• EE 387 - Signal Processing

• Assignment 1

Problem 1. Determine the fundamental period of the following signals

(a) $x(t) = 3\cos(10t+1) - \sin(4t-1)$

$x(t) = 3\cos(\omega_1 t + \phi_1) - \sin(\omega_2 t + \phi)$

Fundamental period of $3\cos(10t+1)$

$= T_1$

$= \frac{2\pi}{\omega_1}$

$= \frac{2\pi}{10}$

$= \pi/5$

Fundamental period of $\sin(4t-1)$

$= T_2$

$= \frac{2\pi}{\omega_2}$

$= \frac{2\pi}{4}$

$= \pi/2$

So, Fundamental period of $x(t) = T_0$

$= \text{LCM}(T_1, T_2)$

$= \text{LCM}(\pi/5, \pi/2)$

$= \pi$

$T_0 = \frac{2\pi k}{\omega_0}$

if $k = 1$

$T_0 = \frac{2\pi}{\omega_0}$

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(b) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

$x[n] = 1 + e^{j(4\pi/7)n} - e^{j(2\pi/5)n}$

$x[n] = 1 + e^{j\omega_1 n} - e^{j\omega_2 n}$

$N_0 = \frac{2\pi}{\omega_0}k$

Fundamental Period of $e^{j\left(\frac{4\pi}{7}\right)n}$

N_1

$= \frac{2\pi}{\omega_1}k$

$= \frac{2\pi}{4\pi/7}k$

$= \frac{7}{2}k$

for $k=2 \ni N_1 = 7 \ (\in \mathbb{Z})$

Fundamental period of $e^{j\left(\frac{2\pi}{5}\right)n}$

$= N_2$

$= \frac{2\pi}{\omega_2}k$

$= \frac{2\pi}{2\pi/5}k$

$= 5h$

for $h=1 \ni N_2 = 5 \ (\in \mathbb{Z})$

So, Fundamental period of $x[n] = N_0$

$= LCM(N_1, N_2)$

$= LCM(7, 5) = 35$

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Problem 2. Determine whether the following signals are periodic or aperiodic? If periodic also find the period.

(a) $x(t) = 2\cos(4t + \pi/3)$

$x(t) = A\cos(\omega_0 t + \phi)$

$\omega = 4$

$2\pi f = 4$

$f = \frac{2}{\pi}$

$T = \pi/2$

if periodic $x(t) = x(t+T)$

$x(t+T) = 2\cos\left(4\left(t+\pi/2\right)+\pi/3\right)$

$x(t+T) = 2\cos\left(2\pi + \left(4t+\pi/3\right)\right)$

$x(t+T) = 2\cos\left(4t + \pi/3\right) = x(t)$ $(\because \cos(2\pi+\theta) = \cos\theta)$

So, signal $2\cos(4t+\pi/3)$ is periodic with fundamental period $= T_0 = \pi/2$

(b) $x(t) = \left[\sin\left(2t - \pi/4\right)\right]^2$

$$\frac{1-\cos 2\theta}{2} = \sin^2\theta$$

$x(t) = \sin^2\left(2t - \pi/4\right)$

$x(t) = \dfrac{1 - \cos 2\left(2t - \pi/4\right)}{2} = \frac{1}{2} - \frac{1}{2}\cos\left(4t - \pi/2\right)$

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(b) $x(t) = \frac{1}{2} - \frac{1}{2}\cos\left(4t - \frac{\pi}{2}\right)$

$$\omega = 4 = 2\pi f$$

$$f = \frac{4}{2\pi} = \frac{2}{\pi}$$

$$T = \frac{1}{f} = \frac{\pi}{2}$$

$x(t+T)$

$$= \frac{1}{2} - \frac{1}{2}\cos\left(4\left(t+\frac{\pi}{2}\right) - \frac{\pi}{2}\right)$$

$$= \frac{1}{2} - \frac{1}{2}\cos\left(4t - \frac{\pi}{2} + 2\pi\right)$$

$$= \frac{1}{2} - \frac{1}{2}\cos\left(2\pi + \left(4t - \frac{\pi}{2}\right)\right) \quad (\because \cos(2\pi+\theta) = \cos\theta)$$

$$= \frac{1}{2} - \frac{1}{2}\cos\left(4t - \frac{\pi}{2}\right)$$

$$= x(t)$$

So, signal $x(t) = \left[\sin\left(2t - \frac{\pi}{4}\right)\right]^2$ is periodic with fundamental period $T_0 = \frac{\pi}{2}$

(c) $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$

$$\omega = \frac{6\pi}{7} = 2\pi f$$

$$f = \frac{3}{7} = \frac{k}{N} \text{ is Rational}$$

$x(n+N)$

$$= \sin\left(\frac{6\pi}{7}(n+N)+1\right)$$

$$= \sin\left(\frac{6\pi}{7}n + \frac{6\pi}{7}N + 1\right)$$

So, $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$ is periodic with fundamental period $N = 7$ "

