1.

$Pr(A) = 0.8$

$Pr(\neg A) = 0.2$

$$\begin{tikzcd} & A \arrow[dl] \arrow[dr] & \\ B \arrow[dr] & & C \arrow[dl] \\ & D & \end{tikzcd}$$

$Pr(B|A) = 0.4$

$Pr(B|\neg A) = 0.9$

$Pr(\neg B|A) = 0.6$

$Pr(\neg B|\neg A) = 0.1$

$Pr(C|A) = 0.6$ $\quad Pr(\neg C|A) = 0.4$

$Pr(C|\neg A) = 0.2$ $\quad Pr(\neg C|\neg A) = 0.8$

$Pr(D|B,C) = 0.8$, $\cancel{Pr(\neg D|B,C) =}$.

$Pr(D|B,\neg C) = 0.5$

$Pr(D|\neg B, C) = 0.6$

$Pr(D|\neg B,\neg C) = 0.3$.

$$Pr(C \wedge D) = Pr(C=1, D=1).$$

$$= Pr(A=a, B=b, C=1, D=1)$$

$$= \sum_{a,b} Pr(A=a).P(B=b|A=a).P(C=1|A=a).P(D=1|B=b,C=1)$$

$$= \{P(A=1).P(B=1|A=1).P(C=1|A=1).P(D=1|B=1,C=1)\} +$$
$$\{P(A=1).P(B=0|A=1).P(C=1|A=1).P(D=1|B=0,C=1)\} +$$
$$\{P(A=0).P(B=1|A=0).P(C=1|A=0).P(D=1|B=1,C=1)\} +$$
$$\{P(A=0).P(B=0|A=0).P(C=1|A=0).P(D=1|B=0,C=1)\}$$

$$= \{0.8 \times 0.4 \times 0.6 \times 0.8\} + \{0.8 \times 0.6 \times 0.6 \times 0.6\} +$$
$$\{0.2 \times 0.9 \times 0.2 \times 0.8\} + \{0.2 \times 0.1 \times 0.2 \times 0.6\}.$$

$$= 0.1536 + 0.1728 + 0.0288 + 0.0024$$

$$= 0.3576 \quad \text{(Ans)}.$$

2. $P(HIV) = 0.05\% = 0.0005$, $P(\neg HIV) = 0.9995$

$P(+|HIV) = 0.98$ $P(-|HIV) = 0.02$

$P(+|\neg HIV) = 0.03$ $P(-|\neg HIV) = 0.97$

$P(HIV|+) = ?$

$$P(HIV|+) = \frac{P(+|HIV) \cdot P(HIV)}{P(+)}$$

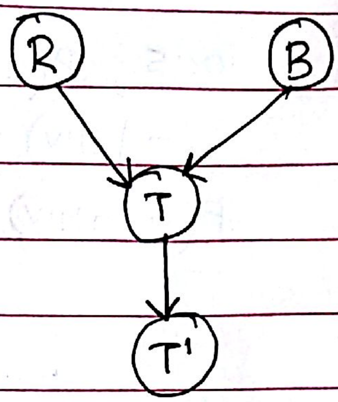
$$\begin{aligned} P(+) &= P(+|HIV) \cdot P(HIV) + P(+|\neg HIV) \cdot P(\neg HIV). \\ &= (0.98 \times 0.0005) + \{(0.03) \times (0.9995)\} \\ &= 0.00049 + 0.029985 \\ &= 0.030475. \end{aligned}$$

$$\therefore P(HIV|+) = \frac{0.98 \times 0.0005}{0.030475}$$

$$= \frac{0.00049}{0.030475}$$

$$= 0.016 \text{ (Ans).}$$

3.



(i) $R \perp\!\!\!\perp B$

$$P(R,B) = P(R=1, B=1, T=t, T'=t')$$
$$= \sum_{t,t'} P(R).P(B).P(T|R,B).P(T'|T)$$

As $P(R,B)$ ~~can~~ be expressed in terms of R and B, they are conditionally independent.

(ii) $R \perp\!\!\!\perp B | T$.

Using rules of d-seperation, the path from $R \rightarrow B$ is not blocked. as node $\rightarrow T \leftarrow$ is a conditioning node.
Thus R and B are not d-seperated by T.
Thus R and B are not conditionally independent given T.

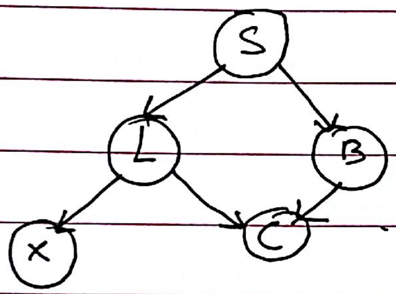
(iii) $R \perp\!\!\!\perp B | T'$

Using rules of d-sepeation, the path from $R \rightarrow B$ is not blocked as node $\rightarrow T \leftarrow$ descends into conditioning node
Thus R and B are not d-seperated by T'
Thus R and B are not conditionally independent given T'

4.(V) $L \perp\!\!\!\perp B \mid T, R$

Using rules of d-seperation, the path from $L \rightarrow B$ is blocked by $\rightarrow R \rightarrow$ which is a conditioning node. Thus L and B are d-seperated by $\{T, R\}$. Thus L and B are <u>conditionally independent</u> given T, R.

6.



(i) given S, pairs that are conditionally independent are:-

a) L and B

b) B and X

(ii) given L, pairs that are conditionally independent are:-

a) S and X

b) B and X

c) C and X

(iii) given $\{L, B\}$, pairs that are conditionally independent are:-

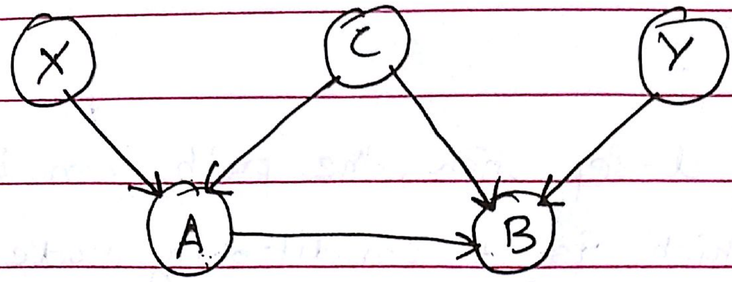
a) S and C

b) S and X

c) C and X.

classmate

8.



Sets of nodes that d-seperate X and Y are $\{A\}$, $\{C\}$, $\{C, A\}$.