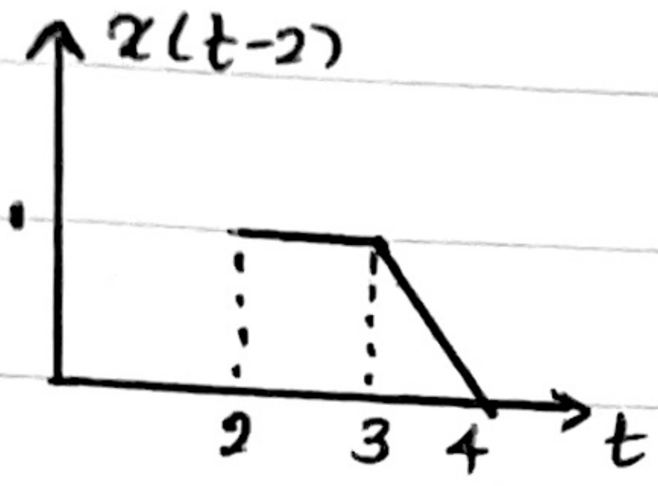
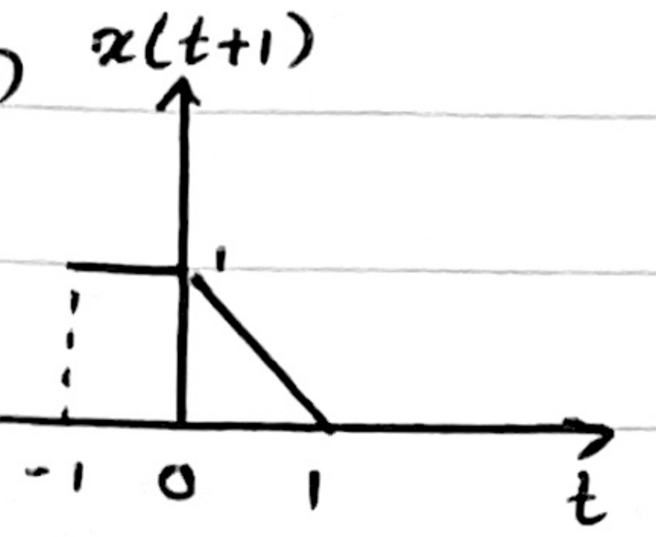
Tutorial 1

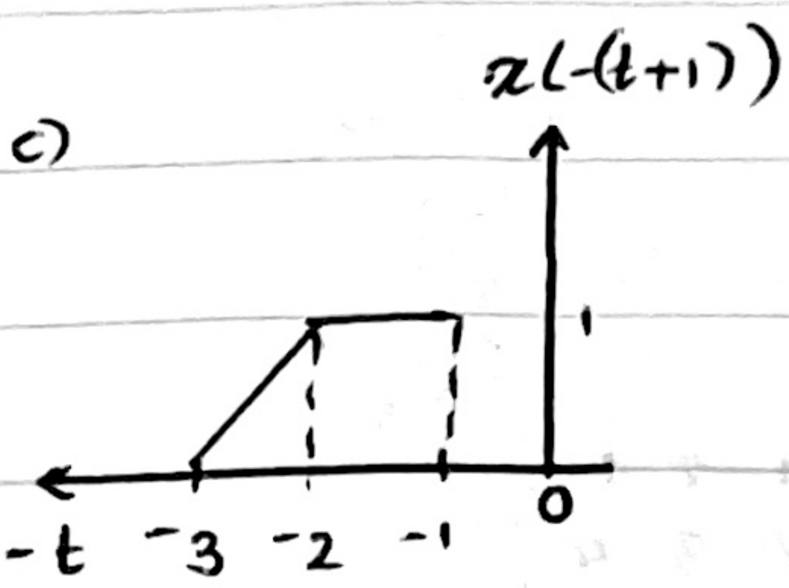
1) a)



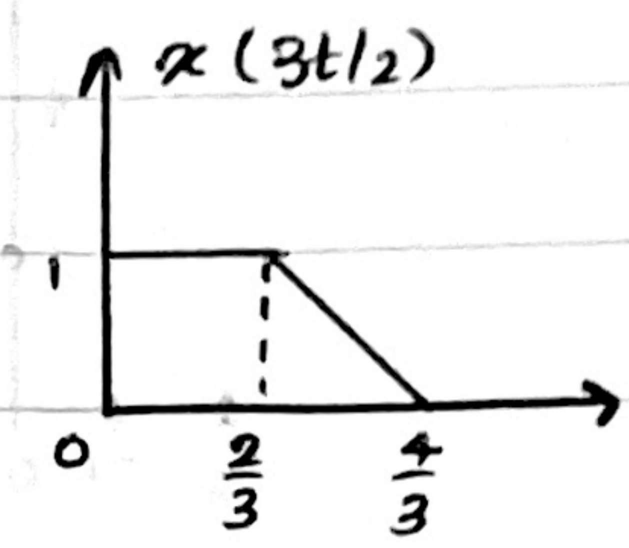
b)



