Default Title  
(AUTONOMOUS)  
B.TECH VI SEMESTER (R20) REGULAR / SUPPLEMENTARY EXAMINATIONS - JUN 2024  
Default Subject  
Time: 3 Hours Max. Marks: 70  
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\*\*MATHEMATICS QUESTION PAPER\*\*  
  
\*\*PART A (10 x 2 = 20 Marks)\*\*  
  
1. Define the rank of a matrix.  
2. Write the echelon form of the matrix `[[1, 2], [2, 4]]`.  
3. State Cayley-Hamilton theorem.  
4. Find the Jacobian of `u = x + y`, `v = x - y`.  
5. Find the Taylor series expansion of `sin(x)` around `x = 0`.  
6. Change the order of integration of `∫∫ f(x, y) dx dy` from `y=0` to `y=1` and `x=0` to `x=y`.  
7. Evaluate `∫∫ dx dy` where R is the region bounded by `x=0`, `y=0` and `x+y =1`.  
8. Define a solenoidal vector.  
9. Find the gradient of `f(x, y, z) = x^2 + y^2 + z^2` at the point (1, 1, 1).  
10. State Green's theorem in the plane.  
  
\*\*PART B (5 x 10 = 50 Marks)\*\*  
  
\*\*UNIT 1\*\*  
  
(a) Find the eigen values and eigen vectors of the matrix `[[1, 1], [3, -1]]`.  
  
\*\*OR\*\*  
  
(b) Determine the rank of the matrix `[[1, 2, 3], [2, 4, 7], [3, 6, 10]]` and hence solve the system of equations `x + 2y + 3z = 0`, `2x + 4y + 7z = 0`, `3x + 6y + 10z = 0`.  
  
\*\*UNIT 2\*\*  
  
(a) Find the maxima and minima of the function `f(x, y) = x^3 + y^3 - 3xy`.  
  
\*\*OR\*\*  
  
(b) Expand `f(x, y) = e^x cosy` in a Taylor series about the point (0, 0) up to the terms of degree 3.  
  
\*\*UNIT 3\*\*  
  
(a) Evaluate `∫∫ (x^2 + y^2) dx dy` over the region bounded by the circle `x^2 + y^2 = a^2`.  
  
\*\*OR\*\*  
  
(b) Evaluate `∫∫∫ (x + y + z) dx dy dz` over the region bounded by `0 ≤ x ≤ 1`, `0 ≤ y ≤ 1`, `0 ≤ z ≤ 1`.  
  
\*\*UNIT 4\*\*  
  
(a) Find the directional derivative of `f(x, y, z) = x^2y + y^2z + z^2x` at the point (1, 0, 1) in the direction of the vector `i + 2j + 2k`.  
  
\*\*OR\*\*  
  
(b) Find the curl and divergence of the vector field `F = (x^2yz)i + (xy^2z)j + (xyz^2)k`.  
  
\*\*UNIT 5\*\*  
  
(a) Verify Green's theorem in the plane for `∫ (x^2 - xy^3)dx + (y^2 - 2xy)dy` where C is the square with vertices (0,0), (2,0), (2,2), and (0,2).  
  
\*\*OR\*\*  
  
(b) Verify Gauss divergence theorem for `F = x^2i + y^2j + z^2k` taken over the cube bounded by `x = 0`, `x = a`, `y = 0`, `y = a`, `z = 0`, `z = a`.