EM314 - NUMERICAL METHODS ASSIGNMENT - 2

E/15/366
THINESH.S

Q 1)

ii. error = $|\cancel{x} - x_*|$

Suppose that $f \in C[a,b]$ and $f(a) \cdot f(b) < 0$.
Then the sequence $\{x_k\}$ generated by the bisection method satisfies.

$$e_k = |x_k - x_*|$$

But $|x_k - x_*| \leq \dfrac{b-a}{2^{k+1}}$

T - Tolerance

But, $e_k < T$

$|x_k - x_*| < T$

$$\frac{b-a}{2^{k+1}} < T$$

$$\frac{b-a}{T} < 2^{k+1}$$

$$\log_2\left(\frac{b-a}{T}\right) < \log_2 2^{k+1}$$

$$\log_2\left(\frac{b-a}{T}\right) < k+1$$

$$k > \log_2\left(\frac{b-a}{T}\right) - 1$$

2)

Q2.

(a). $g(x) = e^{-x}$ $\quad G = [\ln 1.1, \ln 3]$

if $g(x)$ is a Contraction on a closed interval $[a,b]$ if all $p, q \in [a,b]$ $p \neq q$

$$| f(p) - f(q) | < (p-q)$$

$$|g'(x)| < 1$$

$g'(x) = -e^{-x}$ always $g'(x) \leq 0$

$| g'(x) | = e^{-x}$

$x = \ln 1.1$

$$|g'(x)| \leq e^{-\ln 1.1} = 1/1.1 < 1$$

$$|g'(x)| < 1 \quad //$$

g is contraction on G //

(b).

$\ln 1.1 = 0.0953101798$ $\qquad$ $|f(p) - f(q)| < (p-q)$ //

$\ln 3 = 1.098612289$

$g(\ln 1.1) = e^{-\ln 1.1} = 0.9092$

$g(\ln 3) = e^{-\ln 3} = 0.3334$

$|g(\ln 1.1) - g(\ln 3)| < (\ln(3) - \ln 1.1)$

$\therefore \quad g : G \longrightarrow [\, g(\ln(3)),\ g(\ln(1.1))\,] \subset G$ //

$x_{k+1} = g(x_k)$

$x_k \in G$ for any $x_0 \in G$

$g(x(k)) = e^{-x_k}$

$x_{k+1} = e^{-x_k}$

let: $x_0 = 1$

$x_1 = e^{-1} = 0.36788$

$x_2 = e^{-0.36788} = 0.69220$

$x_3 = e^{-0.69220} = 0.49708$ //

Q3 → part (a)

a. $x_{k+1} = g(x_k)$

$g(x) = \tan^{-1}(2x)$.

$x=0$ is a fixed point of $g(x)$.

Consider the $g'(x) = \frac{2}{1+4x^2}$

let's take $x_k \in [-1/2, 1/2]$

$x_* = 0$

$e_{k+1} = |x_{k+1} - x_*|$

$= |x_{k+1} - 0|$ $\because x_* = 0$ (known)

$= |g(x_k) - g(0)|$ $\because x_{k+1} = g(x_k)$

$= |g'(\xi)||x_k - 0|$

$= |g'(\xi)| e_k$

$\xi \in (-0.5, 0.5)$

if them converge to x_*

$g'(x) < 1$

But in this cause

$(\xi) \in [-1/2, 1/2]$ $g'(x) > 1$ and $e_{k+1} > e_k$

$\therefore$ not converge to $x_* = 0$ //.

Part (b)

b). fixed point x_* near x,

$x = 1.16.$

(i). $x_0 = 2$

iteration - 1

$x_1 = \tan^{-1}(2 \times 2)$

$= 1.3258$

$e_1 = |x_0 - x_1|$

$= 2 - 1.3258$

$= 0.6742$

iteration 2

$x_2 = \tan^{-1}(2 \times x_1)$

$= \tan^{-1}(2 \times 1.3258)$

$= 1.2102$

$e_2 = |1.3258 - 1.2102|$

$= 0.1156.$

Newtons method.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$f(x_k) = \tan^{-1}(2x_k)$

