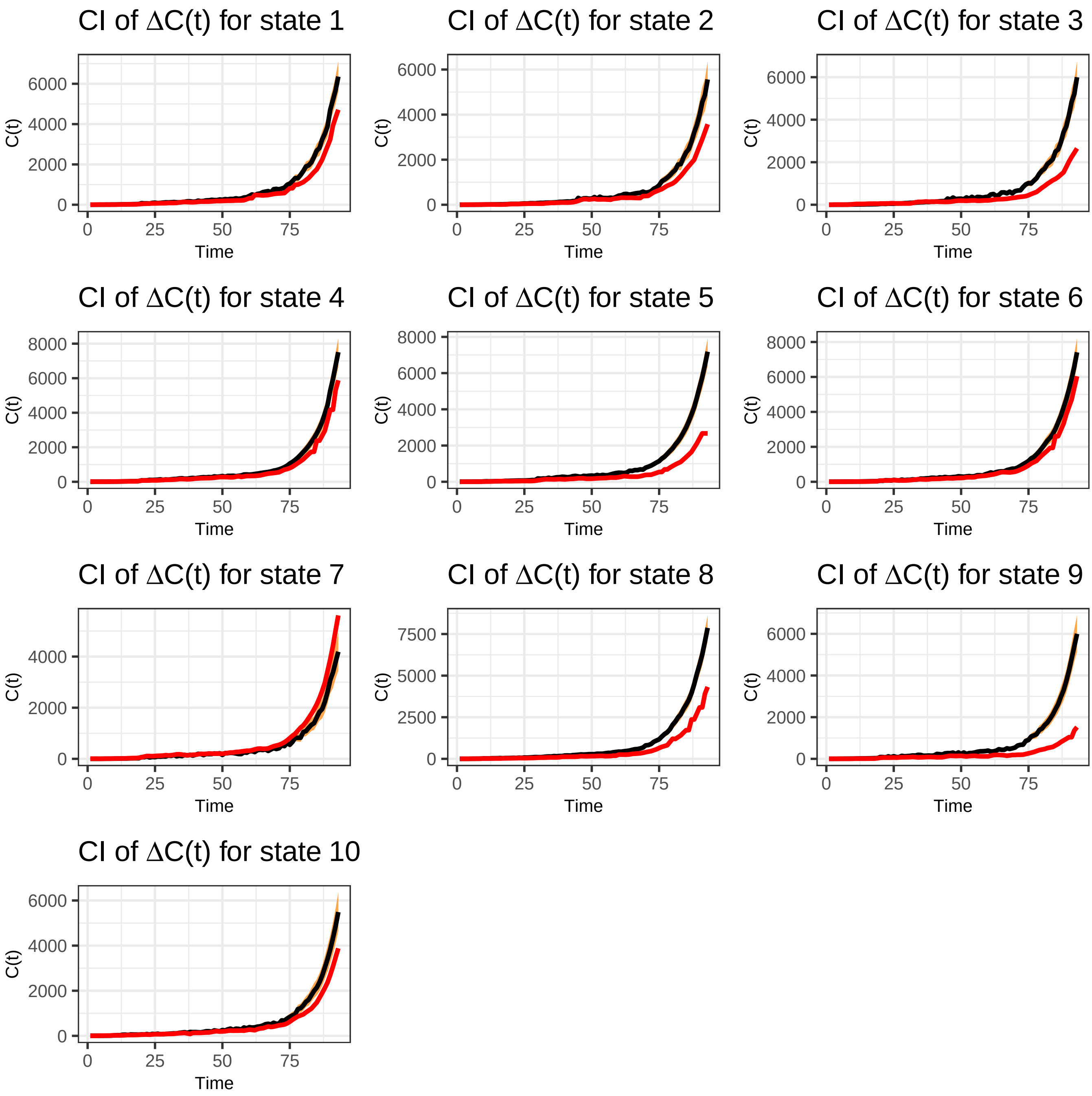


Figure 59: Pointwise Confidence Interval for ΔC_t (true smoothed ΔC_t and $(\hat{\theta}(t) + \hat{\gamma})\hat{A}_t$); mean is in black and true is in red

