

EM314 - NUMERICAL METHODS ASSIGNMENT - 2

E/15/366
THINESH.S

Q1)

11. $error = |x - x_0|$

Suppose that $f \in C[a, b]$ and $f(a) \cdot f(b) < 0$.
Then the sequence $\{x_k\}$ generated by the bisection method satisfies.

$$e_k = |x_k - x_0|$$

$$\text{But } |x_k - x_0| \leq \frac{b-a}{2^{k+1}}$$

T - Tolerance

$$\text{But, } e_k < T$$

$$|x_k - x_0| < T$$

$$\frac{b-a}{2^{k+1}} < T$$

$$\frac{b-a}{T} < 2^{k+1}$$

$$\log_2 \left(\frac{b-a}{T} \right) < \log_2 2^{k+1}$$

$$\log_2 \left(\frac{b-a}{T} \right) < k+1$$

$$k > \log_2 \left(\frac{b-a}{T} \right) - 1$$

2)

02.

a) $g(x) = e^{-x}$ $G = [\ln 1.1, \ln 3]$

if $g(x)$ is a contraction on a closed interval $[a, b]$ if all $p, q \in [a, b]$ $p \neq q$

$$|f(p) - f(q)| < (p - q)$$

$$|g'(x)| < 1$$

$$g'(x) = -e^{-x} \quad \text{always } g'(x) \leq 0$$

$$|g'(x)| = e^{-x}$$

$$x = \ln 1.1$$

$$|g'(x)| \leq e^{-\ln 1.1} = \frac{1}{1.1} < 1$$

$$|g'(x)| < 1$$

g is contraction on G //

(b).

$$\ln 1.1 = 0.0953101798$$

$$\ln 3 = 1.098612289$$

$$|f(p) - f(q)| < (p - q) //$$

$$g(\ln 1.1) = e^{-\ln 1.1} = 0.9092$$

$$g(\ln 3) = e^{-\ln 3} = 0.3334$$

$$|g(\ln 1.1) - g(\ln 3)| < (\ln 3) - \ln 1.1$$

$$\therefore g : G \rightarrow [g(\ln 3), g(\ln 1.1)] \subset G //$$

$$x_{k+1} = g(x_k)$$

$$x_k \in G \quad \text{for any } x_0 \in G$$

$$g(x_k) = e^{-x_k}$$

$$x_{k+1} = e^{-x_k}$$

$$\text{let: } x_0 = 1$$

$$x_1 = e^{-1} = 0.36788$$

$$x_2 = e^{-0.36788} = 0.69220$$

$$x_3 = e^{-0.69220} = 0.49708 //$$

$$a. \quad x_{k+1} = g(x_k)$$

$$g(x) = \tan^{-1}(2x).$$

$x=0$ is a fixed point of $g(x)$.

Consider the $g'(x) = \frac{2}{1+4x^2}$

let's take $x_k \in [-\frac{1}{2}, \frac{1}{2}]$

$$x_* = 0$$

$$e_{k+1} = |x_{k+1} - x_*|$$

$$= |x_{k+1} - 0| \quad \because x_* = 0 \quad (\text{known})$$

$$= |g(x_k) - g(0)| \quad \because x_{k+1} = g(x_k)$$

$$= |g'(c)| |x_k - 0|$$

$$= |g'(c)| e_k$$

$$c \in (-0.5, 0.5)$$

if then converges to x_*

$$g'(c) < 1$$

But in this case

$$(c) \in [-\frac{1}{2}, \frac{1}{2}] \quad g'(c) > 1 \quad \text{and} \quad e_{k+1} > e_k$$

\therefore not converge to $x_* = 0$ //

b). fixed point x_* near x ,
 $x = 1.16$.

