EE387 - Assignment 01.

E/15/366
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problem 01 :-

(a) $x(t) = 3\cos(10t+1) - \sin(4t-1)$

$\cos(10t+1)$, Time period $T_1 = \frac{2\pi}{\Omega_1} = 2\pi/10$

$= \pi/5$

$\sin(4t-1)$, Time period $T_2 = 2\pi/\Omega_2$

$= 2\pi/4$

$= \pi/2$

$\Rightarrow T = T_1/T_2 = \frac{\pi/5}{\pi/2} = 2/5$ sec.

$T = 5T_1 = 2T_2$

$T = 5\,\pi/5 = 2\pi/2$

$T = \pi$ sec

$\Rightarrow$ fundumental periodic of $x(t)$ will be the LCM of $(2\pi/10, 2\pi/4) = 2$

$T = 2\pi/2$

$T = \pi$ //.

(b) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

period of (1) in RHS $= 1$

period of $e^{j4\pi n/7}$ in RHS $= m \cdot \frac{2\pi}{4\pi/7}$

$= 7$ when $m = 2$

period of $e^{j2\pi n/5}$ in RHS $= m\left(\frac{2\pi}{2\pi/5}\right)$

$= 5$ when $m = 1$

Therefor overall signal $x[n]$ is periodic which is the ~~leest~~ least common multiple of the periods of the three terms in $x[n]$,

$= 35$

//.

problem 02:-

(a). $x(t) = 2\cos(4t + \pi/3)$.

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{4} = \pi/2 \;//$$

(b). $x(t) = [\sin(2t + \pi/4)]^2$

$$= 1 - [\cos(2t - \pi/4)]^2$$

$$= 1/2 - 1/2 \cos(4t - \pi/2)$$

$\omega = 4$

$T = 2\pi/\omega = \pi/2 \;//$

$x(t+T)$

$$= 1/2 - 1/2 \cos(4(t + \pi/2) - \pi/2)$$

$$= 1/2 - 1/2 \cos(2\pi + (4t - \pi/2))$$

$$= 1/2 - 1/2 \cos(4t - \pi/2) \quad ; \quad \cos(2\pi + \theta) = \cos\theta$$

$$= x(t)$$

Signal is periodic with period $\pi/2$ //

problem 02

(c) $x[n] = \sin\left(\frac{6\pi}{7} n + 1\right)$

$x(n+N)$

$\omega = 6\pi/7$

$= \sin\left[\frac{6\pi(n+N)}{7} + 1\right]$

$T = 2\pi/\omega$

$= \sin\left[\frac{6\pi}{7} n + \frac{6\pi}{7} N + 1\right]$

$f = 3/7 \quad \in \mathbb{R}$

$\therefore \quad x[n] = \sin\left(\frac{6\pi}{7} n + 1\right)$ is periodic with $n = 7$

(d) $x[n] = \cos\left[\frac{\pi}{8} n^2\right]$

$x[n+N] = \cos\left[\frac{\pi}{8} (N+n)^2\right]$

$= \cos\left[\frac{\pi}{8} n^2 + \frac{\pi}{8} N^2 + \frac{\pi}{8}(2nN)\right]$

if $x[n]$ periodic $x(n) = x(N+n)$

$\therefore$ only possible $\frac{\pi}{8} N^2 = 2\pi m$

$\frac{\pi}{8}(2nN) = 2\pi k$

$\Rightarrow \quad m = 4, \; k = 1, \; N = 8$

$= \cos\left[8\pi + \frac{\pi}{8} n^2 + 2\pi n\right]$

$= \cos\left[2\pi n + \frac{\pi}{8} n^2\right]$

$= \cos\left[\frac{\pi}{8} n^2\right] \quad ; \quad \cos[2n\pi + \theta] = \cos\theta$

$n \in \mathbb{Z}$

$N = 8$ Signal $x(n) = \cos\left[\frac{\pi}{8} n^2\right]$ is periodic.

