

## Exercice 2

1)  $x$  n'est pas libre dans  $\Phi'$

$$\begin{array}{c}
 \frac{\Phi', \Phi \vdash \Phi'}{\Phi' \vdash \Phi \Rightarrow \Phi'} \text{ax} \\
 \frac{\Phi' \vdash \Phi \Rightarrow \Phi'}{\Phi' \vdash \exists x. \Phi \Rightarrow \Phi'} \exists_0 \\
 \frac{\Phi \vdash \Phi', \Phi}{\vdash \Phi \Rightarrow \Phi', \Phi} \text{ax} \\
 \frac{\vdash \Phi \Rightarrow \Phi', \Phi}{\vdash \exists x. \Phi \Rightarrow \Phi', \Phi} \Rightarrow_D \\
 \frac{\vdash \exists x. \Phi \Rightarrow \Phi', \Phi}{\vdash \exists x. \Phi \Rightarrow \Phi', \forall x. \Phi} \exists_D \\
 \frac{(\forall x. \Phi) \Rightarrow \Phi' \vdash \exists x. \Phi \Rightarrow \Phi'}{\vdash ((\forall x. \Phi) \Rightarrow \Phi') \Rightarrow \exists x. \Phi \Rightarrow \Phi'} \Rightarrow_G
 \end{array}$$

2)

$$\begin{array}{c}
 \frac{\Phi', \Phi \vdash \Phi'}{\Phi \Rightarrow \Phi', \Phi \vdash \Phi'} \text{ax} \\
 \frac{\Phi \Rightarrow \Phi', \Phi \vdash \Phi'}{\Phi \Rightarrow \Phi', \forall x. \Phi \vdash \Phi'} \Rightarrow_G \\
 \frac{\Phi \Rightarrow \Phi', \forall x. \Phi \vdash \Phi'}{(\exists x. \Phi \Rightarrow \Phi'), \forall x. \Phi \vdash \Phi'} \forall_G \\
 \frac{(\exists x. \Phi \Rightarrow \Phi'), \forall x. \Phi \vdash \Phi'}{(\exists x. \Phi \Rightarrow \Phi') \vdash (\forall x. \Phi) \Rightarrow \Phi'} \exists_G \\
 \frac{(\exists x. \Phi \Rightarrow \Phi') \vdash (\forall x. \Phi) \Rightarrow \Phi'}{\vdash (\exists x. \Phi \Rightarrow \Phi') \Rightarrow (\forall x. \Phi) \Rightarrow \Phi'} \Rightarrow_D
 \end{array}$$

## Exercice 3

$$\begin{array}{c}
 \frac{P(x), Q(x) \vdash Q(x)}{P(x) \Rightarrow Q(x), P(x) \vdash Q(x)} \text{ax} \\
 \frac{P(x) \Rightarrow Q(x), P(x) \vdash Q(x)}{\forall x. P(x) \Rightarrow Q(x), P(x) \vdash \exists x. Q(x)} \Rightarrow_D \\
 \frac{\forall x. P(x) \Rightarrow Q(x), P(x) \vdash \exists x. Q(x)}{\forall x. P(x) \Rightarrow Q(x), \exists x. P(x) \vdash \exists x. Q(x)} \forall_G \\
 \frac{\forall x. P(x) \Rightarrow Q(x), \exists x. P(x) \vdash \exists x. Q(x)}{\vdash (\forall x. P(x) \Rightarrow Q(x)) \Rightarrow (\exists x. P(x) \Rightarrow \exists x. Q(x))} \exists_D
 \end{array}$$

formule pôle:  $(\forall x. P(x) \Rightarrow Q(x)) \Rightarrow (\exists y. P(y)) \Rightarrow \exists z. Q(z)$   
 formule première:  $\exists z. \exists x. \forall y. (P(x) \Rightarrow Q(x)) \Rightarrow P(y) \Rightarrow Q(z)$