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(d) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$

$$x[n+N] = \cos\left(\frac{\pi}{8}(n+N)^2\right)$$

$$x[n+N] = \cos\left(\frac{\pi}{8}n^2 + \frac{\pi}{8}N^2 + \frac{\pi}{8}(2nN)\right)$$

If signal $x[n]$ to be periodic $x[n+N] = x[n]$

i.e $\cos\left(\frac{\pi}{8}n^2 + \frac{\pi}{8}N^2 + \frac{\pi}{8}(2nN)\right) = \cos\left(\frac{\pi}{8}n^2\right)$

This is only possible if

$$\frac{\pi}{8}N = 2\pi m \text{ and } \frac{\pi}{8}N^2 = 2\pi k$$

$(\because \cos(2\pi + \theta) = \cos\theta)$

$\Rightarrow m = 1, k = 4 \quad N = 8$

Hence,

$$x[n+N]$$

$$= \cos\left(\frac{\pi}{8}n^2 + \frac{\pi}{8}8^2 + \frac{\pi}{8}(16n)\right)$$

$$= \cos\left(8\pi + \frac{\pi}{8}n^2 + 2\pi n\right)$$

$$= \cos\left(2\pi n + \pi/8\, n^2\right) \quad (\because n \in \mathbb{Z}, \cos(2n\pi + \theta) = \cos\theta)$$

$$= \cos\left(\frac{\pi}{8}n^2\right)$$

$$= x[n]$$

Then, signal $x(t) = \cos\left(\frac{\pi}{8}n^2\right)$ is periodic with a fundamental period of $N = 8$

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(e) $x(t) = \sin\left(\frac{\pi}{8}t^2\right)$

$$x(t+T) = \sin\left(\frac{\pi}{8}(t+T)^2\right)$$

$$x(t+T) = \sin\left(\frac{\pi}{8}t^2 + \frac{\pi}{8}T^2 + \frac{\pi}{8}(2tT)\right)$$

For above equation, For signal $x(t)$ to be periodic $x(t+T)$ should be equal to $x(t)$

i.e

$$\sin\left(\frac{\pi}{8}t^2 + \frac{\pi}{8}T^2 + \frac{\pi}{8}(2tT)\right) = \sin\left(\frac{\pi}{8}t^2\right)$$

This is only possible if

$$T = \frac{2\pi}{\pi/8} = 16$$

$$T = 16 \Rightarrow \sin\left(\frac{\pi}{8}t^2 + \frac{\pi}{8}\cdot 16^2 + \frac{\pi}{8}(2\times t\times 16)\right)$$

$$= \sin\left(\frac{\pi}{8}t^2 + 32\pi + 4\pi t\right)$$

$$= \sin\left(\frac{\pi}{8}t^2\right) \qquad (\sin(2\pi n + \theta) = \sin\theta)$$

So, $x(t) = \sin\left(\frac{\pi}{8}t^2\right)$ is periodic with fundamental period $T = 16$.

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(f) $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

$x[n+N] = \cos\left(\frac{\pi}{2}(n+N)\right)\cos\left(\frac{\pi}{4}(n+N)\right)$

$x[n+N] = \cos\left(\frac{\pi}{2}n + \frac{\pi}{2}N\right)\cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N\right)$

From the above equation for the signal to be periodic $\cos\left(\frac{\pi}{2}n + \frac{\pi}{2}N\right)\cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N\right) = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

That is $x[n+N] = x[n]$

This is only possible if,

$\frac{\pi}{2}N_1 = 2\pi m_1$, $\frac{\pi}{4}N_2 = 2\pi m_2$ $(\because \cos(2\pi+\theta) = \cos\theta)$

Where N_1 and N_2 are the periods of $\cos\left(\frac{\pi}{2}n\right)$ and $\cos\left(\frac{\pi}{4}n\right)$ respectively. and $m_1, m_2 \in \mathbb{Z}^+$

$\Rightarrow N_1 = 4$, $m_1 = 1$, $N_2 = 8$, $m_2 = 1$

Let $x_1[n] = \cos\left(\frac{\pi}{2}n\right)$ and $x_2[n] = \cos\left(\frac{\pi}{4}n\right)$ then

$x_1[n+N_1] = \cos\left(\frac{\pi}{2}n + \frac{\pi}{2}N_1\right) = \cos\left(2\pi + \frac{\pi}{2}n\right) = \cos\left(\frac{\pi}{2}n\right) = x_1[n]$

$x_2[n+N_2] = \cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N_2\right) = \cos\left(2\pi + \frac{\pi}{4}n\right) = \cos\left(\frac{\pi}{4}n\right) = x_2[n]$

Hence, the fundamental period of the signal $x[n]$ will be L.C.M (N_1, N_2) that is LCM $(4, 8) = 8$

Therefore, The given signal $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$ is periodic with a fundamental period of $N = 8$

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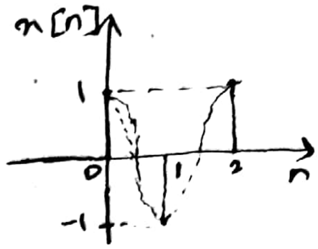
Problem 3. Classify each of the signals below as a power signal or an energy signal. In addition, find the power or the energy of the signal.