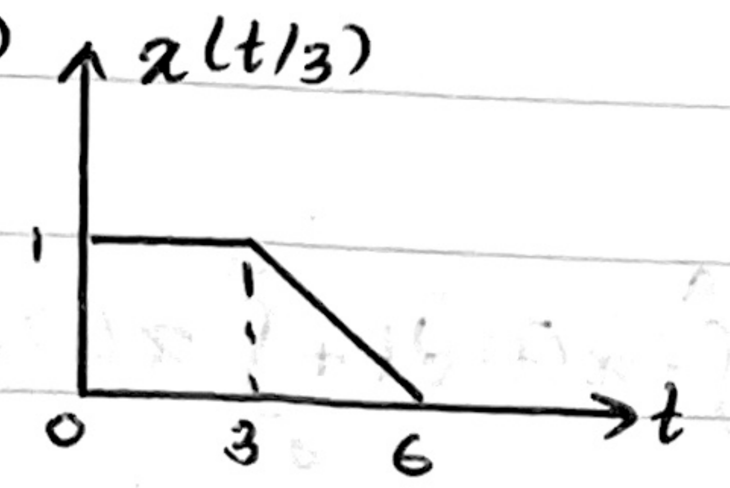
c)



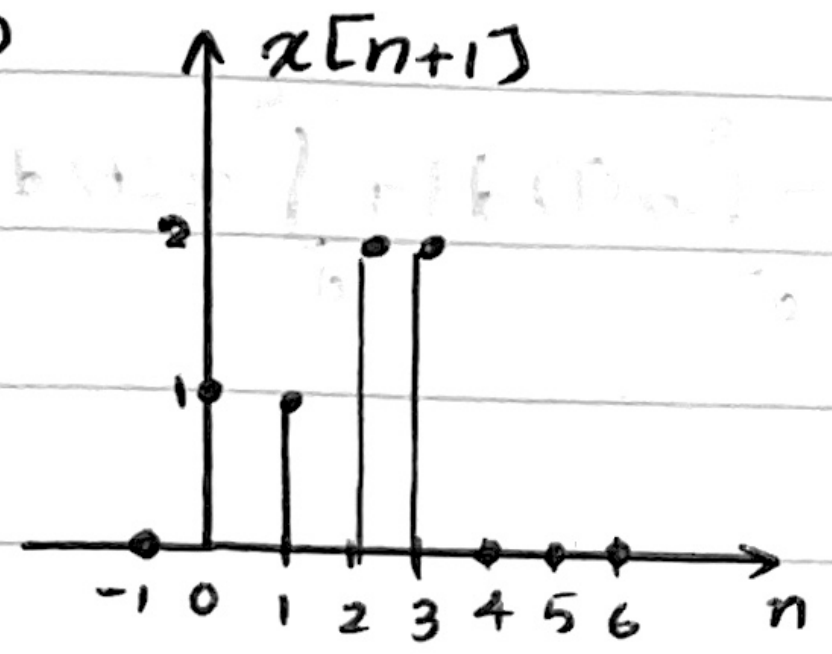
d)



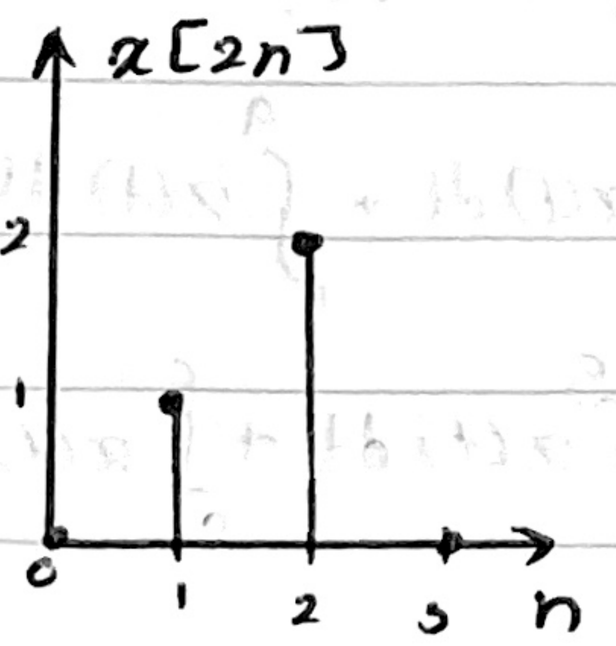
e)



