The method of maximum likelihood (MLE)

Definition:

Let X_1, X_2, ... , X_n be a random sample from a distribution with probability distribution $f(x,\ \theta)$, and let $L(\boldsymbol{x},\ \theta) = \Pi_{i=1}^{n} f(x_i,\ \theta)$ be the likelihood function of the sample. If $t = u(x_1, x_2, ..., x_n)$ is the value of θ that maximizes $L(\text{x}, \theta)$ then $T = U(X_1, X_2, ..., X_n)$ is the maximum likelihood estimator (MLE) of θ.

- To maximize $L(\text{x}, \theta)$ use $\frac{\partial}{\partial x}\{L(\text{x}, \theta)\} = 0$.
- Rather than maximizing the likelihood function it is usually easier to maximize its natural logarithm, since the results are the same. Note that logarithm is a monotonic function. Therefore for an iid sample,

$$L(\boldsymbol{x},\ \theta) = ln\,(\Pi_{i=1}^{n} f(x_i, \theta)) = \Sigma_{i=1}^{n} ln\ f(x, \theta)$$

Example 1. Find the MLE of the Poisson distribution with parameter λ.

Example 2. If $X_i \sim N(\mu, \sigma^2)$; $i = 1, 2, ..., n$. Find the MLE's of μ and σ^2.

Example 3. Let X_1, X_2, ... , X_n be independent random variables each Bernoulli with probability of success θ. Find the MLE's of $E(X)$ and $Var(X)$. The probability mass function of the Bernoulli distribution is

$$p(x, \theta) = \theta^x(1 - \theta)^{1-x};\ x = 0, 1.$$