Multivariate Calulus & PDE

1. Sketch the graph of the following
   (a) $2x + 4y + 3z = 12$
   (b) $x^2 + 2y^2 + z^2 - 4x + 4y - 2z + 3 = 0$
   (c) $f(x, y) = \sqrt{16 - 4x^2 - y^2}$
2. Given the function $f(x, y) = \sqrt{8 + 8x - 4y - 4x^2 - y^2}$, find the level curve corresponding to $z = 0$ and sketch the level curves for $z = 1, 2$ and 3.
3. Let $w = f(x, y, z) = x^2 + y^2 - z^2$.
   Sketch the level surfaces $w = -1, w = 0, w = 1$.
4. Find the domain of the function $f(x, y) = \sqrt{1 - \frac{x^2}{9} - \frac{y^2}{4}}$ and state whether it is open, closed or neither.
5. Prove that a subset in $\mathbb{R}^2$ is closed if and only if its complement in $\mathbb{R}^2$ is open.
6. Prove that any isolated point is a boundary point.
7. Let $\mathrm{R} = \left\{\frac{1}{n} \mid n \in \mathbb{N}\right\} \cup \{0\} \subset \mathbb{R}^1$ show that 0 is a boundary point of R but not an isolated point of R.
8. Let $\mathrm{R} = \left\{(x, y) \mid x^2 + y^2 < 1\right\} \cup \{(5, 5)\} \subset \mathbb{R}^2$ show that R is neither open nor closed in $\mathbb{R}^2$.