Assignment - 1

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***Abstract*—This document contains the solution to Exercise 3.18 (a) of Oppenheimer.**

Problem 1. A causal LTI system has the system function

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)(1 - z^{-1})} \tag{1}$$

Find the impulse response $h(n)$ of the system.
Solution: We substitute $x := z^{-1}$ and carry out long division

$$\begin{array}{r} -2 \\ -\frac{1}{2}x^2 - \frac{1}{2}x + 1) \overline{\quad x^2 + 2x + 1} \\ \underline{-x^2 \quad -x + 2} \\ x + 3 \end{array}$$

Thus, on exapnding using partial fractions,

$$H(z) = -2 + \frac{z^{-1} + 3}{\left(1 + \frac{1}{2}z^{-1}\right)(1 - z^{-1})} \tag{2}$$

$$= -2 + \frac{1}{3}\left(\frac{1}{1 + \frac{1}{2}z^{-1}}\right) + \frac{8}{3}\left(\frac{1}{1 - z^{-1}}\right) \tag{3}$$

We know that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \tag{4}$$

and

$$\delta(n) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{5}$$

Thus, using (4) and (5) in (3),

$$h(n) = -2\delta(n) + \frac{1}{3}\left(8 + \left(-\frac{1}{2}\right)^n\right)u(n) \tag{6}$$

The plot of $h(n)$ has been generated using the Python code `codes/1_1.py`.

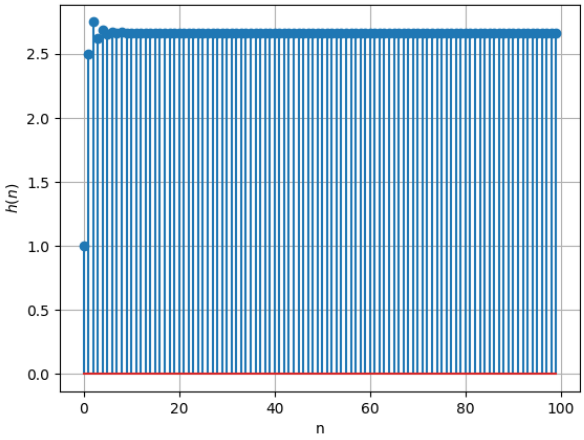

Fig. 1: Plot of $h(n)$ against n