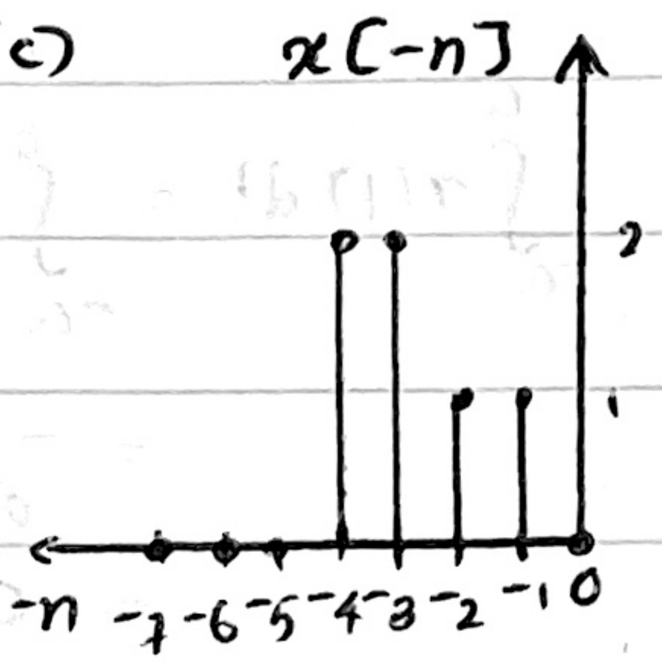
2) a)



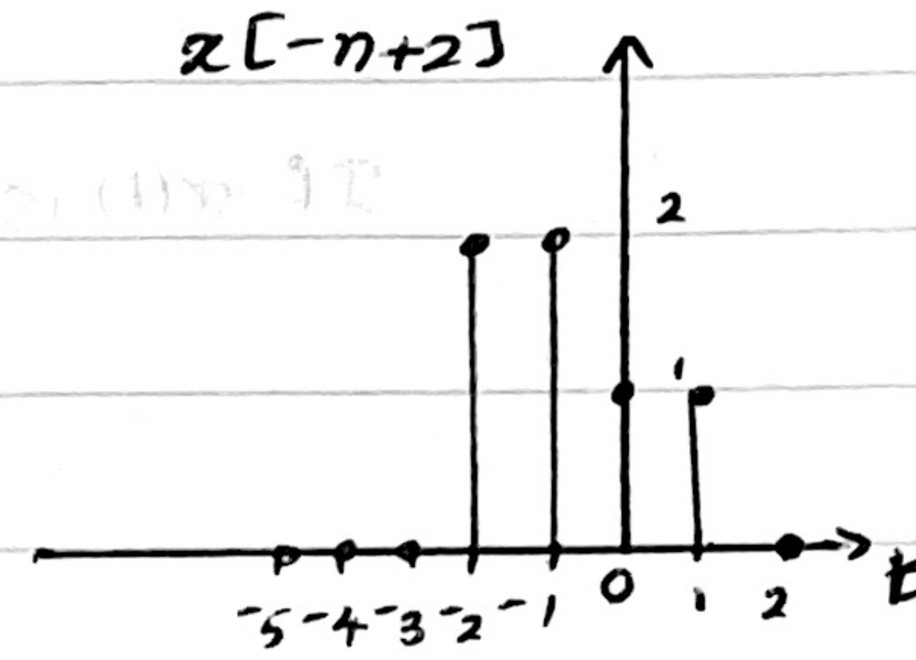
b)



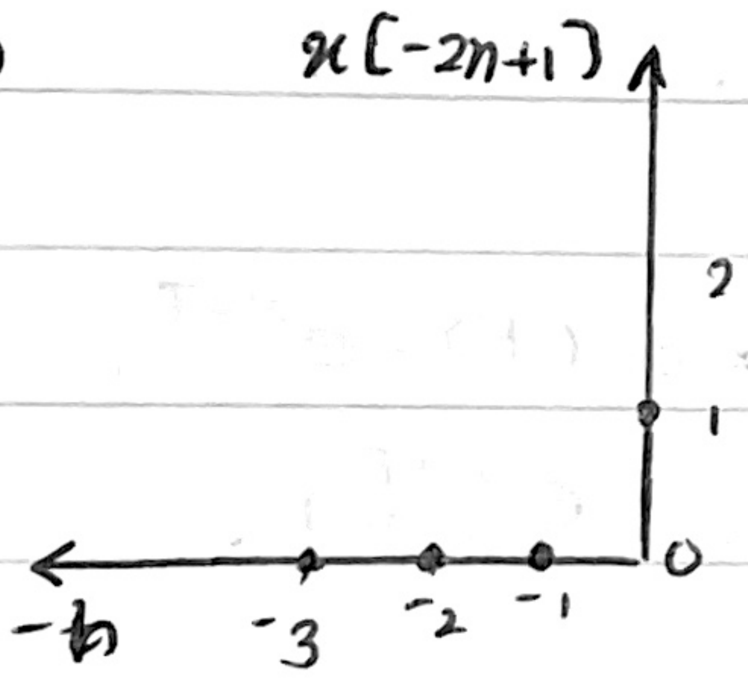
c)



