- Markov's Inequality
- Chebyshev's Inequality

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What if you could get a negative score?

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What if you could get a negative score?

No bound!

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Rearranging gives

$$P(X \geq k) \leq \frac{E[X]}{k}$$

Chebyshev's Inequality

Chebyshev's Inequality: Let X be any random variable, with mean $\mu = E[X]$ and (finite) variance. Let $\alpha > 0$.

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Chebyshev's Inequality (picture for Gaussian)

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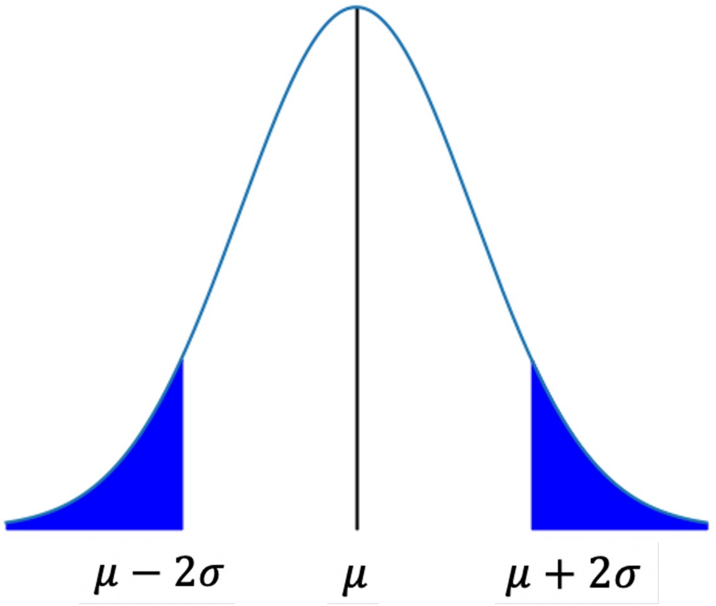

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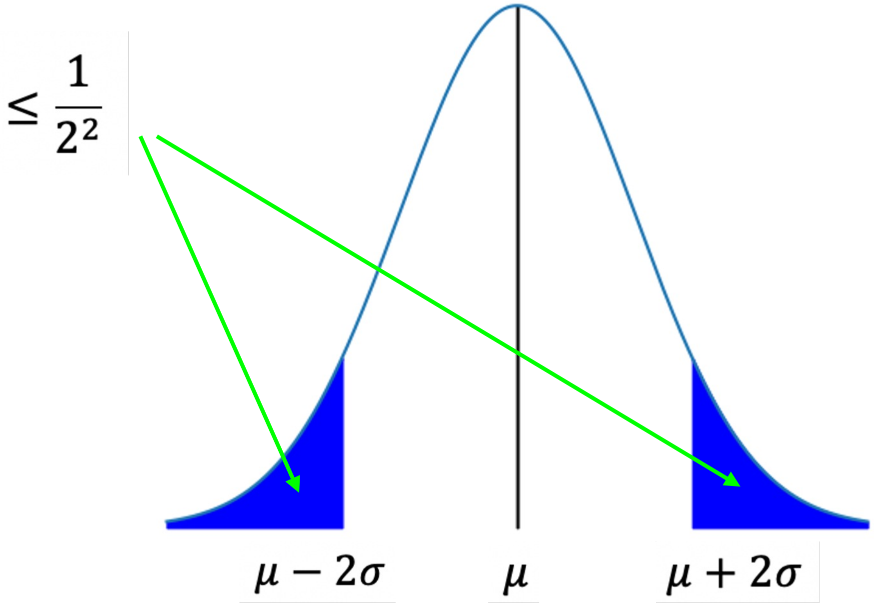

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$$\leq \frac{E[(X - \mu)^2]}{\alpha^2} \text{ [Markov]}$$

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$$\begin{aligned} P(|X - \mu| \geq \alpha) &= P((X - \mu)^2 \geq \alpha^2) \\ &\leq \frac{E[(X - \mu)^2]}{\alpha^2} \text{ [Markov]} \\ &= \frac{Var(X)}{\alpha^2} \end{aligned}$$