(e) $x(t) = \sin\left(\frac{\pi}{8} t^2\right)$

$$x(t) = \sin\left[\frac{\pi}{8} t^2\right]$$

$$x(t+T) = \sin\left[\frac{\pi}{8}(t+T)^2\right]$$

$$= \sin\left[\frac{\pi}{8} t^2 + \frac{\pi}{8} T^2 + \frac{\pi}{8}(2tT)\right]$$

$x(t)$ to be periodic $x(t+T) = x(t)$.

possible T

$$T = \frac{2\pi}{\pi/8}$$

$$= 16.$$

$T = 16 \Rightarrow$

$$= \sin\left[\frac{\pi}{8} t^2 + \frac{\pi}{8} \times 16^2 + \frac{\pi}{8}\, 2 \times t \times 16\right]$$

$$= \sin\left[2\pi(16 + 2t) + \frac{\pi}{8} t^2\right]$$

$$\sin\left(\frac{\pi}{8} t^2\right) \quad \because \sin(2\pi + \theta) = \sin\theta.$$

$x(t)$ is periodic with $T = 16$ //

problem 03.

(a) . $x[n] = \begin{cases} \cos(\pi n) & n \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \left[\cos^2 (\pi n) \right] \qquad \cos^2 \theta = \frac{1+\cos 2\theta}{2}$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \left[\frac{1+\cos 2\pi n}{2} \right]$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum \left[\tfrac{1}{2} \right] + \underbrace{\lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \tfrac{1}{2} \cos 2\pi n}_{= 0}$$

$$= \frac{1}{2N+1} \lim_{N \to \infty} \tfrac{1}{2} \sum_{-N}^{N} 1$$

$P_\infty = 1/2$ (finite).

$$E_\infty = \lim_{n \to \infty} \sum_{-N}^{N} \left| x[n] \right|^2 = \sum_{-\infty}^{\infty} \left| x[n] \right|^2$$

$$= \sum_{-\infty}^{\infty} \left| \cos^2 (\pi n) \right|$$

$$= \sum_{0}^{\infty} \frac{1+\cos 2\pi n}{2}$$

$$= \tfrac{1}{2} \sum_{0}^{\infty} 1 + \tfrac{1}{2} \underbrace{\sum_{0}^{\infty} \cos(2\pi n)}_{= 0}$$

$= \infty$ infinite

$x[n]$ is a power signal with finite power $1/2$ and infinite ~~darful~~ energy.

(b).
$$x(t) = \begin{cases} \tfrac{1}{2}\cos(\omega t+1) & -\pi/\omega \le t \le \pi/\omega \\ 0 & \text{otherwise.} \end{cases}$$

$$E_x = \lim_{t\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

$$E_x = \int_{-\pi/\omega}^{\pi/\omega} \left[\tfrac{1}{2} (\cos(\omega t+1)\right]^2 dt.$$

$$= \tfrac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} \left[\cos^2(\omega t) + 1 + 2\cos\omega t\right] dt$$

$$= \tfrac{1}{4} \int_{-\pi/\omega}^{\frac{\pi}{\omega}} \left[\frac{(1+\cos 2\omega t)}{2} + 2\cos\omega t + 1\right] dt.$$

$$= \tfrac{1}{8} \int_{-\pi/\omega}^{\pi/\omega} \left[\cos 2\omega t + 4\cos\omega t + 3\right] dt.$$

$$= \tfrac{1}{8} \left[\underbrace{\int_{-\pi/2\omega}^{\pi/2\omega} \cos(2\omega t)\, dt}_{=0} + \underbrace{\int_{-\pi/2\omega}^{\pi/2\omega} \cos(\omega t)\, dt}_{=0} + \int_{-\pi/\omega}^{\pi/\omega} 3 \, dt \right]$$

$$= \tfrac{1}{8} \, 3 \times t \Big|_{-\pi/\omega}^{\pi/\omega} = \frac{3\pi}{4\omega},$$

$< \infty$ finite.