d)



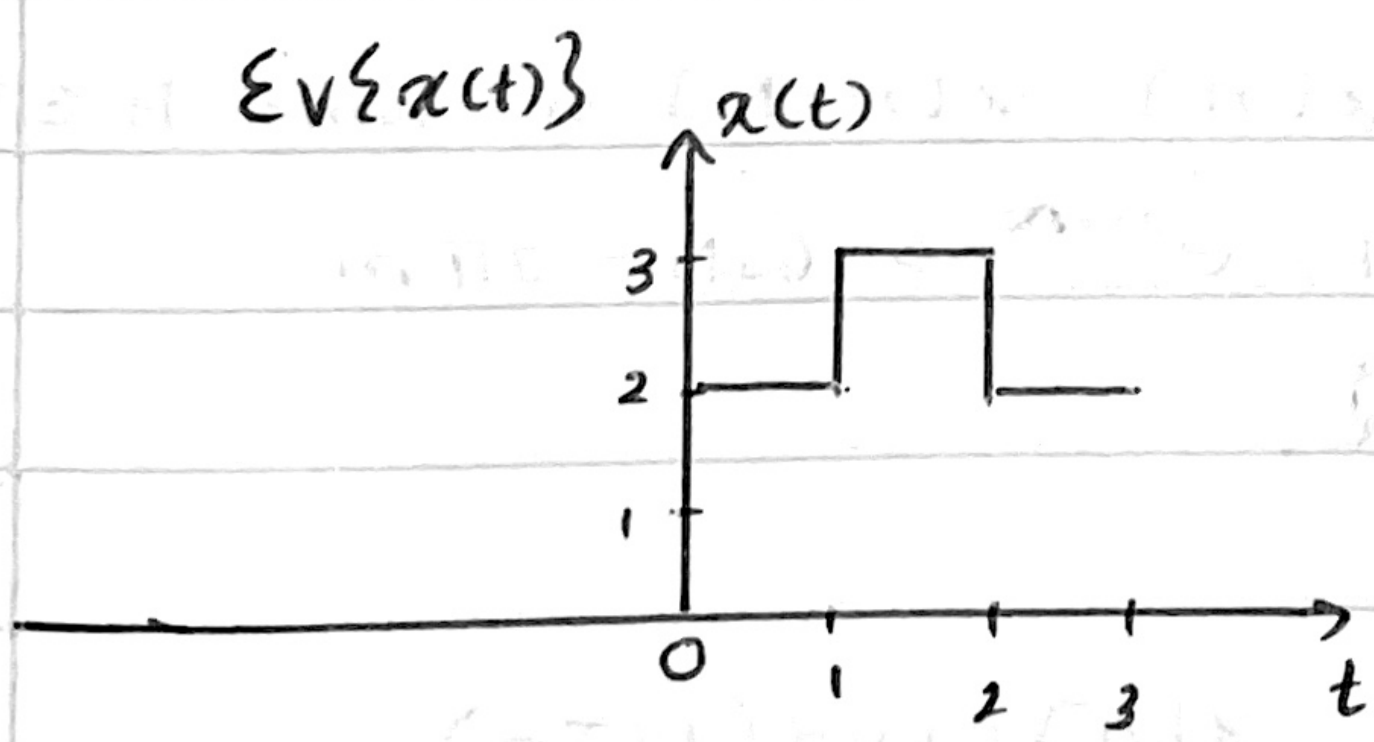
b)

