Home Assignemnt-2

1. A map uses a linear transformation represented by $A = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$ to stretch regions vertically. If the original area is 12 square units, calculate the area of the transformed region.
2. State the implication of two feature vectors having a zero-dot product.
3. Compute the determinant of the matrix Bif the eigenvalues of a square matrix are 15,0,3.
4. Determine the rank of the feature set and identify the redundant feature(s) for the vectors $x_1 = (1,1,1)$, $x_2 = (2,2,2)$, $x_3 = (0,1,1)$.
5. Compute the singular values of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -2 \end{bmatrix}$.
6. Explain why high-dimensional data often contains redundant information.
7. An industrial plant monitors machine health using multiple sensors. At a given machine, the following features are collected:

   f_1: Motor vibration amplitude
   f_2: Root mean square (RMS) vibration
   f_3: Machine temperature
   f_4: Thermal stress index
   f_5: Power consumption
   f_6: Electrical load factor
   The plant wants to minimize data processing cost by eliminating highly correlated features. Analyze which features are likely redundant and identify the essential features to retain.
8. A grayscale image is represented by $A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$. Obtain the SVD of A and determine the rank of A, also construct the rank-1 approximation using the largest singular value.
9. Consider the following matrices: **(a)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} -1 & -2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$ For each matrix A, (i) compute its Singular Value Decomposition (SVD) and express it in the form $A = U\,\Sigma\,V^T$.(ii) Clearly specify the dimensions of the matrices U, Σ, and V.(iii)Identify the singular values and determine the rank of each matrix.(iv) Comment on how the shape of matrix A(rectangular vs square) affects the structure of its SVD.
10. A grayscale image is represented by the following pixel-intensity matrix $B = \begin{bmatrix} 5 & 4 \\ 3 & 2 \\ 1 & 0 \end{bmatrix}$

    Using the above matrix, (i) compute the Singular Value Decomposition (SVD) of the matrix B, i.e., express $B = U\Sigma V^T$,(ii) determine the rank of the matrix B, (iii) construct a rank-1 approximation of the matrix B, also explain the significance of Singular Value

Decomposition in image compression, emphasizing how dimensionality reduction helps preserve the most important visual information while reducing data size.