Low Earth Orbit Collision Risk

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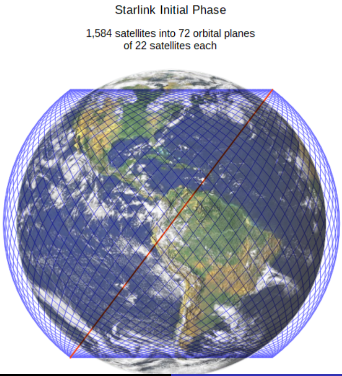
Introduction to the Problem

- Low Earth Orbit describes orbits between 160km and 1000km above the Earth's surface.
- The risk of collision between satellites is increasing with more and more satellites being deployed, e.g. in satellite constellations
- Repeat collisions will make the orbit shells unstable (Kessler effect)
- **Aim:** to estimate the frequency and probability of collisions with satellites from constellation deployments



Approaches Considered

Item	Name	Foster-Estes	Chan	Patera	Alfano	Serra et al.	Pelayo-Ayuso
1	Characteristic Main Feature	Polar Coordinates	Rice Integral/MECSA	"Line" Integral	Error Function	Laplace Transform	Hermite Polynomials
2	Solution Nature	Numerical	Analytical	Numerical	Numerical	Analytical	Analytical
3	No. of Terms if Analytical	N/A	4	N/A	N/A	15	2
4	Accuracy (Digits)	8 to 9	6 to 7	5 to 6	5 to 6	6 to 7	1 to 2
5	Percentage Error	10^{-6} %	10^{-5} %	10^{-4} %	10^{-4} %	10^{-5} %	10%
6	Computation Times (ms)	1.1	0.00018	0.08	0.003	0.00016	0.00014

Figure: Comparison of the Existing Approaches for Computing Collision Probability

Model for collision of two satellites

Foster-Estes model, 1992

$$P = \beta \int_0^{r_A} r \int_0^{2\pi} e^{-\frac{1}{2} r^2 \left[\left(\frac{\sin\theta}{\sigma_{x'}} \right)^2 + \left(\frac{\cos\theta}{\sigma_{z'}} \right)^2 \right] + r x_e \left[\frac{\sin\theta \sin\varphi}{\sigma_{x'}^2} + \frac{\cos\theta\cos\varphi}{\sigma_{z'}^2} \right]} \, d\theta \, dr$$

where

$$\beta = \frac{1}{2\pi \sigma_{x'} \sigma_{z'}} e^{-\frac{1}{2} x_e^2 \left[\left(\frac{\sin\varphi}{\sigma_{x'}} \right)^2 + \left(\frac{\cos\varphi}{\sigma_{z'}} \right)^2 \right]}$$

The variables have the following meanings:

- P = Probability of $0 \leq r \leq r_A$.
- $\sigma_{x'}$ = Uncertainty in the x-direction.
- $\sigma_{z'}$ = Uncertainty in the z-direction.
- r_A = Threshold for calculating collision probability, distance between the centers of the two satellites
- r and θ = **Actual** relative spatial position when the satellites are the closest (distance and angle).
- x_e and φ = **Anticipated** miss location (distance and angle).

Our Model

Kepler's Third law: Computation of a period of an orbit of the Earth

$$T = \sqrt{\frac{4\pi^2 r_W^3}{GM}}$$

where

- G = gravitational constant
- M = mass of the Earth
- r_W = radius of the orbit, i.e. radius of the Earth + height of the orbit above the surface of the Earth.

Expected number of collisions $= 2\frac{N}{T}\alpha \mathrm{P}$, where

- α = scaling factor depending on the period of time considered
- N = total number of target orbital planes.
- multiplication by 2 as two distinct orbits cross twice

Monte Carlo Method

Assumption 1

The only contribution to the probability of a collision comes from the closest satellite in the target orbital.

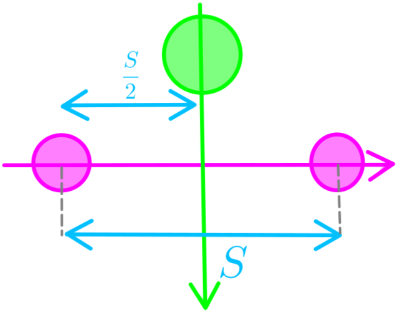


Figure: Closest satellite lies in the range [0,S/2]

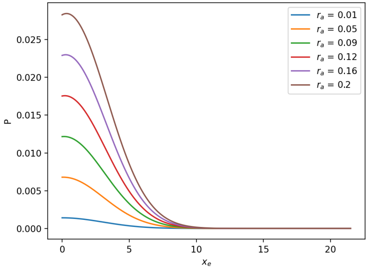


Figure: Probability becomes negligible outside [0,S/2] range

Monte Carlo Method

Assumption 2

The expected miss distance x_e follows the uniform distribution: $x_e \sim \mathrm{U}([0, S/2])$.