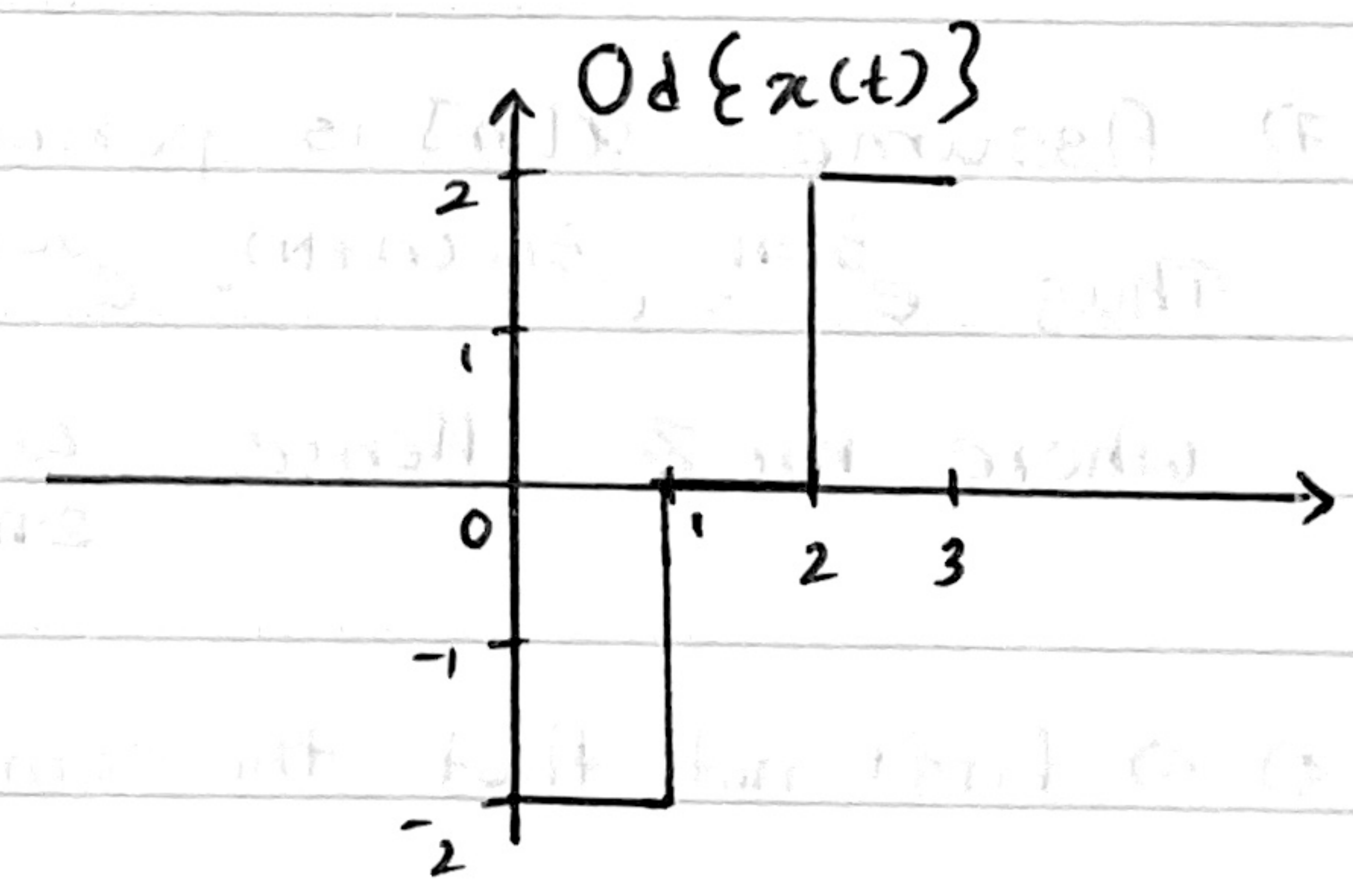


3) $x(t) = \mathcal{E}v\{x(t)\} + \mathcal{O}d\{x(t)\}$, $\mathcal{E}v\{x(t)\} = \frac{1}{2}[x(t) + (x(-t)]$