$\therefore$ power $P_x = 0$

This is an energy signal with finite total energy.

problem 04.

$x(t)$ with the period 4:

$$x(t) = \begin{cases} \sin(\pi t) & 0 \le t \le 2 \\ 0 & 2 < t \le 4 \end{cases}$$

$T = 4$

$$a_0 = \frac{1}{T} \int_T x(t) e^{-jk2\pi/T t} \, dt$$

$$= \frac{1}{4} \int_0^2 \sin(\pi t) e^{-j \cdot 0 \cdot (2\pi/4) t} \, dt$$

$$= \frac{1}{4} \int_0^2 \sin(\pi t) \, dt$$

$$= 0.$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk2\pi/T t} \, dt$$

$$= \frac{1}{4} \int_0^2 \sin(\pi t) e^{-jk(\pi/2)t} \, dt$$

$$= \frac{1}{8j} \int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right) e^{-jk(\pi/2)t} \, dt$$

$$= \frac{1}{8j} \left[\frac{e^{j\pi(2-k)} - 1}{j\pi(1-k/2)} + \frac{e^{j\pi(2+k)} - 1}{j\pi(1+k/2)} \right]$$

$$= \frac{e^{-j\pi k} - 1}{\pi(k^2-4)} = \cancel{\frac{(-1)^k + e^{j}}{\pi(k^2-4)}}.$$

$$F.S \Rightarrow = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} = \sum_{k=-\infty}^{\infty} \left[\frac{(-1)^k + e^{-j\pi k}}{\pi(k^2-4)} e^{jk(\pi/2)t} \right] \quad (k \neq 0)$$

problem 05.:-

$T = \frac{1}{2}$

$x(t) = \cos(4\pi t)$

$y(t) = \sin(4\pi t)$

$z(t) = x(t)\,y(t).$

(a). $x(t) = \cos(4\pi t) = \frac{1}{2}e^{4\pi t} + \frac{1}{2}e^{-4\pi t}.$

So that F.S coefficient of $x(t)$ are

$a_1 = a_{-1} = \frac{1}{2}$

(b). $y(t) = \sin(4\pi t) = \frac{1}{2j}e^{4\pi t} - \frac{1}{2j}e^{-4\pi t}$

F.S coefficient $y(t)$ are

$b_1 = \frac{1}{2}j, \quad b_{-1} = -\frac{1}{2}j$

(C). Using multiplication property.

$$z(t) = x(t).y(t). \xleftrightarrow{FS} C_k = \sum_{t=-\infty}^{\infty} a_l b_{k-l}$$

$$C_k = a_k * b_k = \sum_{t=-\infty}^{\infty} a_l b_{k-l} = \frac{1}{4j}\delta[k-2] - \frac{1}{4j}\delta[k+2]$$

F.S coefficients of $z(t)$ are $C_2 = \frac{1}{4j}, \quad C_{-2} = -\frac{1}{4j}$

(d) $z(t) = \sin(4\pi t)\cos(4\pi t) = \frac{1}{2}\sin(8\pi t) = \frac{1}{4j}e^{8\pi t} - \frac{1}{4j}e^{-8\pi t}$

nonzero Fourier series Coefficient of $z(t)$ are

$C_2 = \frac{1}{4j}, \quad C_{-2} = -\frac{1}{4j}$ same with (C).

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problem 07.

(a). $x_1(t) = x(1-t) + x(-1-t)$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

. $x_1(t) = x(1-t) + x(-1-t)$

Apply reversal property

$$x(-t) \xleftrightarrow{FT} X(-j\omega)$$

Apply shifting property

$$x(-t+1) \xleftrightarrow{FT} e^{j\omega} X(-j\omega)$$

$$x(-t-1) \xleftrightarrow{FT} e^{-j\omega} X(-j\omega)$$

apply linearity property to F.T signal $x_1(t)$,

$$X_1(j\omega) = e^{j\omega} X(-j\omega) + e^{-j\omega} X(-j\omega)$$

$$X_1(j\omega) = (e^{j\omega} + e^{-j\omega}) X(-j\omega)$$

$$\Rightarrow \quad X_1(j\omega) = 2\cos\omega X(-j\omega).$$

//.

