ASSIGNMENT 4

CEE/ENGMT 5980 Decision Analysis

Instructions

In this assignment, you will:

- Examine the implications of using an *a priori* method
- Utilize an advanced visualization tool to explore tradeoffs in the GAA dataset
- Bonus: conduct an OAT and global sensitivity analysis

Please turn in a separate PDF document containing the answers to each of the parts of this assignment

Note: Though your work will only be seen by those grading the course and will not be used or shared outside the course, you should take care to obscure any information you feel might be of a sensitive or confidential nature.

*You can complete each assignment exercise as you progress through the unit. Wait to submit the assignment until all exercises are complete. A **Submit Assignment** button can be found on the assignment page. Information about the grading rubric is available on any of the assignment pages online. Do not hesitate to contact your TA if you have any questions about the assignment.*

Part One (35 points)

Examining solutions from the GAA

You are a consultant working with the American Aviation Company (AAC) to design a general aviation aircraft (GAA). You ran a multi-objective optimization using the 10-objective formulation described by Woodruff et al., (2013). A description of each of the 10 objectives can be found in the table below.

Table 3 GAA Objectives

Name	Description	Unit	Preference
Takeoff noise (NOISE)	Measurement of noise at takeoff	dB	Minimize
Empty weight (WEMP)	Weight of the aircraft without passengers	lb	Minimize
Direct operating cost (DOC)	Cost of flying the aircraft	1970$/hr	Minimize
Ride roughness (ROUGH)	Measure of flight roughness	ratio	Minimize
Fuel weight (WFUEL)	Weight of fuel	lb	Minimize
Purchase price (PURCH)	Price of purchase	1970$	Minimize
Range (RANGE)	Flight range	nmi	Maximize
Lift-drag ratio (LDMAX)	A measure of flight performance	ratio	Maximize
Cruising speed (VCMAX)	Aircraft cruising speed	Knots	Maximize
Product Family Penalty Function (PFPF)	A measure of commonality across aircraft families (2, 4 and 6 seats)	Unitless	Minimize

Here are some things to note before you get started:

- To make things simpler, all objectives that are being maximized (RANGE, LDMAX, and VCMAX) have been made negative in the attached "GAA_pset.csv" file, so that <u>all objectives should be minimized</u>.
- It's recommended to use Excel, Python or MATLAB for parts (a) to (c).
- We recommend using the J3 Visualization Tool to generate all the plots required in parts (d) and (e). You can also utilize the following alternative tools:
    - Categorical Parallel Axis Plot tool
    - Plotly Chart Studio
    - Tableau
    - Microsoft Power BI

a) Use the goal programming methodology described in Woodruff et al., (2013) (Equations (1) and (2) shown below) to identify the solution with the best aggregate performance.

The authors define the aggregate objective metric z as a combination of the multiple objectives into a single metric. A simplified version of this metric, which does not separate the 2, 4, and 6 seaters, is:

$$z = \sum_{j=1}^{7} \frac{d_j}{7}$$

Where d_j is the goal deviation for the jth objective, defined as:

$$d_j = \left\{ \frac{attained_j - goal_j}{attained_j},\ 0 \right\}\ for\ minimization\ objectives$$

$$d_j = \left\{ \frac{goal_j - attained_j}{goal_j},\ 0 \right\}\ for\ maximization\ objectives$$

As noted on Page 2 of this assignment, all objective scores in “GAA_pset.csv” are intended to be minimized. <u>These equations can be used as-is even with objective values that have negative sign.</u>