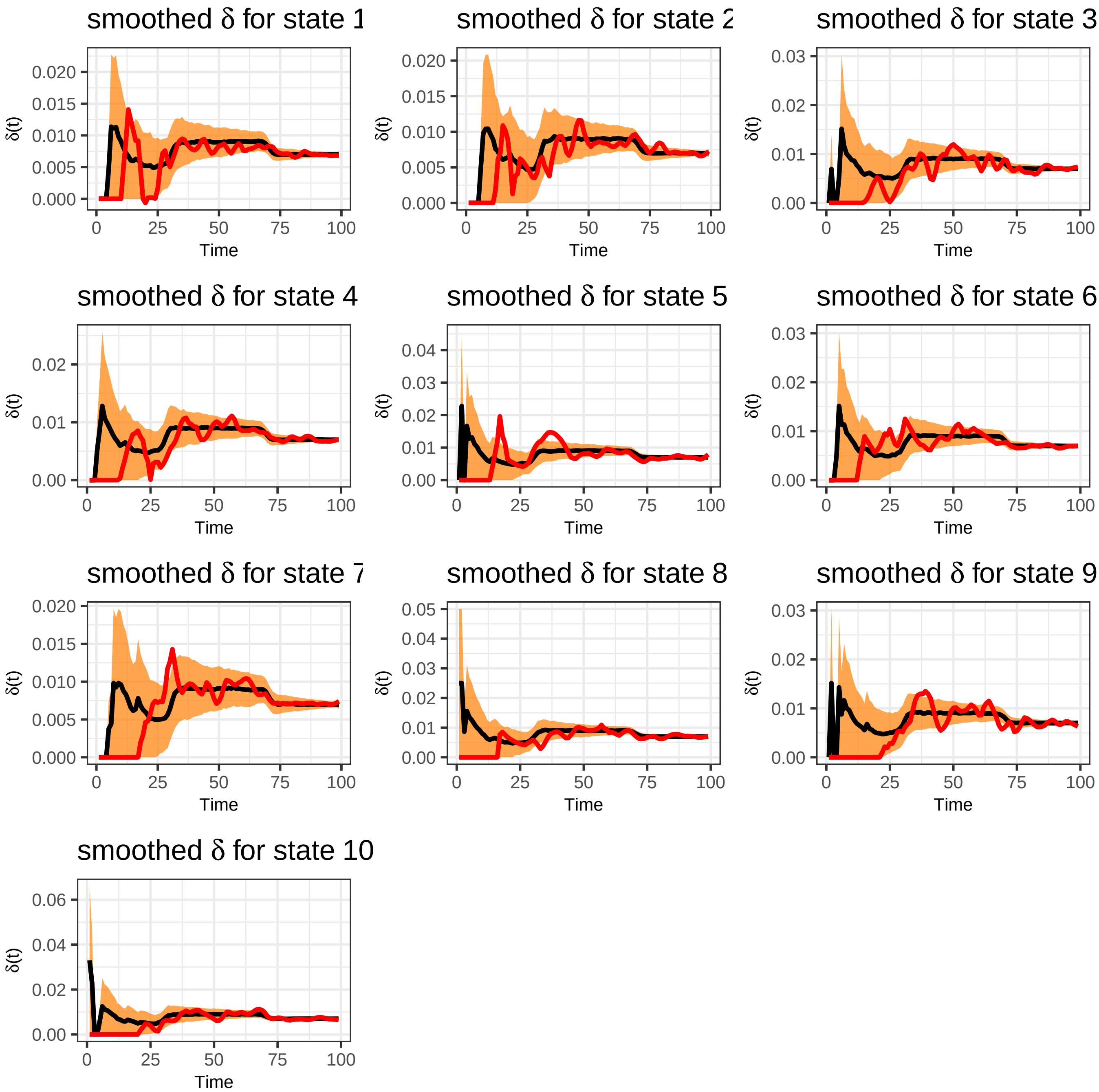


Figure 60: Sim3: Pointwise Confidence Interval for smoothed $\delta(t)$; mean is in black and true is in red