(a)

$$x[n] = \begin{cases} \cos(\pi n) & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



$$E_\infty = \lim_{N \to \infty} \sum_{-N}^{N} |x[n]|^2 = \sum_{-\infty}^{\infty} |x[n]|^2$$

$2\cos^2\theta = 1 + \cos 2\theta$

$$E_\infty = \sum_{-\infty}^{\infty} |\underbrace{\cos(\pi n)}_{x[n]}|^2$$

$$E_\infty = \sum_{-\infty}^{\infty} \underbrace{\cos^2(\pi n)}_{x[n]} = \sum_{-\infty}^{-1} \underbrace{0}_{0} + \sum_{0}^{\infty} (\cos^2 \pi n)$$

$$E_\infty = \sum_{0}^{\infty} \frac{1 + \cos(2\pi n)}{2}$$

$$E_\infty = \frac{1}{2} \sum_{0}^{\infty} (1 + \cos 2\pi n)$$

$$E_\infty = \frac{1}{2} \sum_{0}^{\infty} 1 + \frac{1}{2} \underbrace{\sum_{0}^{\infty} \cos(2\pi n)}_{0}$$

$2\cos^2\theta = 1 + \cos 2\theta$

$$E_\infty = \infty \quad (\text{infinite})$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} [\cos^2(\pi n)]$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \left(\frac{1 + \cos 2\pi n}{2} \right)$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \left(\frac{1}{2} + \frac{\cos 2\pi n}{2} \right)$$

$$P_\infty = \frac{1}{2N+1} \lim_{N \to \infty} \left(\sum_{-N}^{N} \frac{1}{2} + \underbrace{\sum_{-N}^{N} \frac{\cos 2\pi n}{2}}_{0} \right)$$

$$P_\infty = \frac{1}{2N+1} \lim_{N \to \infty} \left(\frac{1}{2} \sum_{-N}^{N} 1^N \right) \qquad \left(\because \sum_{-N}^{N} 1^n = N_2 - N_1 + 1 = 2N + 1 \right)$$

$$P_\infty = \frac{1}{2N+1} \times \frac{1}{2} \times \lim_{N \to \infty} (2N+1)$$

$$P_\infty = \frac{1}{2} \quad (\text{finite})$$

$\therefore$ Above $x[n]$ is a power signal with finite average power $\frac{1}{2}$ and infinite total energy.

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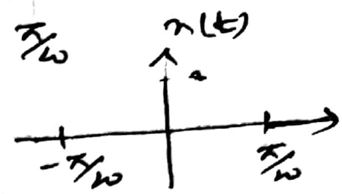
(b)

$$x(t) = \begin{cases} \frac{1}{2}(\cos(\omega t)+1) & -\pi/\omega < t \leq \pi/\omega \\ 0 & \text{otherwise} \end{cases}$$



Continuous time sinusoidal signals are always periodic. So, This $x(t) = \frac{1}{2}(\cos(\omega t)+1)$ is periodic.

$$E_\infty = \lim_{t \to \infty} \int_{-T}^{T} |x(t)|^2 \cdot dt = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

$\Downarrow$ periodic

$$E_\infty = \int_{-\pi/\omega}^{\pi/\omega} x\left[\frac{1}{2}(\cos(\omega t)+1)\right]^2 \cdot dt.$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$E_\infty = \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega} (\cos^2(\omega t) + 2\cos\omega t + 1) \cdot dt$$

$$E_\infty = \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega} \left(\frac{1+\cos(2\omega t)}{2} + 2\cos\omega t + 1\right) \cdot dt$$

$$E_\infty = \frac{1}{8}\int_{-\pi/\omega}^{\pi/\omega} (\cos(2\omega t) + 4\cos\omega t + 3) \cdot dt$$

$$E_\infty = \frac{1}{8}\left[\underbrace{\int_{-\pi/\omega}^{\pi/\omega} \cos(2\omega t) \cdot dt}_{0} + 4\underbrace{\int_{-\pi/\omega}^{\pi/\omega} \cos(\omega t) \cdot dt}_{0} + \int_{-\pi/\omega}^{\pi/\omega} 3 \cdot dt\right]$$

$$E_\infty = \frac{1}{8} \times 3[t]_{-\pi/\omega}^{\pi/\omega} = \frac{3\pi}{4\omega} J < \infty \text{ (finite)}$$

This is an energy signal with finite total energy $\frac{3\pi}{4\omega}$ J and average power $P_\infty = 0$ "

(area under the curve)

$$\left(\because P = \lim_{T \to \infty} \frac{E}{T}\right)$$