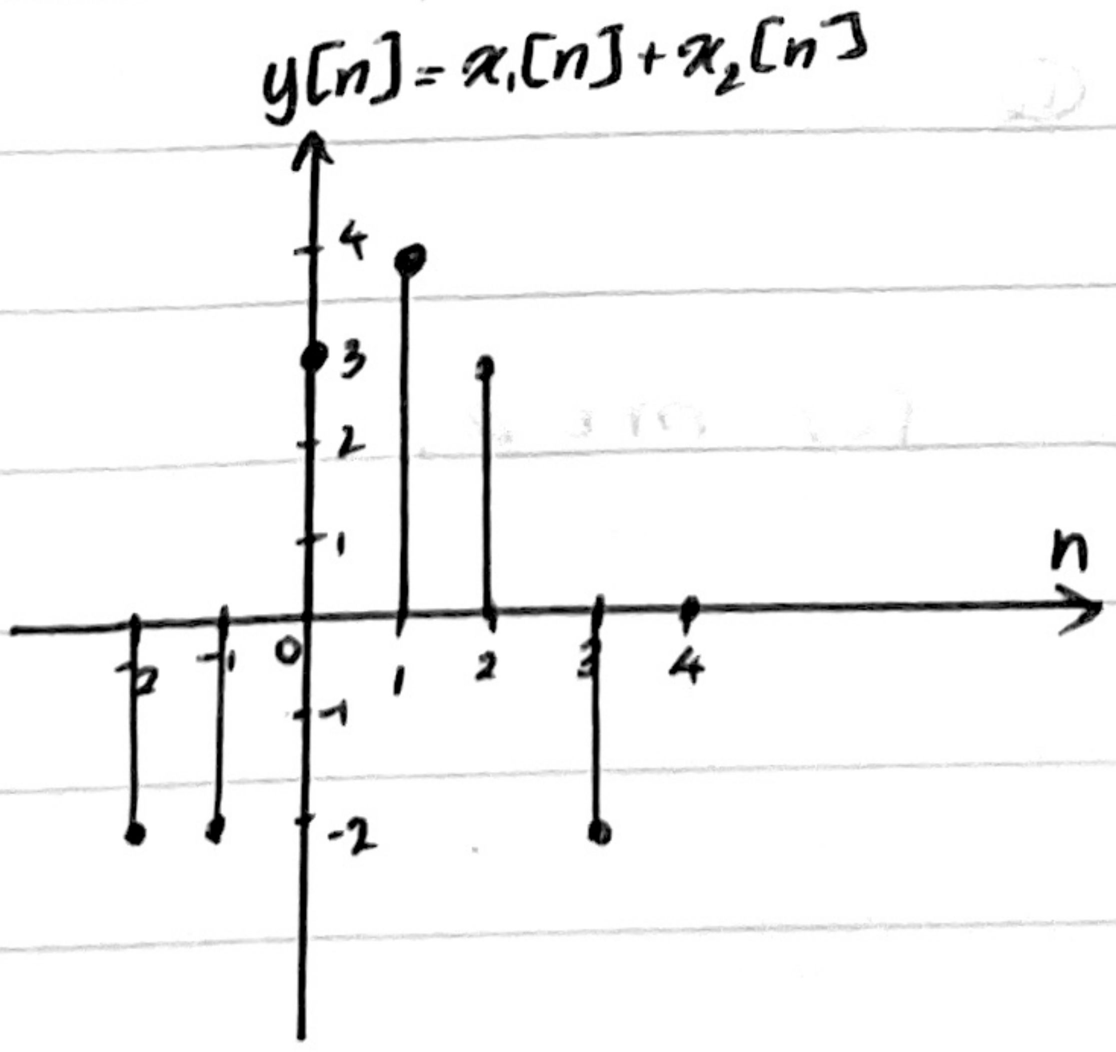
$\mathcal{O}d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$.

