

## Probability and Statistics

1. Which of the following illustrates probability? statistics?
  - (a) Studying past sales data to estimate next month's revenue for a company.
  - (b) Predicting the chance that it will rain on a given day based on weather models.
  - (c) Determining the chance that tails will result when a coin is tossed.
  - (d) Determining the amount of waiting time required to check out at a certain grocery store
2. List the elements of each of the following sample spaces:
  - (a) The set of integers between 1 and 50 divisible by 8.
  - (b) The set is  $S = \{x \mid x^2 + 4x - 5 = 0\}$ .
  - (c) The set of outcomes when a coin is tossed until a tail or three heads appear.
  - (d) The set is  $S = \{x \mid 2x - 4 \geq 0 \text{ and } x < 1\}$ .
3. A research team wanted to investigate the video gaming habits. To explore that, researchers gathered data from 200 high school students and each participant reported the number of hours they spent playing video games per week. The goal was to analyze the gaming behavior of high school students overall. Identify each of the following as specifically as possible.
  - (a) Population of Interest
  - (b) Variable of Interest
  - (c) Sample
4.
  - (a) How many even four-digit numbers can be formed from the digits 0, 1, 2, 5, 6, and 9 if each digit can be used only once?
  - (b) In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there?
  - (c) A young boy asks his mother to get 5 Game-Boy TM cartridges from his collection of 10 arcades and 5 sports games. How many ways are there that his mother can get 3 arcades and 2 sports games?
5. The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.99. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01. If a person tests positive, what is the probability that the person has the disease?

6. A company that manufactures shoes has three factories. Factory 1 produces 25% of the company shoes, Factory 2 produces 60%, and Factory 3 produces 15%. One percent of shoes produced by Factory 1 are mislabeled, 0.5% of those produced by Factory 2 are mislabeled, and 2% of those produced by Factory 3 are mislabeled. Given that you have bought a mislabeled pair of shoes, find the probability that it is produced by Factory 3.
7. In a certain assembly plant, three machines B1, B2, and B3 make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.
- (a) What is the probability that it is defective?
- (b) If a product was chosen randomly and found to be defective, what is the probability that it was made by machine B3?

## Multivariate Calculus & PDE

1. Check whether the following limits exist or not.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2+y^2}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{2x^2-3y^2}$

(c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy \cos(y)}{3x^2+y^2}$

2. Prove the following.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{10xy^2}{x^2+y^2} = 0$

(b)  $\lim_{(x,y) \rightarrow (0,0)} x \sin\left(\frac{1}{y}\right) = 0$

(c)  $\lim_{(x,y) \rightarrow (2,1)} (2xy - 3y^2) = 1$

3. Find the iterated limits  $f(x, y) = \frac{(y-x)(1+x)}{(y+x)(1+y)}$  as  $(x, y) \rightarrow (0,0)$  for  $(x+y) \neq 0, x > -1, y < 1$  and check whether they are equal.