Probability & Statistics

1. A group of 13 students are studying in Colombo for five weeks. As part of their study of the local economy, they each purchased an oriental rug and arranged shipment to the United States. The shipping time, in days, for each rug was

   $$81, 31, 42, 39, 72, 43, 74, 30, 28, 36, 37, 35, 45$$

   You have been asked to analyze the shipping experience of these students to provide advice to future students. In your analysis, answer the following questions.

   (a) What are the mean and median for this data set?

   (b) What are the sample variance and the sample standard deviation?

   (c) Is the distribution symmetric or skewed? If skewed state whether it is negatively or positively skewed.

   (d) Estimate the percentage of days that are within two standard deviations of the mean.

2. In a survey on examination qualifications, 50 people were asked,

   How many subjects are listed on your certificate?

   The frequency distribution of their responses is recorded in the table below.

Number of subjects	1	2	3	4	5	6	7
Number of people	5	3	7	8	9	10	8

   (a) Calculate the mean and standard deviation of the distribution.

   (b) *A Normal Distribution has approximately 68% of its data values within one standard deviation of the mean.*
   Use your answers to part (a) to check if the given distribution satisfies this property of a Normal Distribution. Show your work clearly.

3. A university professor conducted a test for three different groups of students using three different exam papers (P1, P2, and P3). He wants to choose the paper that follows a symmetrical distribution in terms of the students' marks. The marks obtained by the students for each paper are as follows:

   $$\text{P1: } 62, 48, 55, 71, 80, 42, 90, 37, 33, 60, 85$$
   $$\text{P2: } 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75$$
   $$\text{P3: } 50, 18, 22, 27, 35, 48, 41, 39, 78, 73, 29$$

   (a) Determine whether each dataset is normally distributed or skewed. If skewed, state whether it is positively or negatively skewed.

   (b) Identify which exam paper should be selected based on statistical reasoning.

4. A company evaluates job applicants based on two different skill assessment tests. The first test, which measures Technical Aptitude, has a national average score of 70 with a standard deviation of 10. The second test, which measures Problem-Solving Ability, has a national average score of 50 with a standard deviation of 12. An applicant took both tests and scored 85 on the first test and 62 on the second test. Assuming a normal distribution, determine in which test the applicant performed better relative to other applicants.

Multivariate Calulus & PDE

1. Let $z = f(x, y) = 2x^2 - xy$.

   (a) Find Δz, then use your result to find the change in z if (x, y) changes from (1, 1) to (0.98, 1.03).

   (b) Compute the value of dz, and compare your result with the value of Δz.

2. Approximate the value of $\sqrt{(3.02)^2 + (4.01)^2}$ using differentials.

3. A storage tank has the shape of a right circular cylinder. Suppose that the radius and height of the tank are measured at 1.5ft and 5ft, respectively, with a possible error of 0.05ft and 0.1ft, respectively. Use differentials to estimate the maximum error in calculating the capacity of the tank.

4. (a) Let $u = f(x, y)$, where $x = r\cos(\theta)$ and $y = r\sin(\theta)$. If $u = x^2 y + y^2$, find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial \theta}$.

   (b) If $T = x^3 - xy + y^3$, $x = p\cos(\Phi)$, $y = p\sin(\Phi)$ find $\frac{\partial T}{\partial p}$ and $\frac{\partial T}{\partial \Phi}$

   (c) Let $w = f(x, y, z) = e^{x^2 y} + e^{y^2 z}$, where $x = t$, $y = t^2$ and $z = t^3$. Find $\frac{dw}{dt}$.

5. If $x = 2r - s$ and $y = r + 2s$, find $\frac{\partial^2 U}{\partial y \partial x}$ in terms of derivatives with respect to r and s. Where second order partials of U exists and are continuous.

6. Let $f(x, y, z) = \ln(x^2 + y^2 + z^2)$.

   (a) Find the gradient vector $\nabla f(x, y, z)$.

   (b) Directional derivative of $f(x, y, z)$ at (1, 1, 1) in the direction of $\mathbf{v} = \langle -1, 1, 0 \rangle$.

7. The function

$$f(x, y, z) = x^2 y + y^2 z + z^2 x$$

has a gradient vector

$$\nabla f(x, y, z) = \langle 2xy + z^2, x^2 + 2yz, y^2 + 2zx \rangle.$$

At the point $(1, 2, 1)$, find the direction in which the directional derivative of $f(x, y, z)$ is minimized. What is the minimum value of the directional derivative?

8. Find the equation of the tangent plane and normal line at the point $(-2, 1, -3)$ to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$