TD2 :

Exercice 1 : /

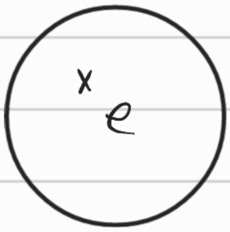
Exercice 2 :

① • $[\![P(a)]\!]^{I}_{p} = T$

$[\![\exists x. P(x)]\!]^{I}_{p} = T$

#1

$DI = \{e\}$



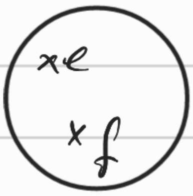
$I(a) = e$
$I(p) = \{(e, T)\}$

#2 ème façon

$D_I = \mathbb{N}$
$I(a) = 0$
$I(p) = \{n \in \mathbb{N} \mid n \bmod 2 = 0\}$

• $[\![P(a)]\!]^{I}_{p} = F$

$[\![\exists x. P(x)]\!]^{I}_{p} = T$

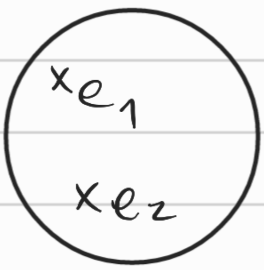


$I(p) = \{(e, F), (f, T)\}$
$I(a) = e$

• $[\![P(a)]\!]^{I}_{p} = T$

$[\![\exists x. P(x)]\!]^{I}_{p} = F$

$I(a) = e_1$
$I(p) = \{(e_1, T), (e_2, F)\}$



$I(P)(I(a)) = T$

$\vee$ $I(P)($[illegible]$) = F$

$$\bigvee_{v \in D} I(P)(v) = \ldots$$

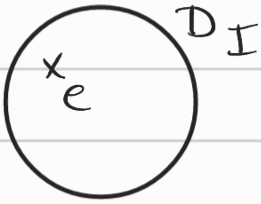
$$= I(P)(v_a) \vee_B \bigvee_{v \in D \setminus \{v_a\}} I(P)(v)$$

$$= T \vee_B \_\_\_$$

$$= T$$

Dans D il y a forcément la valeur v_a qui interprète a : $I(a) = v_a \in D_I$

- $\_\_\_ = F$
  $\_\_\_ = F$



$I(a) = e$
$I(p) = \{(e, F)\}$

$D_I = \{e\}$

(2)

$D_I = \{e\}$ $\quad F_1 = (\exists x . P(x)) \Rightarrow P(a)$
$I(a) = e$
$I(p) = \{(a, T)\}$ $\quad$ • $[\![F_1]\!]^I_\rho = T$

• $[\![\forall x . P(x)]\!]^I = I(P)(e) = T$

$D_I = \{e, f\}$ $\quad$ • $[\![F_2]\!]^I_\rho = T \Rightarrow_\beta F = F$
$I(a) = e$
$I(p) = \{(e, F), (f, T)\}$

• $[\![\forall x . P(x)]\!]^I_\rho = I(P)(e) \wedge I(P)(f) = F \wedge T = F$

$D_I = \{e\}$ $\quad$ • $[\![F_3]\!]^I = F \Rightarrow F = T$

$I(a) = e$

$I(P) = \{(e, F)\}$ $\quad$ • $[\![\forall x . P(x)]\!]^I_\rho = I(P)(e) = F$

$[\![P(a) \Rightarrow \exists x . P(x)]\!]^I_\rho$ Soit I une interprétation quelconque du vocabulaire on a forcément un élément v_a dans D_I qui interprête a, c'est à dire :

$I(a) = v_a \in D_I$

$$I(P)(v_a) \Rightarrow_\beta \left(I(P)(v_a) \vee_\beta \bigvee_{v \in D \setminus \{v_a\}} I(P)(v_a) \right)$$

Par cas:

- Soit $I(P)(v_a) = F$ alors $(i) = F \Rightarrow_\beta \_\_\_ = T$
- Soit $I(P)(v_a) = T$ alors $(i) = T \Rightarrow_\beta \left(T \vee_\beta \bigvee_{v \in D_I \setminus \{v_a\}} I(P)(v) \right)$

Donc $P(a) \Rightarrow \exists x . P(x)$ est valide.

Exercice 3:

(4) $(\forall x . P(x) \wedge Q(x)) \Rightarrow (\forall x . P(x)) \wedge (\forall x . Q(x)) = A$

$$[\![A]\!]^I_\rho = [\![\forall x . P(x) \wedge Q(x)]\!]^I_\rho \Rightarrow_\beta \left([\![\forall x . P(x)]\!]^I_\rho \wedge_\beta [\![\forall x . Q(x)]\!]^I_\rho \right)$$

Soit il existe $v_0 \in D_I$ tel que $I(P)(v_0) = F$

ou $I(Q)(v_0) = F$

$$[\![A]\!]^I_\rho = \bigwedge_{v \in D_I} \left(I(P)(v) \wedge_\beta I(Q)(v) \right) \Rightarrow_\beta \ldots$$

$$= \bigwedge_{v \in D_I \setminus \{v_0\}} \left(I(P)(v) \wedge_\beta I(Q)(v) \right) \wedge_\beta \left(I(P)(v_0) \wedge_\beta I(Q)(v_0) \right) \Rightarrow_\beta \ldots$$

$$\underbrace{v \in D_I \setminus \{v_0\}}_{F}$$

$$[\![A]\!]^I_p = F \Rightarrow \ldots = T$$

Sinon, pour tout $v \in D_I$ on a : $I(P)(v) = I(Q)(v) = T$

$$[\![A]\!]^I_p = \cancel{T} \Rightarrow_\beta \left(\bigwedge_{v \in D_I} I(P)(v)\right) \wedge_\beta \left(\bigwedge_{v \in D_I} I(Q)(v)\right)$$

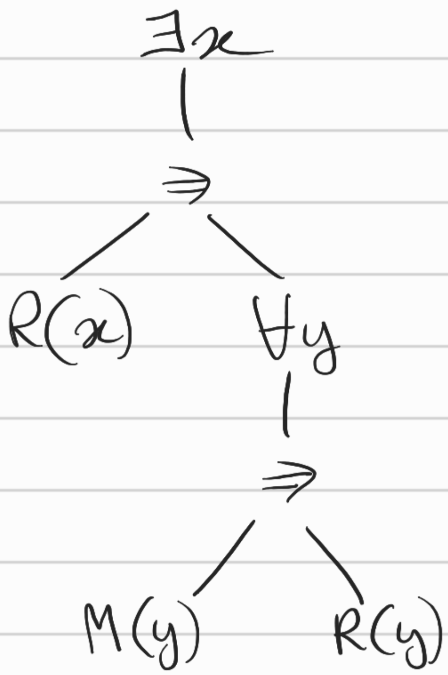
$$= \left(\bigwedge_{v \in D_I} I(P)(v)\right) \wedge_\beta \left(\bigwedge_{v \in D_I} I(Q)(v)\right)$$

$$= \left(\bigwedge_{v \in D_I} T\right) \wedge_\beta \left(\bigwedge_{v \in D_I} T\right) = T$$

Exercice 4 :

1

$R(x) =$ " x résoud ce problème "
$M(x) =$ " x est mathématicien "
$C =$ " Cabot "



$$\left(\exists x . R(x)\right) \Rightarrow \forall y . M(y) \Rightarrow R(y)$$

$$\left(\forall x . R(x) \Rightarrow \forall y . M(y) \Rightarrow R(y)\right) \Rightarrow M(c) \land \neg R(c) \Rightarrow \forall z . R(z)$$

valide?