(b). $x_2(t) = x(3t-6)$.

$= x[3(t-2)]$

Apply time shifting property

$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega).$$

$$x(t-2) \xleftrightarrow{FT} e^{-2j\omega} X(j\omega).$$

Apply time scale property.

$$x(at) \xleftrightarrow{FT} 1/a \; e^{-2j\omega} X(j\omega/a)$$

therefore $\Rightarrow$ $x[3(t-2)] \xleftrightarrow{FT} \frac{1}{3} e^{-2j\omega} X\left(\frac{j\omega}{3}\right)$

//.

(c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$.

Apply time shifting property.

$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(-j\omega)$$

$$x(t-1) \xleftrightarrow{FT} e^{-j\omega} X(j\omega)$$

Apply differentiation property

$$\frac{dx(t)}{dt} \xleftrightarrow{FT} j\omega X(j\omega)$$

$$\frac{dx(t-1)}{dt} \xleftrightarrow{FT} e^{-j\omega} j\omega X(j\omega)$$

$$\frac{d^2 x(t)}{dt^2} \xleftrightarrow{FT} j^2\omega^2 X(j\omega) = -\omega^2 X(j\omega)$$

$$\Rightarrow \quad \frac{d^2 x(t-1)}{dt^2} \xleftrightarrow{FT} -e^{-j\omega} \omega^2 X(j\omega) \quad //.$$

problem $\cancel{08}$:-

$$e^{-|t|} \xleftrightarrow{F} \frac{2}{1+\omega^2}$$

(a). Apply differentiation property.

$$t\, e^{-|t|} \xleftrightarrow{FT} j \frac{d}{d\omega} \left[\frac{2}{1+\omega^2}\right]$$

$$t\, e^{-|t|} \xleftrightarrow{FT} j \left[\frac{-2}{[1+\omega^2]^2} 2\omega\right]$$

$$t \cdot e^{-|t|} \xleftrightarrow{F.T} \frac{-4j\omega}{(1+\omega^2)^2}$$

//.

(b). $\dfrac{4t}{(1+t^2)^2}$

$$t \cdot e^{-|t|} \xleftrightarrow{F.T} X(j\omega) = \frac{-4j\omega}{(1+\omega^2)^2}$$

$$t \cdot e^{-|t|} = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} \frac{-4j\omega}{(1+\omega^2)^2} e^{j\omega t} \, d\omega \quad \text{—(1)}.$$

(1) $\times 2\pi$, $t \to \omega$

$$\Rightarrow 2\pi \omega e^{-|\omega|} = \int_{-\alpha}^{\alpha} \frac{\cancel{+}4jt}{(1+t^2)^2} e^{-j\omega t} \, dt.$$

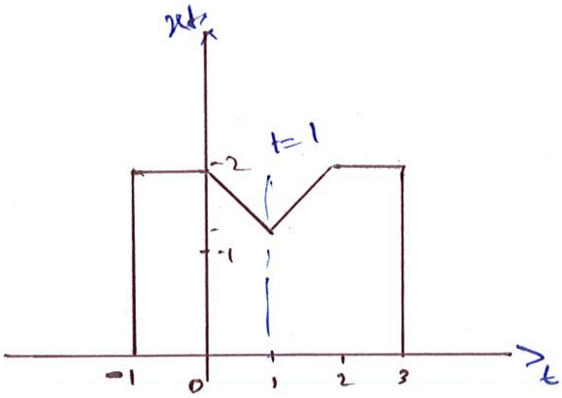
$$2\pi\omega e^{-|\omega|} = \int_{-\alpha}^{\alpha} \frac{\cancel{+}4jt}{(1+t^2)^2} \cdot e^{-j\omega t} \cdot dt \, (j^{-1})$$

$$j.2\pi.\omega\, e^{-|\omega|} = \int_{-\alpha}^{\alpha} \frac{4t}{(1+t^2)^2} e^{-j\omega t} \, dt$$

F.T of $\dfrac{4t}{(1+t^2)^2} = 2\pi\omega j \cdot e^{-|\omega|}$

//.

problem 09.