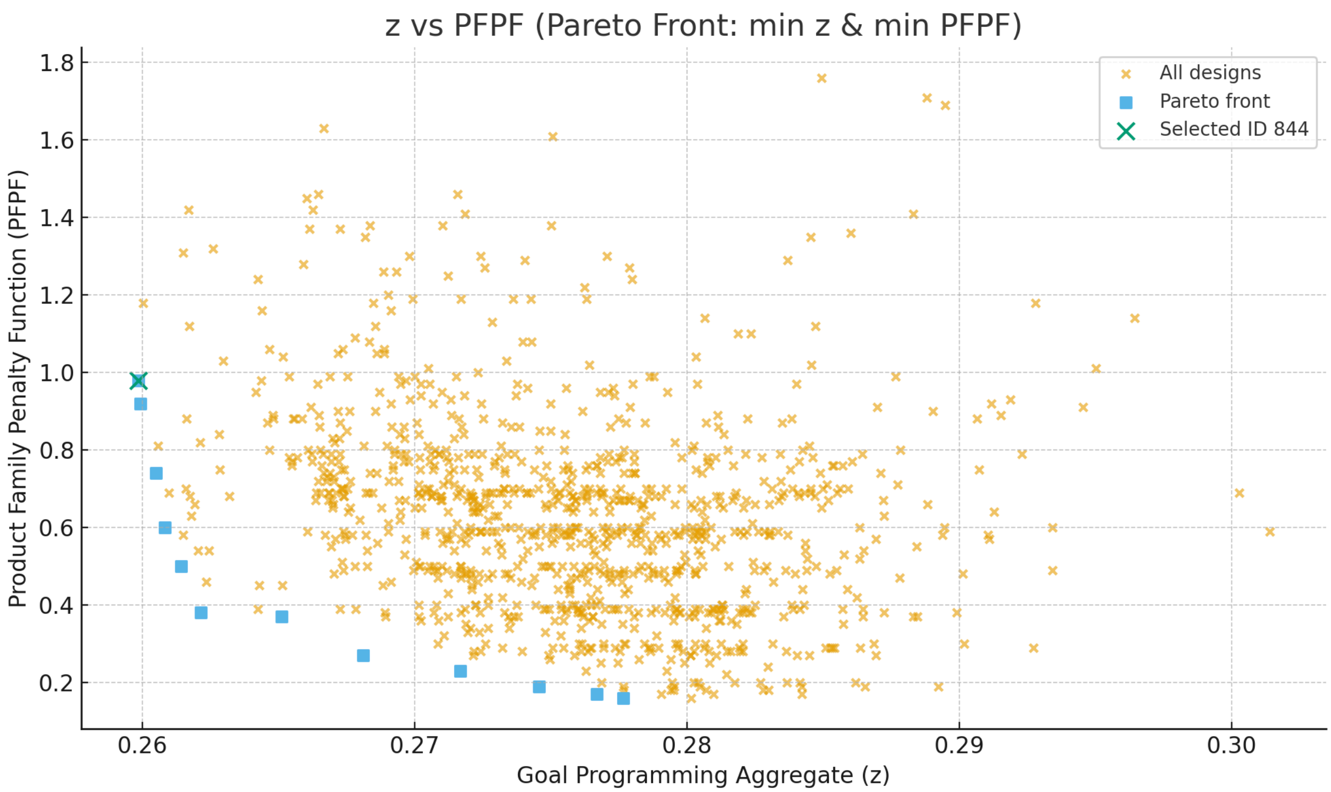
Using this method, select a solution from the Pareto set using the following goal values. Ignore objectives that do not have goals. You do not need to worry about separating the 2, 4 and 6 seat metrics.

Objective Name:	NOISE	WEMP	DOC	ROUGH	WFUEL	PURCH	RANGE	LDMAX	VCMAX	PFPF
Goal:	NA	<1950	<60	NA	<400	<42000	<-2300	<-16	<-200	NA

What ID number is selected? What might this solution look like? Describe this solution based on how it performs in its objectives (i.e., this plane will have low fuel efficiency as it has a high WFUEL).
(5 points)

Aircraft ID #844 achieved the lowest aggregate deviation score (z = 0.26), making it the best overall performer. It meets goals for empty weight, fuel weight, and range but slightly misses on cost and aerodynamic metrics (L/D max and cruise speed). This suggests a light, fuel-efficient, long-range aircraft that trades affordability and minor aerodynamic performance for overall efficiency and endurance.

b) Plot the goal programming z-value performance against the PFPF objective and note the location of the solution selected in part (a). Highlight the Pareto Front. Is there a tradeoff between the two performance measures? **Using Python**
(5 points)



From the frontier, there's a limited tradeoff: moving left to slightly smaller z doesn't automatically worsen PFPF, but the best joint improvements sit along the lower-left boundary; many interior points are dominated.

c) Does the goal programming solution lie on the Pareto front? What does its location say about the *a priori* method?
(2 points)

Yes, the goal programming solution (ID #844) lies on the Pareto front, meaning it is an efficient design that balances both objectives without being dominated by others. This indicates the a priori goal-setting method was effective in identifying a solution consistent with the true optimal trade-offs between performance and commonality.

d) Examine the entire 10-objective Pareto set within the “GAA_pset.csv” dataset using the visualization software of your choice (see Page 2 of this assignment).

