

TD5

Exercice 1

Technique 1: Forme prénexe

$\forall F, \exists F'$ tq $F' \equiv F$ et F' est sous forme prénexe

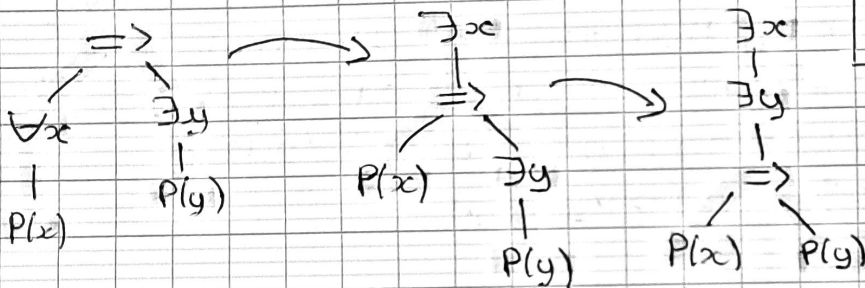
IP n'y a pas unicité

$(\forall x F) \vee G \equiv \forall x (F \vee G)$ si G ne contient pas de variable libre x

1) $(\forall x. P(x)) \Rightarrow \exists y. P(y)$ (formule polie)

$\exists x. P(x) \Rightarrow \exists y. P(y)$

$\exists x. \exists y. P(x) \Rightarrow P(y)$



$$\begin{aligned} (\forall x. F) \Rightarrow G \\ &\equiv \neg (\forall x. F) \vee G \\ &\equiv (\exists x \neg F) \vee G \\ &\equiv \exists x (\neg F \vee G) \\ &\equiv \exists x (F \Rightarrow G) \end{aligned}$$

2) $(\forall x. \exists y. R(x, y)) \Rightarrow \exists x. \forall y. R(x, y)$ (formule non polie)

$\equiv (\forall x. \exists y. R(x, y)) \Rightarrow \exists w. \forall z. R(w, z)$ (formule polie)

$\equiv \exists w. (\forall x. \exists y. R(x, y)) \Rightarrow \forall z. R(w, z)$

$\equiv \exists w. \exists x. (\exists y. R(x, y)) \Rightarrow \forall z. R(w, z)$

$\equiv \exists w. \exists x. \forall y. \forall z. R(x, y) \Rightarrow R(w, z)$

5) $(\exists x. \forall y. (\exists z. S(x, y, z)) \wedge R(x, y)) \Rightarrow \exists y. (\forall x. S(x, y, z)) \wedge \exists x. R(x, y)$ (formule non polie)

$\equiv (\exists x. \forall y. (\exists z. S(x, y, z)) \wedge R(x, y)) \Rightarrow \exists u. (\forall v. S(v, u, w)) \wedge \exists t. R(t, u)$

$\equiv \exists u. (\exists x. \forall y. (\exists z. S(x, y, z)) \wedge R(x, y)) \Rightarrow \exists t. \forall v. S(v, u, w) \wedge R(t, u)$

$\equiv \exists u. \exists t. \forall x. \exists y. ((\exists z. S(x, y, z)) \wedge R(x, y)) \Rightarrow \forall v. S(v, u, w) \wedge R(t, u)$

$\equiv \exists u. \exists t. \forall x. \exists y. \forall z. \forall v. S(x, y, z) \wedge R(x, y) \Rightarrow S(v, u, w) \wedge R(t, u)$