(a). find $\angle X(j\omega)$

$A(j\omega)$, $\theta(j\omega)$ } Real.

$$X(j\omega) = A(j\omega)\, e^{j\theta(j\omega)}$$

$$= |X(j\omega)|\, e^{j(\angle X(j\omega))}$$

$x(t)$ is symmetric about $t=1$
a signal $g(t)$ is symmetic about $(t=0)$

$$g(t) = x(t+1)$$
$$x(t) = g(t-1)$$

$$X(j\omega) = G(j\omega)\, e^{-j\omega(1)}$$
$$= A(j\omega)\, e^{j\theta(j\omega)}$$

$\angle G(j\omega) = \pm\pi$ [$\because$ magnitude >0]

$A(j\omega)$ is real $\Rightarrow$ $X(j\omega) = G(j\omega)\, e^{-j\omega(1)}$

$$X(j\omega) = G(j\omega)\, e^{-j\omega(1)}$$
$$= A(j\omega)\, e^{j\theta(\omega)}$$

$\therefore$

$A(j\omega) = G(j\omega)$, and $e^{j\theta(\omega)} = e^{-j\omega(1)}$

$$\theta(\omega) = (-\omega)$$

$$\angle X(j\omega) = (-\omega)$$

//

(b) find $X(j0)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$\omega = 0 \Rightarrow \quad X(j0) = \int_{-\infty}^{\infty} x(t)\, e^{0} \cdot dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot dt \Rightarrow [\text{total area}]$$

$$X(j0) = [3-(-1)] \times 2 - \left[\tfrac{1}{2} \times 2 \times 1\right]$$

$$= 7$$

//

(c). find $\int_{-\infty}^{\infty} X(j\omega)\, d\omega$

$$x(t) = \frac{1}{2\pi} \int_{-\infty} (j\omega)\, d\omega$$

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} X(j\omega)\, d\omega = x(0) \times 2\pi$$

$$= 2\pi \times 2$$

$$= 4\pi$$

//

(d) evaluate $\int_{-\alpha}^{\alpha} |X(j\omega)|^2 \, d\omega$

Using parseval's theorem

$$\int_{-\alpha}^{\alpha} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} |x(j\omega)|^2 \, d\omega$$

$$\int_{-\alpha}^{\alpha} |X(j\omega)|^2 \, d\omega = 2\pi \cdot \int_{-\alpha}^{\alpha} |x(t)|^2 \cdot dt.$$

$$= 2\pi \left[\int_{-1}^{0} (2^2) \, dt + \int_{0}^{1} (2-t)^2 \, dt + \int_{1}^{2} t^2 \, dt + \int_{2}^{3} 2^2 \, dt. \right.$$

$$= 2\pi \left[4 + \frac{(2-t)^3}{-3} \Big|_0^1 + \frac{t^3}{3} \Big|_1^2 + 4 \right]$$

$$= 2\pi \times \left[4 - \left[\frac{1}{3} - \frac{8}{3} \right] + \left[\frac{8}{3} - \frac{1}{3} \right] + 4 \right]$$

$$= \frac{76\pi}{3} \quad //.$$

(e). sketch the inverse f. transform of $R\{X(j\omega)\}$

$$Ev\{x(t)\} = Re\{X(j\omega)\}$$

$$= \frac{1}{2}\left[x(t) + x(-t)\right]$$