(i). $x_0 = 2$

iteration - 1

$$\begin{aligned} x_1 &= \tan^{-1}(2x_0) \\ &= 1.3258 \end{aligned}$$

$$\begin{aligned} e_1 &= |x_0 - x_1| \\ &= 2 - 1.3258 \\ &= 0.6742 \end{aligned}$$

iteration 2

$$\begin{aligned} x_2 &= \tan^{-1}(2x_1) \\ &= \tan^{-1}(2 \times 1.3258) \\ &= 1.2102 \end{aligned}$$

$$\begin{aligned} e_2 &= |1.3258 - 1.2102| \\ &= 0.1156 \end{aligned}$$

Newton's method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$f(x_k) = \tan(2x_k)$$

$$f'(x_k) = \frac{2}{1+4x_k^2}$$

$$\begin{aligned} f'(x_0) &= \frac{2}{1+4 \times 2^2} \\ &= \frac{2}{9} \end{aligned}$$

$$x_0 = 2$$

$$x_2 = 1$$

$$x_1 = 2 - \frac{\tan^{-1}(4)}{f'(\tan^{-1}(4))}$$

$$= 2 - \frac{1.3258 \times 2}{9}$$

$$x_1 = 1.70537 //$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.70537$$

$$e_1 = |x_0 - x_1|$$

$$= |2 - 1.70537|$$

$$= 0.2946 //$$

$$x_2 = 1.70537 - \frac{\tan^{-1}(2 \times 1.70537)}{0.2557}$$

$$= 0.98700$$

$$e_2 = |x_1 - x_2|$$

$$= 0.71837 //$$

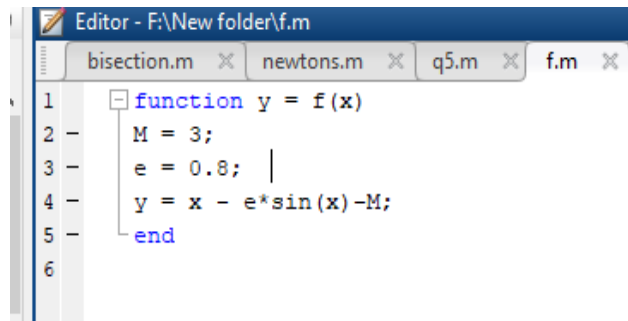
```
Editor - F:\New folder\newtons.m
bisection.m x newtons.m x q5.m x f.m x q6.m x df.m x df4.m x c
1 function [zero, res, nter]=newtons(f,df,x0,tol,nmax)
2     nter=0;
3     x=x0-f(x0)/df(x0);
4
5     while abs(x0-x)>=tol && nter <=nmax
6         x0=x;
7         x=x0-f(x0)/df(x0);
8         nter=nter+1;
9     end
10    if nter>nmax
11        fprintf('newtons method support without convergence');
12    end
13    format long;
14    zero=x;
15    res=f(x);
```

Newton's function

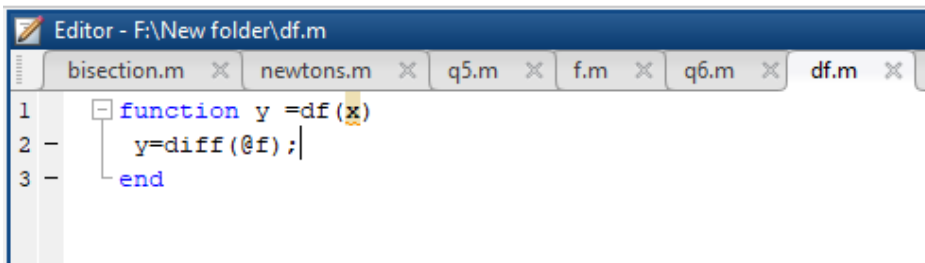
```
Editor - F:\New folder\bisection.m
bisection.m x newtons.m x q5.m x f.m x q6.m x df.m x df4.m x q4a.m x +
1 function [zero,res,niter]= bisection(f,a,b,tol,nmax)
2     x=[a (a+b)/2 b];y=f(x);niter =0;I=(b-a)/2;
3     if y(1)*y(3)>0
4         error('The signs of the function at the extrema must be opposite');
5     elseif y(1)==0
6         zero =a ; res=0; return
7     elseif y(3)==0
8         zero =b ; res=0; return
9     end
10    while (I>=tol && niter <=nmax)
11        if sign(y(1))*sign(y(2))<0
12            x(3)=x(2); x(2)=(x(1)+x(3))/2; y=f(x); I=(x(3)-x(1))/2;
13        elseif sign(y(2))*sign(y(3))<0
14            x(1)=x(2); x(2)=(x(1)+x(3))/2; y=f(x); I=(x(3)-x(1))/2;
15        else
16            x(2)=x(find(y==0)); I=0;
17        end
18        niter = niter+1;
19    end
20    if niter>nmax
21        fprintf('bisection method exited without convergence');
22    end
23    zero =x(2);
24    res = f(x(2));
```