An executive at the firm would like an aircraft that meet the following criteria:

Objective Name:	NOISE	WEMP	DOC	ROUGH	WFUEL	PURCH	RANGE	LDMAX	VCMAX	PFPF
Criteria	<73.6	<1960	<68.9	<1.9	<423	<43,250	>2217	>15.4	>195.1	<1

Pro tip #1: Parallel axis plots are great tools for visualizing high-dimensional datasets.
Pro tip #2: The ranges in your visualization do not have to be exact, merely approximate.

How many aircraft meet these criteria? Given the number of aircraft that meet these criteria, what would you advise the executive to do? Why?
(3 points)

Only two aircraft (ID 269 and 463) meet all of the executive’s criteria. Both designs perform well across most objectives, with low noise, lightweight construction, and a strong range, demonstrating high overall efficiency and balance. Because so few aircraft satisfy every requirement, the executive should consider loosening one or two constraints, such as purchase cost or fuel weight, to increase viable options.

e) Your clients are interested in releasing three models of aircraft for different market segments:
1. Model 1: An **economy** model that has low operating cost (DOC) and a low purchase price (PURCH)
2. Model 2: A **comfort** model that has low NOISE and low roughness (ROUGH)
3. Model 3: A **performance** model that has high RANGE, high LDMAX and high VCMAX.

They would <u>like all airplanes to have low PFPF</u> to facilitate the manufacturing process.

Make one set of visualizations each for Models 1, 2 and 3. Each set should include at least one of the following:
- 3D plot
- 2D plot
- Parallel axis plot

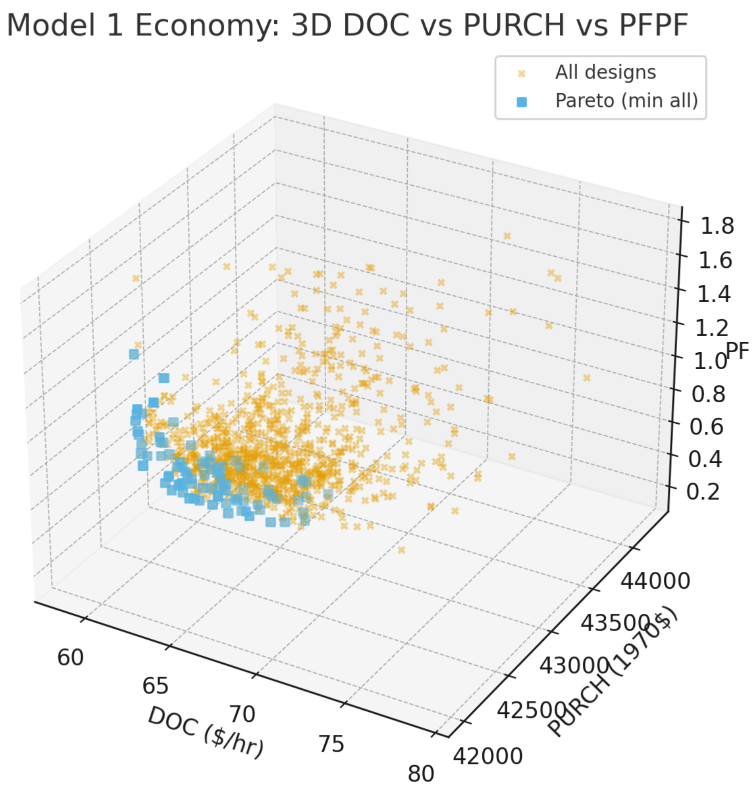
- The settings that correspond to the visualizations.