$\mathcal{E}v\{x(t)\}$

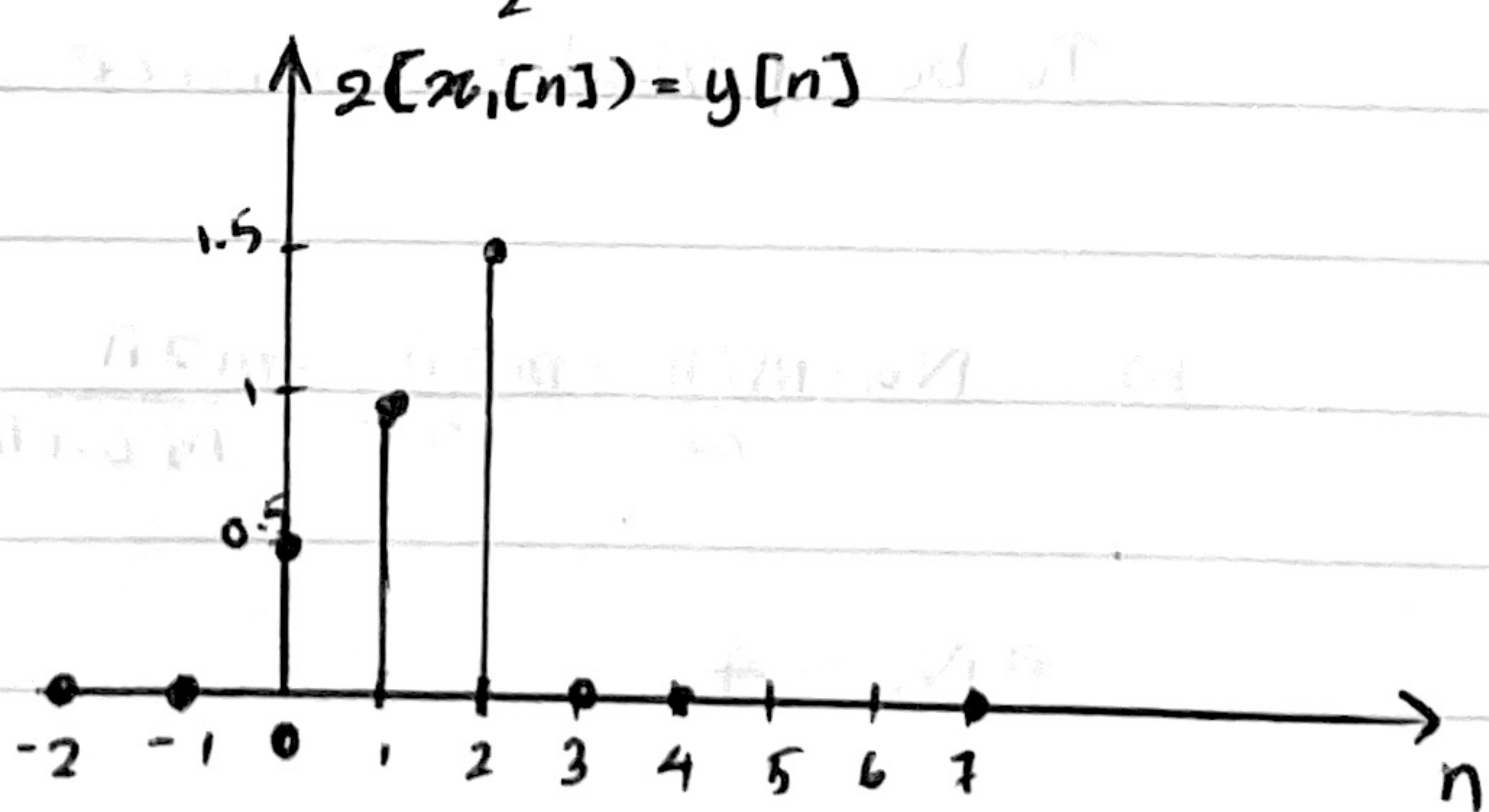




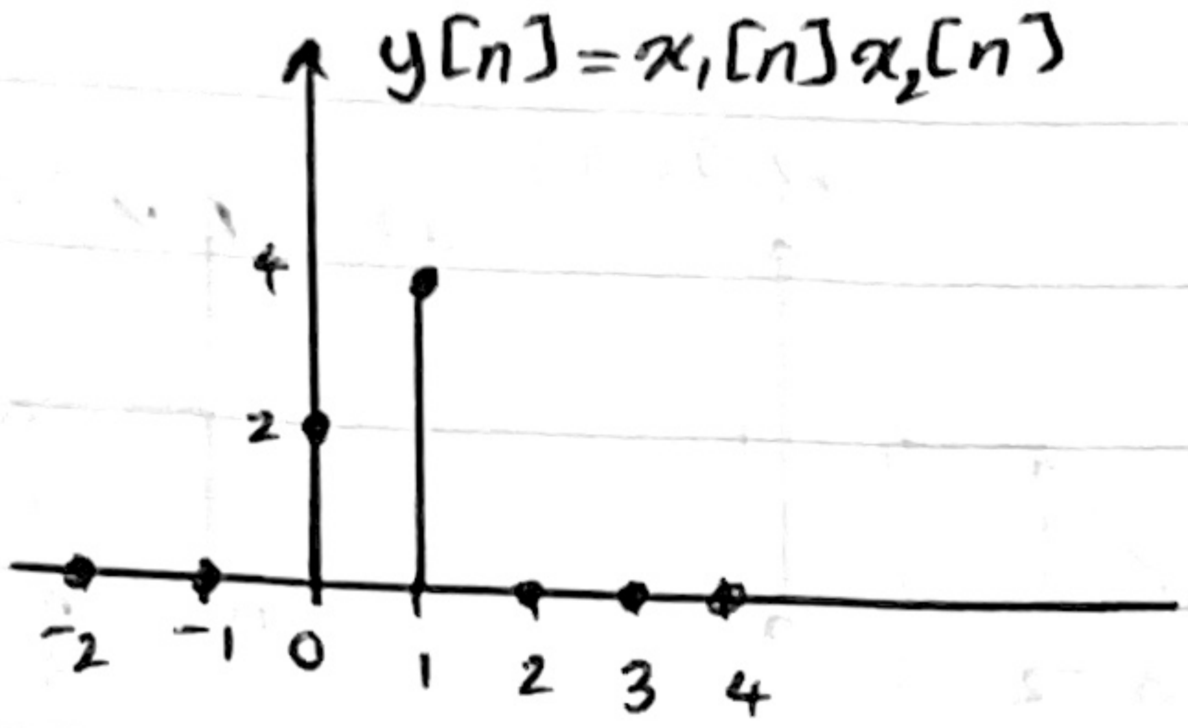
4) a)



b)



c)