Bisection function

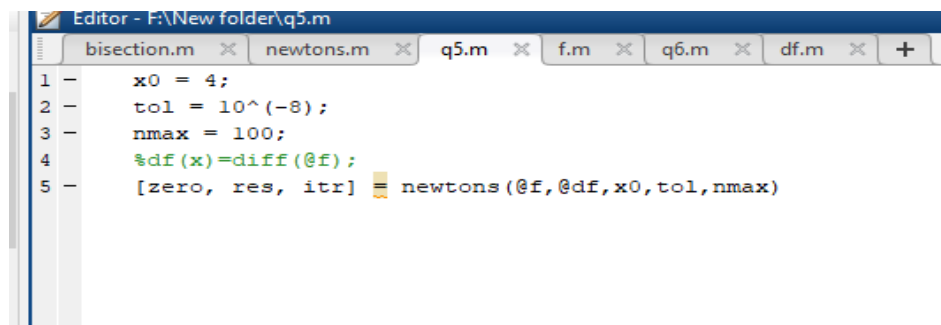
Q5) Applications



```
Editor - F:\New folder\f.m
bisection.m x newtons.m x q5.m x f.m x
1 - function y = f(x)
2 -     M = 3;
3 -     e = 0.8;
4 -     y = x - e*sin(x)-M;
5 - end
6
```



```
Editor - F:\New folder\df.m
bisection.m x newtons.m x q5.m x f.m x q6.m x df.m x
1 - function y =df(x)
2 -     y=diff(@f);
3 - end
```



```
Editor - F:\New folder\q5.m
bisection.m x newtons.m x q5.m x f.m x q6.m x df.m x +
1 - x0 = 4;
2 - tol = 10^(-8);
3 - nmax = 100;
4 - %df(x)=diff(@f);
5 - [zero, res, itr] = newtons(@f,@df,x0,tol,nmax)
```

