ASSIGNMENT 3
EM314

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E/15/366

Q 1)

$f(x) = \ln(x)$

$x_0 = 1$

$x_1 = 2$

$x_2 = 3$

$x_3 = 4$

interval $[1, 4]$

$P(x) = \ln(x)$.

points $x: [1, 2, 3, 4]$

Lagrange interpolation

$$L_k(x) = \prod_{i \neq k} \frac{x - x_i}{x_k - x_i}$$

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) \cdot \left(\frac{x - x_2}{x_0 - x_2}\right) \left(\frac{x - x_3}{x_0 - x_3}\right)$$

$$= \frac{(x-2)(x-3)(x-4)}{-6}$$

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right) \left(\frac{x - x_2}{x_1 - x_2}\right) \left(\frac{x - x_3}{x_1 - x_3}\right)$$

$$= \frac{(x-1)(x-3)(x-4)}{2}$$

$$l_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$= \frac{(x-1)(x-2)(x-4)}{-2}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(x-1)(x-2)(x-3)}{6}$$

$$P(x) = \sum_{i=0}^{n} f(x_i)\, L_i(x)$$

$$\cancel{= \ln(1)\frac{(x-2)(x-3)(x-4)}{-6}} + \cancel{\frac{(x-1)(x-3)(x-4)}{2}\ln(2)} + \cancel{\frac{(x-1)(x-2)(x-4)}{-2}\ln}$$

$$= \frac{(x-2)(x-3)(x-4)}{-6} f(x_0) + \frac{(x-1)(x-3)(x-4)}{2} f(x_1) + \frac{(x-1)(x-2)(x-4)}{-2} f(x_2)$$

$$+ \frac{(x-1)(x-2)(x-3)}{6} f(x_3)$$

$$f(x) = \ln(x)$$

$$= \frac{(x-2)(x-3)(x-4)}{-6}\ln(1) + \frac{(x-1)(x-3)(x-4)}{2}\ln(2) + \frac{(x-1)(x-2)(x-4)}{-2}\ln(3)$$

$$+ \frac{(x-1)(x-2)(x-3)}{6}\ln(4).$$

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Q 2)

$$L_k(x) = \prod_{i \neq k} \frac{x - x_i}{x_k - x_i}$$

for any $x \in \mathbb{R}$

n integer

Set points $x_1, x_2, x_3 \ldots x_n$

Solution :

$$P(x) = \sum_{k=1}^{n} f(x_k)\, L_k(x) =$$

$$\begin{bmatrix} l_0(x_0) & l_1(x_0) & \cdots & l_n(x_0) \\ l_0(x_1) & l_1(x_1) & \cdots & l_n(x_1) \\ \vdots & & & \\ l_0(x_n) & l_1(x_n) & \cdots & l_1(x_n) \end{bmatrix} \begin{pmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix} = \sum_{k=1}^{n} L_k(x)$$

for any $x_1, x_2 \ldots x_n$ interpolated by zeroth order polynomial $P(x) = f(x) = 1$

Interpolation polynomial is unique

$$1 = P(x) = \sum_{k=1}^{n} L_k(x) \qquad \text{any } x.$$

Q 3)

bisection.m | newtons.m | q5.m | f.m | q6.m | df.m | df4.m | q4a.m | q4new.m | interpolant

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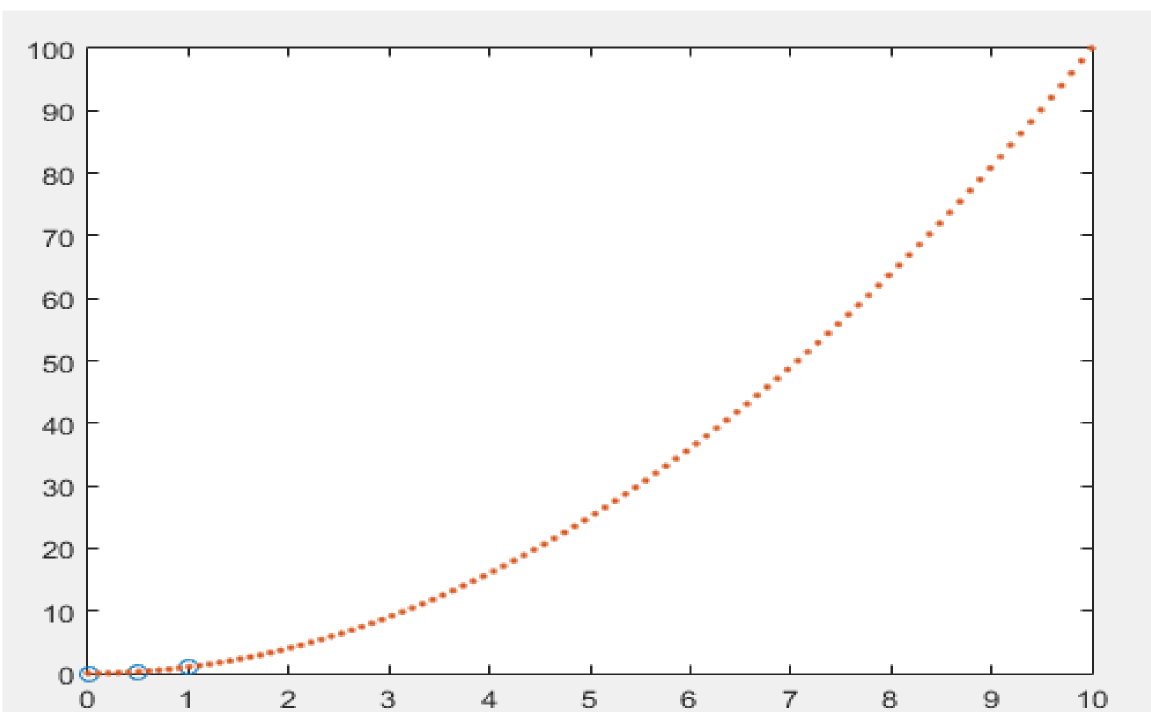
function y=lagrange(x,pointx,pointy)

n=size(pointx,2);
L=ones(n,size(x,2));
if (size(pointx,2)~=size(pointy,2))
  % fprintf(1,'\nERROR!\nPOINTX and POINTY must have the same number of elements\n');
   y=NaN;
else
   for i=1:n
      for j=1:n
         if (i~=j)
            L(i,:)=L(i,:).*(x-pointx(j))/(pointx(i)-pointx(j));
         end
      end
   end
   y=0;
   for i=1:n
      y=y+pointy(i)*L(i,:);
   end
end
```

```
>> x=[0,0.5,1];
>> y=[0,0.25,1];
>> xx = linspace(0,10);
>> yy = lagrange(xx,x,y);
>> plot(x,y,'o',xx,yy,'.')
>>
```



Expected answer :

x :	0	$\frac{1}{2}$	1
y :	0	$\frac{1}{4}$	1

$$l_0(x) = 2\left(x-\tfrac{1}{2}\right)(x-1)$$

$$l_1(x) = 4x(1-x)$$

$$l_2(x) = 2x\left(x-\tfrac{1}{2}\right)$$

$$P(x) = y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x)$$

$$= 0 + \tfrac{1}{4}\, 4x(1-x) + 2x\left(x-\tfrac{1}{2}\right)$$

$$= x^2$$

$$P(x) = x^2$$

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Bchvbvsbvzbvncv   bvhbvnbvhbv