5) $$\int_{-a}^{a} x(t)\,dt = \begin{cases} 2\int_0^a x(t)\,dt & ; \text{ if } x(t) \text{ is even} \\ 0 & ; \text{ if } x(t) \text{ is odd} \end{cases}$$

If $x(t)$ is even, $\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_0^a x(t)\,dt = \int_0^a -x(\bar{t})\,d\bar{t} + \int_0^a x(t)\,dt$

$$= 2\int_0^a x(t)\,dt.$$

If $x(t)$ is odd, $\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_0^a x(t)\,dt = -\int_0^a -x(\bar{t})\,d\bar{t} + \int_0^a x(t)\,dt$

$$= -\int_0^a x(t)\,dt + \int_0^a x(t)\,dt$$

$$= 0$$

Hence, $$\int_{-a}^{a} x(t)\,dt = \begin{cases} 2\int_0^a x(t)\,dt & ; \text{ is } x(t) \text{ is even} \\ 0 & ; \text{ if } x(t) \text{ is odd} \end{cases}$$

6) We compute, $x(t+T) = e^{j\omega(t+T)} = e^{j\omega T} e^{j\omega t} = x(t) \cdot e^{j\omega T}$.

Then $x(t+T) = x(t)$ if we choose T, s.t., $e^{j\omega T} = 1$.

i.e., $T = \frac{2\pi}{\omega} \cdot k$, where $k \in \mathbb{Z}$.

Thus the fundamental period $T_0 = \frac{2\pi}{\omega}$.

7) Assume $x[n]$ is periodic, i.e., $x[n] = x[n+N]$ for some $N \in \mathbb{Z}$

Thus $e^{j\omega n} = e^{j\omega(n+N)} = e^{j\omega n} e^{j\omega N} \Rightarrow 1 = e^{j\omega N} \Rightarrow \omega N = 2\pi m$

where $m \in \mathbb{Z}$. Hence, $\frac{\omega}{2\pi} = \frac{m}{N} \in \mathbb{Q}$.

8) a) First note that the sample signal, $x[n] = \cos(15Tn)$.

To be periodic sequece, $\frac{15T}{2\pi} \in \mathbb{Q}$

b) $N_0 = \frac{m2\pi}{\omega_0} = \frac{m2\pi}{15T} = \frac{m2\pi}{15\,0.1\pi} = m\frac{4}{3}$ for $m \in \mathbb{Z}$.

$\Rightarrow N_0 = 4.$

9) a) $x(t) = 2e^{j(t+\pi/4)}$

periodic, $T_0 = \frac{2\pi}{1} = 2\pi$

b) $x[n] = e^{j(\pi/4)n}$

periodic, $N_0 = \frac{2\pi}{\pi/4} = 8$

c) $x(t) = \cos(t+\pi/4)$

periodic, $T_0 = \frac{2\pi}{1} = 2\pi$

d) $x(t) = \cos(t) + \sin(\sqrt{2}\,t)$

Not periodic as $\nexists\, n_1, n_2 \in \mathbb{Z}$ s.t. $2\pi n_1 = \sqrt{2}\pi n_2$. (i.e., no common period).

e) $x[n] = \cos^2(\pi n/8)$

$x[n] = \cos^2(\pi n/8) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi n}{4}\right)$

Periodic, $N_0 = \frac{2\pi}{\pi/4} = 8$.

10) a) $x(t) = e^{-at}u(t)$, $a>0$.

$E_\infty = \int_{-\infty}^{\infty} |e^{-at}u(t)|^2 dt = \int_0^{\infty} e^{-2at} dt = \frac{1}{2a} < \infty$

$\Rightarrow$ Energy Signal.

b) $x(t) = A\cos(\omega t + \theta)$

$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [A\cos(\omega t+\theta)]^2 dt = A^2/2$

$\Rightarrow$ Power signal.

c) $x[n] = 3u[n]$

$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (3u[n])^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 9 = 9/2$.

$\Rightarrow$ Power signal.

d) $x[n] = 3e^{j3n}$

$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |3e^{j3n}|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 9 = 9$.

$\Rightarrow$ Power signal.