>> q5

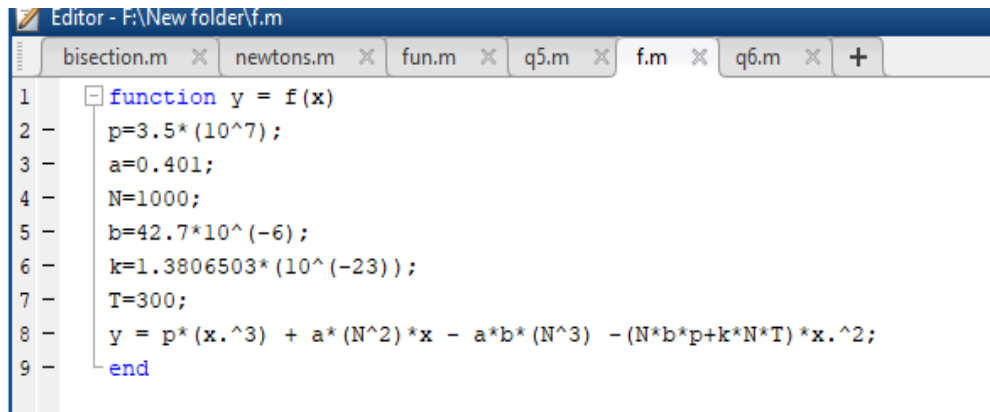
zero = 3.0629

res = 0.0000000088119

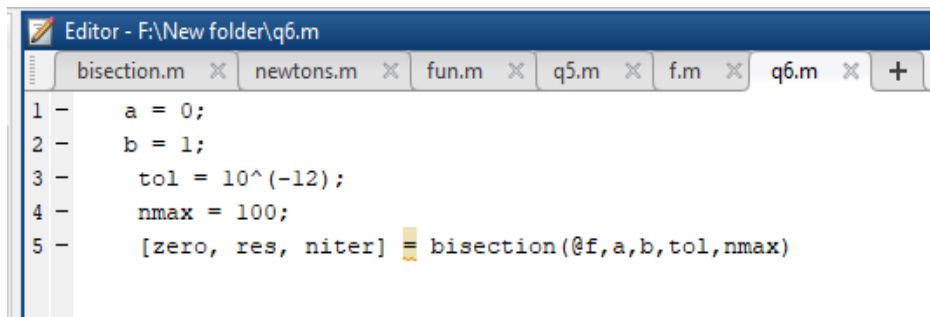
itr = 88

angle E = 3.0629 rad

Q6)



```
Editor - F:\New folder\f.m
bisection.m x newtons.m x fun.m x q5.m x f.m x q6.m x +
1 function y = f(x)
2     p=3.5*(10^7);
3     a=0.401;
4     N=1000;
5     b=42.7*10^(-6);
6     k=1.3806503*(10^(-23));
7     T=300;
8     y = p*(x.^3) + a*(N^2)*x - a*b*(N^3) - (N*b*p+k*N*T)*x.^2;
9 end
```



```
Editor - F:\New folder\q6.m
bisection.m x newtons.m x fun.m x q5.m x f.m x q6.m x +
1 a = 0;
2 b = 1;
3 tol = 10^(-12);
4 nmax = 100;
5 [zero, res, niter] = bisection(@f,a,b,tol,nmax)
```

>> q6

zero = 0.042700

res = 0.00000040785

niter = 39

the volume = 0.0427m3

Q7)

Example

```
bisection.m x newtons.m x q5.m x f.m x q6.m x df.m x
1 - a = 0;
2 - b = pi/2;
3 - tol = 10^(-6);
4 - nmax = 100;
5 - [zero, res, niter] = bisection(@f,a,b,tol,nmax)
```

```
Editor - F:\New folder\f.m
bisection.m x newtons.m x q5.m x f.m
1 - function y = f(x)
2 - % M = 3;
3 - % e = 0.8;
4 - % y = x - e*sin(x)-M;
5 - %y=x*x+4*x-4;
6 - y=x-cos(x);
7 - end
8

Command Window
>> q6

zero =

    0.7391

res =

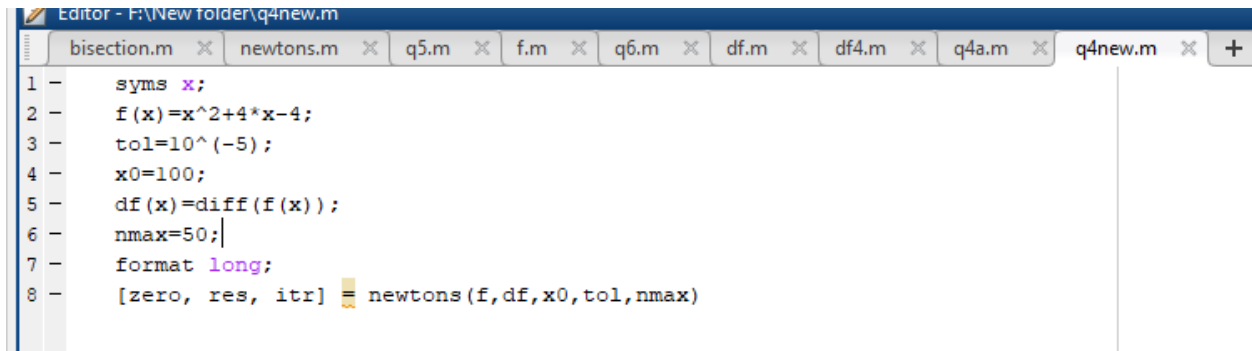
    1.2419e-06

niter =

    20
```

Q4)

Using newtons() function



The image shows a MATLAB Editor window with the title bar "Editor - F:\New folder\q4new.m". The window contains several tabs: "bisection.m", "newtons.m", "q5.m", "f.m", "q6.m", "df.m", "df4.m", "q4a.m", and "q4new.m". The "q4new.m" tab is active, displaying the following MATLAB code:

```
1 - syms x;  
2 - f(x)=x^2+4*x-4;  
3 - tol=10^(-5);  
4 - x0=100;  
5 - df(x)=diff(f(x));  
6 - nmax=50;  
7 - format long;  
8 - [zero, res, itr] = newtons(f,df,x0,tol,nmax)
```

>> q4new

Zero = 0.82843

Res = 0.000000000014795

ltr = 8