

Multivariate Calculus & PDE

1. Determine the region in which the function is continuous.

(a) $f(x, y) = x^2y + 3x^3y^4 - x + 2y$

(b) $f(x, y) = \frac{2x-y}{x^2+y^2}$

(c) $\ln(x^2 + y^2 - 1)$

2. Investigate the continuity of

$$f(x, y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

at $(0,0)$.

3. Is

$$f(x, y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

continuous on \mathbb{R}^2 ?

4. Suppose that the function f is defined by,

$$f(x, y) = \begin{cases} x^2 + y^2 & \text{if } x^2 + y^2 \leq 1 \\ 0 & \text{if } x^2 + y^2 > 1 \end{cases}$$

Discuss the continuity of f .

5. Find all the first and second-order partial derivatives of the following.

(a) $f(x, y) = x \cos(y) + ye^x$

(b) $f(x, y) = \sin(xy^2) + e^{x^3+5y}$

6. Let $z = y^k e^{\frac{-x^2}{4y}}$. Find the value of constant k such that the following condition is satisfied.

$$\frac{\partial z}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^2 \frac{\partial z}{\partial x} \right)$$