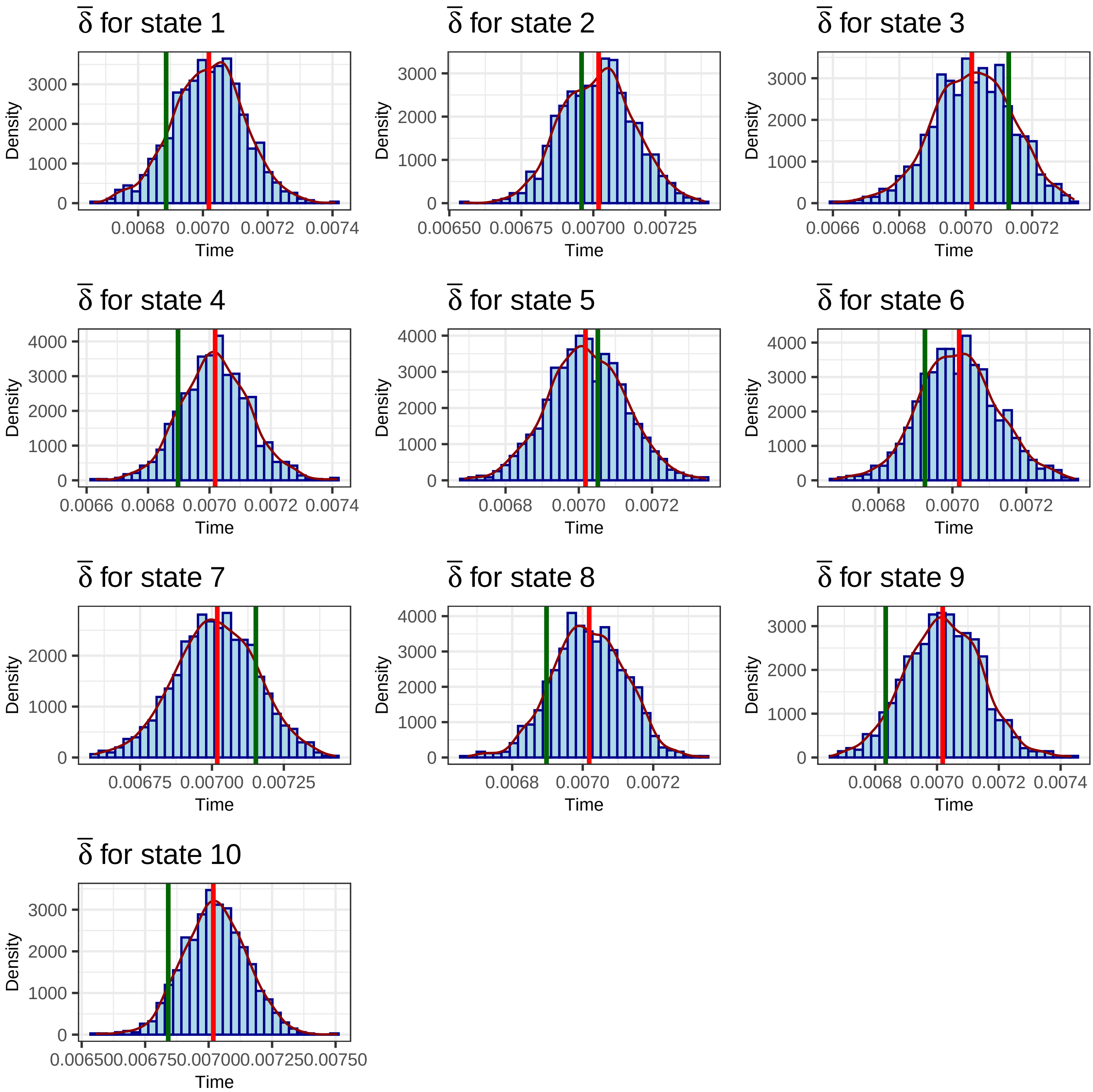


Figure 61: Sim3: Sampling distribution of $\delta(t)$ (estimated by $D \sim \bar{\delta}H$); mean is in red and true is in darkgreen