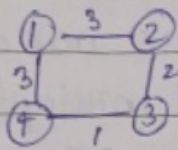


1)

a) The minimum spanning Tree (MST) of a graph is not unique. when the graph contains multiple edges with the same weight. In other words, if there are two or more ~~un~~edges in the graph that have the same weight, then there may be multiple MSTs for that graph. To illustrate this, consider a simple graph with four nodes and four edges. where two edges have same edge weight.



In this the max edge weight exists twice, then the graph does not have a unique MST. You have multiple choices to remove the edge with highest weight to get a MST, so the MST is not unique.

If all ~~edges~~ have distinct weights, then the MST is always unique.

2) To determine, if a graph has multiple MST's

Approach  $\rightarrow$  Kruskal's Algorithm and check if there are any other such sets of edges that would also create a MST. If there are multiple such sets then the graph has multiple MSTs otherwise it has only one.

Pseudocode:

has multiple MSTs (Graph G):

sort the edges of 'G' in non-decreasing order of weight. Initialize a set 'S'.

for each edge 'E' in the sorted list of edges do:

If adding E to S does not create a cycle in S then:

add E to S

else: continue;



If

$S.size() < V-1$  then:

return false.

let  $T$  be the set of edges in  $S$

for each edge  $e$  in  $T$  do:

remove  $e$  from  $T$

use Kruskal's algo

If the weight of new MST is equal to the weight of  $T$  then: continue to the new edge.

else: return true // multiple MST's

return false // one MST.

2). The maximum flow problem is a graph theory problem where the goal is to determine the maximum amount of flow that can be sent through a network, subject to the capacity constraints of its edges. It is commonly used in fields such as transportation and telecommunications to optimise resource allocation and reduce congestion. The problem can be solved using algo such as Ford-Fulkerson, Edmond's-Karp.

Ford-Fulkerson algorithm terminates when no more augmenting path can be found. It is a classical algorithm that it is guaranteed to converge to a maximum flow solution, although the running time can vary depending on the choice of augmenting path ~~search~~ search and flow augmentation methods.

Pseudocode for Ford-Fulkerson algo.

1. Initialize flow  $F$  to 0.
2. While there exists an augmenting path  $p$  in the residual network:

3. Find the min. residual capacity  $c-f(p)$  along  $P$ .  
4. Augment flow along  $P$  by  $(-f(p))$ .  
5. Update residual network with the net flow.  
6. Return  $F$ .

### Time Complexity:

Vary depend on 1). augmenting path search  
2). flow augmentation methods.

Worst Case  $\rightarrow O(E \times C^2)$ .