$$f'(x_k) = \frac{2}{1+4x_k}$$

$f'(x_2) = \frac{2}{1+4\times 2}$

$= \frac{2}{9}$

$x_0 = 2$

$x_2 = $

$$x_1 = 2 - \frac{\tan^{-1}(4)}{f'(\tan^{-1}(4))}$$

$x_2 = x_1 - \frac{f}{f'}$

$$= 2 - \frac{1.3258}{9} \times 2$$

$x_1 = 1.70537$

$=$

$e_1 = |x_0 - x_1|$

$= |2 - 1.70537|$

$= 0.2946$

$$x_2 = 1.70537 - \frac{\tan^{-1}(2\times 1.70537)}{0.2557}$$

$= 0.98700$

$e_2 = |x_1 - x_2|$

$= 0.71837$

Editor - F:\New folder\newtons.m

```matlab
function [zero, res, nter]=newtons(f,df,x0,tol,nmax)
 nter=0;
 x=x0-f(x0)/df(x0);

 while abs(x0-x)>=tol && nter <=nmax
     x0=x;
     x=x0-f(x0)/df(x0);
     nter=nter+1;
 end
 if nter>nmax
     fprintf('newtons method support without convergence');
 end
 format long;
 zero=x;
 res=f(x);
```

Newtons function

Editor - F:\New folder\bisection.m

```matlab
function [zero,res,niter]= bisection(f,a,b,tol,nmax)
x=[a (a+b)/2 b];y=f(x);niter =0;I=(b-a)/2;
if y(1)*y(3)>0
    error ('The signs of the function at the extrema must be opposite');
elseif y(1)==0
    zero =a ;  res=0;  return
elseif y(3)==0
    zero =b ; res=0;   return
end
while (I>=tol && niter <=nmax)
    if sign(y(1))*sign(y(2))<0
        x(3)=x(2);  x(2)=(x(1)+x(3))/2;  y=f(x);  I=(x(3)-x(1))/2;
    elseif sign(y(2))*sign(y(3))<0
          x(1)=x(2); x(2)=(x(1)+x(3))/2; y=f(x); I=(x(3)-x(1))/2;
    else
        x(2)=x(find(y==0)); I=0;
    end
    niter = niter+1;

end
if niter>nmax
    fprintf('bisection method exited without convergence');
end
zero =x(2);
res = f(x(2));
```

Bisection function

Q5) Applications

Editor - F:\New folder\f.m

bisection.m | newtons.m | q5.m | f.m

```matlab
function y = f(x)
M = 3;
e = 0.8;
y = x - e*sin(x)-M;
end
```

Editor - F:\New folder\df.m

bisection.m | newtons.m | q5.m | f.m | q6.m | df.m

```matlab
function y =df(x)
 y=diff(@f);
end
```

Editor - F:\New folder\q5.m

bisection.m | newtons.m | q5.m | f.m | q6.m | df.m

```matlab
x0 = 4;
tol = 10^(-8);
nmax = 100;
%df(x)=diff(@f);
[zero, res, itr] = newtons(@f,@df,x0,tol,nmax)
```

>> q5

zero = 3.0629

res = 0.0000000088119

itr = 88

angle E = 3.0629 rad

Q6)

Editor - F:\New folder\f.m

bisection.m | newtons.m | fun.m | q5.m | f.m | q6.m

```
function y = f(x)
p=3.5*(10^7);
a=0.401;
N=1000;
b=42.7*10^(-6);
k=1.3806503*(10^(-23));
T=300;
y = p*(x.^3) + a*(N^2)*x - a*b*(N^3) -(N*b*p+k*N*T)*x.^2;
end
```

Editor - F:\New folder\q6.m

bisection.m | newtons.m | fun.m | q5.m | f.m | q6.m

```
a = 0;
b = 1;
tol = 10^(-12);
nmax = 100;
[zero, res, niter] = bisection(@f,a,b,tol,nmax)
```

>> q6

zero = 0.042700

res = 0.00000040785

niter = 39

the volume = 0.0427m3

Q7)

Example

bisection.m newtons.m q5.m f.m q6.m df.m

```
a = 0;
b = pi/2;
tol = 10^(-6);
nmax = 100;
[zero, res, niter] = bisection(@f,a,b,tol,nmax)
```

Editor - F:\New folder\f.m

bisection.m newtons.m q5.m f.m

```
function y = f(x)
% M = 3;
% e = 0.8;
% y = x - e*sin(x)-M;
%y=x*x+4*x-4;
y=x-cos(x);
end
```

Command Window

```
>> q6

zero =

    0.7391


res =

   1.2419e-06


niter =

    20
```

Q4)

Using newtons() function

Editor - F:\New folder\q4new.m

bisection.m | newtons.m | q5.m | f.m | q6.m | df.m | df4.m | q4a.m | q4new.m

```
syms x;
f(x)=x^2+4*x-4;
tol=10^(-5);
x0=100;
df(x)=diff(f(x));
nmax=50;
format long;
[zero, res, itr] = newtons(f,df,x0,tol,nmax)
```

>> q4new

Zero = 0.82843

Res =0.000000000014795

Itr = 8