Algorithm 1: Monte Carlo Averaging of Probability of Collision

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Result: $\mathbb{E}[P(\text{collision})|x_e \sim \mathrm{U}([0, S/2])]$
sum $= 0$;
for $n = 0, 1, \ldots, N-1$ **do**
    Sample from $\mathrm{U}([0, S/2])]$;
    Calculate $P(\text{collision})$, with this sample of x_e;
    sum $+= P(\text{collision})$;
end
return sum/N

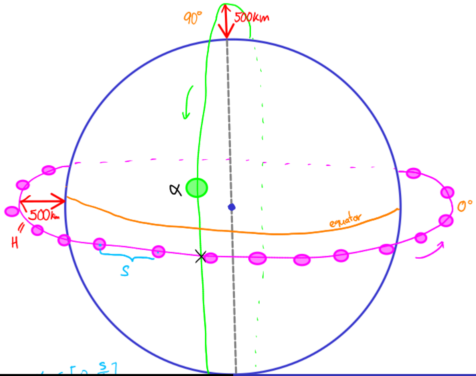
Monte Carlo Method

- We now have $\mathbb{E}[P(\text{collision})|x_e \sim \mathrm{U}([0, \mathrm{S}/2])]$.
- We also know the expected number of interactions per some time period, $2\frac{N}{T}\alpha$.
- We can compute the probability of survival after this time period:

$$P(\text{survival}) = (1 - P(\text{collision}))^{2\frac{N}{T}\alpha}$$

Example 1

- Orbital plane A (green) has one satellite and orbital plane B (purple) has $1,000$ equally spaced satellites, i.e. we assume $N = 1$
- S = distance between the neighbouring satellites on orbital plane B.
- The angle between the 2 orbital planes is 90 degrees ($\varphi = 0.5\pi$).
- We set threshold $r_A = 10$, i.e.: collision will occur if $r \leq 10$ (radius of each satellite (8m) + extra distance (2m)).
- We set the height of the orbits to be 500 km above ground.



Example 2 - Monte Carlo Results

The following result assumes 72 target orbits with 1000 equally spaced satellites each.

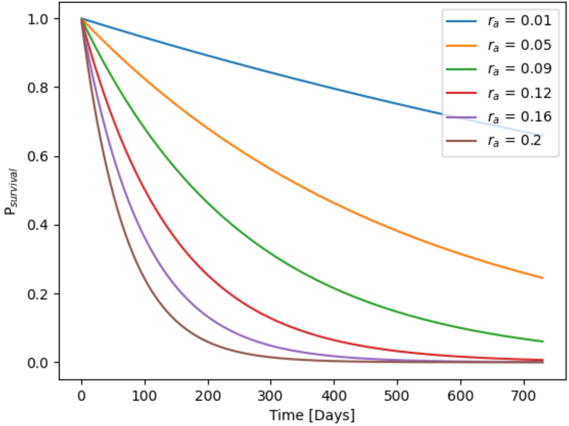


Figure: How the probability of survival changes over the course of 2 years

Example 2 - Monte Carlo Results

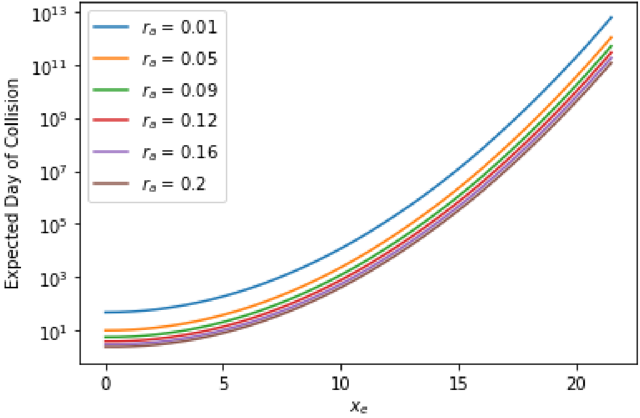


Figure: The dependence of the satellite survival time on the threshold distance between the two satellites.

Example 3

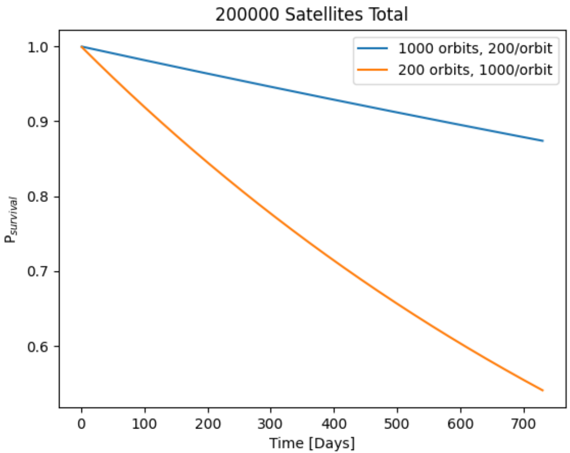


Figure: Is it better to deploy 200000 satellites total on a) 1000 orbits with 200 satellites each or b) on 200 orbits with 1000 satellites each?

Conclusions

- The smaller the satellites deployed are going to be, the smaller the probability of collision (smaller satellite $\rightarrow$ smaller r_A).
- Advise to use less busy orbital shells (i.e. different altitudes).
- Better to populate more orbits with less satellites than less orbits with more satellites.
- Automated collision probability calculation is necessary to protect the space infrastructure in the years to come.

Reference

K. Chan, *Comparison of Methods For Spacecraft Collision Probability Computations*, 2020.

J. L. Foster and H. S. Estes, *A Parametric Analysis of Orbital Debris Collision Probability and Maneuver Rate for Space Vehicles*, NASA/JSC-25898, August 1992.