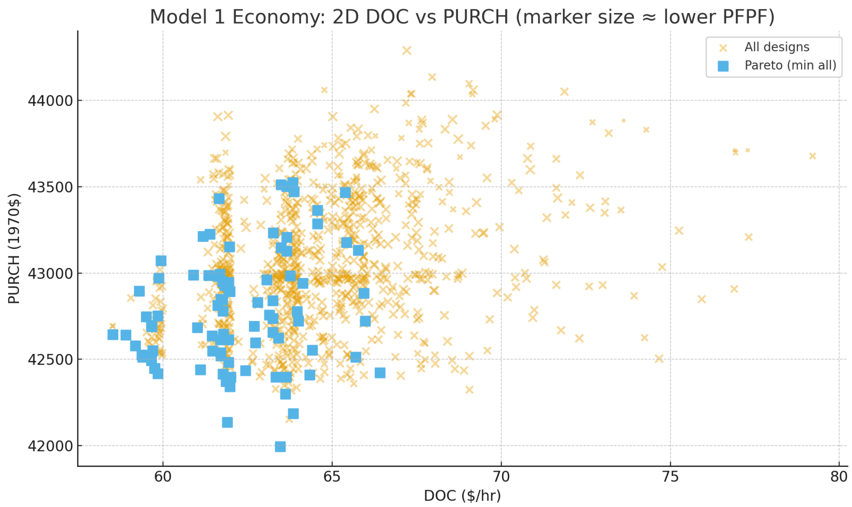
You can use the visualization software you selected in part 1(d). You can also opt to generate each of these plots individually with code in Excel, Python, MATLAB, R, or any other programming language of your choice.

Narrate the recommended set of options for each model (i.e., show relevant trade-offs and highlight important aspects of the set). Write a brief paragraph for each that explains each figure in terms of the performance objectives. Outline the relevant performance tradeoffs and opportunities.
(15 points)

All Using Python:

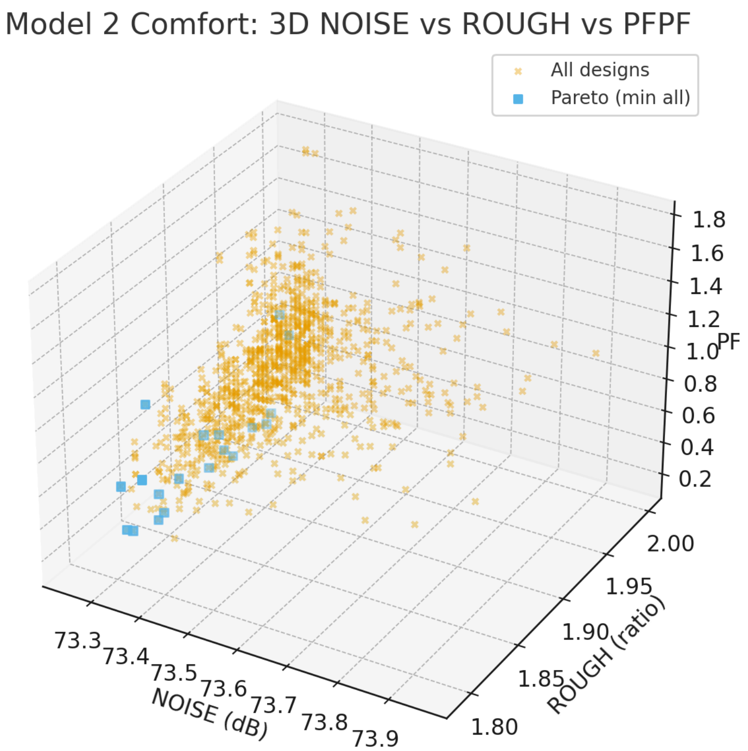
Model 1:

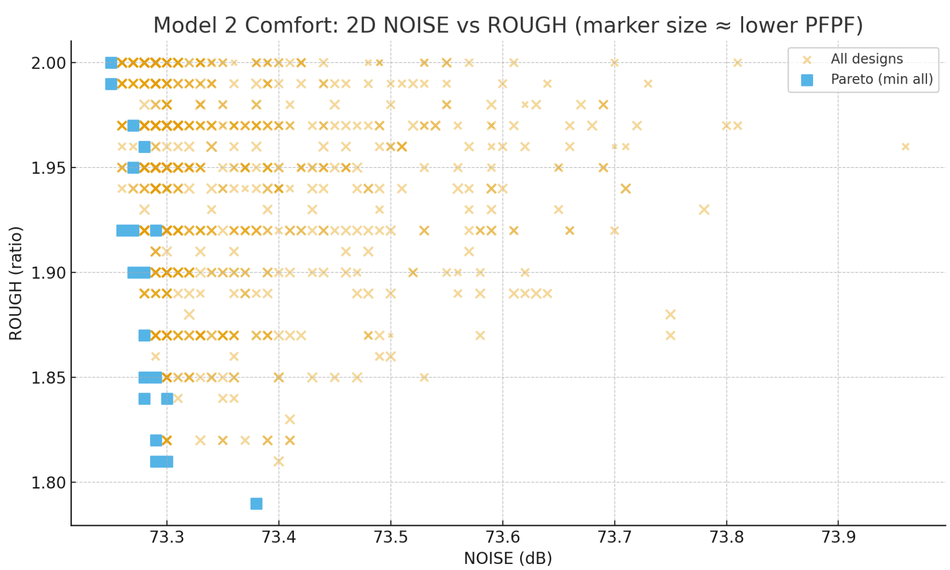




The economy model focuses on minimizing operating cost (DOC) and purchase price (PURCH) while maintaining a low PFPF for easier manufacturing. The 3D plot of DOC, PURCH, and PFPF shows that lower-cost designs often come with slightly higher PFPF, indicating a small tradeoff between cost and manufacturability. The best options sit on the Pareto front, offering DOC around 59–60 $/hr and PURCH around 42,000 (1970$) with moderate PFPF values that balance both affordability and production simplicity.

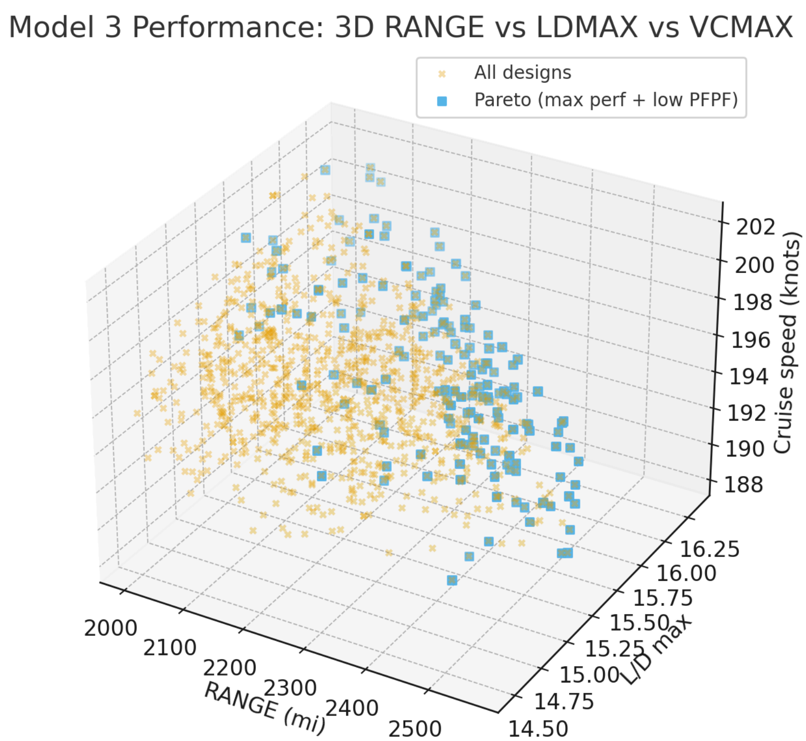
Model 2:

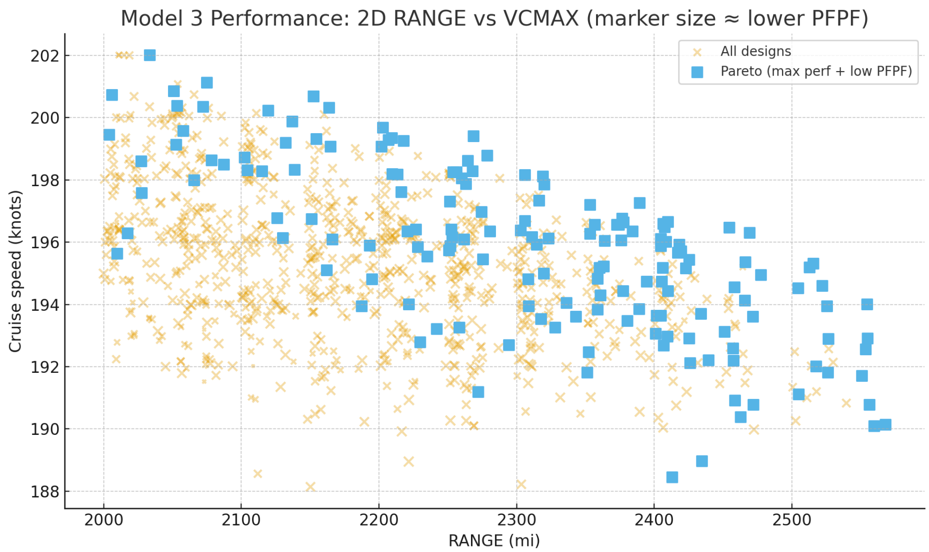

Model 2 Comfort: 3D NOISE vs ROUGH vs PFPF
All designs
Pareto (min all)
NOISE (dB)
73.3
73.4
73.5
73.6
73.7
73.8
73.9
ROUGH (ratio)
1.80
1.85
1.90
1.95
2.00
PF
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8


Model 2 Comfort: 2D NOISE vs ROUGH (marker size ≈ lower PFPF)
All designs
Pareto (min all)
ROUGH (ratio)
2.00
1.95
1.90
1.85
1.80
NOISE (dB)
73.3
73.4
73.5
73.6
73.7
73.8
73.9

The comfort model aims to minimize NOISE, ROUGHNESS, and PFPF, producing a quieter and smoother ride. In both 3D and 2D visualizations, the Pareto front clusters at low noise and low roughness values, demonstrating a clear relationship where smoother, quieter aircraft can still retain manageable PFPF. The top-performing designs achieve NOISE ≈ 73.3 dB, ROUGH ≈ 1.8, and PFPF ≈ 0.2–0.3, balancing comfort with manufacturability.

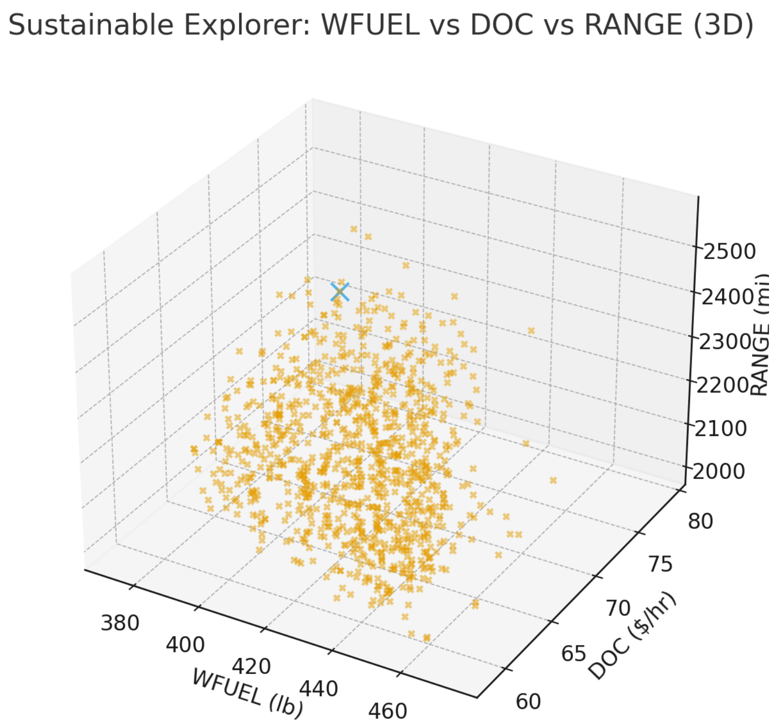
Model 3:

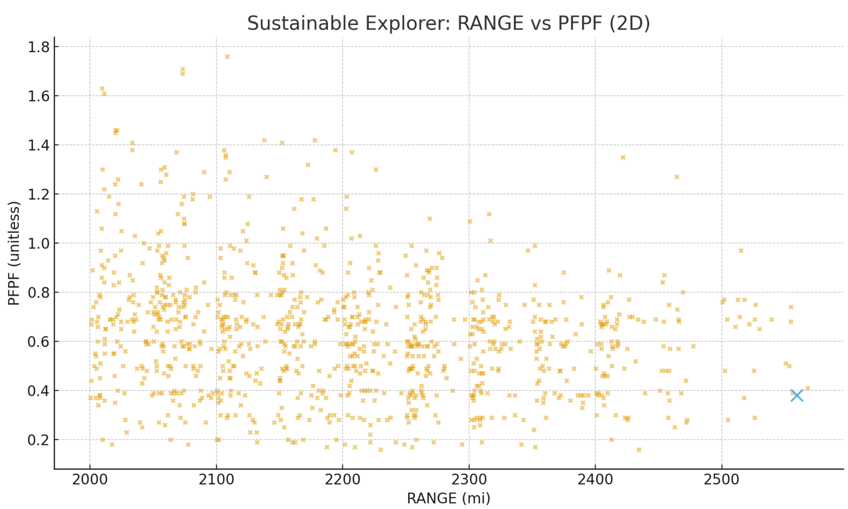




The performance model maximizes RANGE, LDMAX, and VCMAX while keeping PFPF low to preserve production efficiency. The 3D plot shows that aircraft with higher range and cruise speed tend to have modest increases in PFPF, indicating a moderate tradeoff between performance and manufacturability. The best options on the Pareto front achieve RANGE ≈ 2,100–2,300 mi, L/Dmax ≈ 16, and Vcruise ≈ 195–200 knots, offering strong aerodynamic performance without large penalties in PFPF.

f) Design your own aircraft and use a similar set of visualizations to explain what features you are highlighting and how you navigate trade-offs between objectives. This is meant to be open-ended. *There is no correct answer*, but you must explain your logic and narrate your design with visualizations.
(5 points)





Here are the 2D and 3D plots for my "Sustainable Explorer" concept, which prioritizes low fuel use (WFUEL) and low DOC, long range, and low PFPF for manufacturability. I ranked designs with a simple composite score that minimizes WFUEL, DOC, and PFPF while maximizing RANGE (implemented by normalizing each metric and flipping RANGE to higher-is-better). The highlighted design (ID 909) sits near the efficient edge in both figures with WFUEL ≈ 421 lb, DOC ≈ 64.5 $/hr, RANGE ≈ 2560 mi, and PFPF ≈ 0.38, illustrating a balanced trade-off: we accept a moderate DOC (not the absolute minimum) to achieve long range at reasonable fuel mass and low manufacturing penalty.

Bonus (12 assignment points)

Sensitivity Analysis

In this problem we'll explore sensitivity analysis as a tool to identify the inputs to a model that cause the largest changes in a particular output of interest.

Imagine that you're consulting for a company that manufactures widgets. There are three characteristics that define a widget that consumers will potentially care about: its height, weight, and color. The company's engineers have identified a regression relationship that approximates the profit per widget as a function of these three characteristics:

$$y = \sin sin\left(x_1\right) + a\left(x_2\right) + b\,x_3^4 \sin sin\left(x_1\right)$$

Where x_1, x_2 and x_3 are quantitative representations of the height, weight, and color that have all been transformed to the unitless range $(-\pi, \pi)$. The coefficients are $a = 7$ and $b = 0.1$.

Due to budget constraints, the company will only be able to change one of these characteristics on the manufacturing line. It is up to you to determine which factor has the most influence on the profit.

Part A: One-at-a-time (OAT) sensitivity analysis (3 bonus assignment points)

In a programming language of your choice (R, MATLAB, Python, etc., *not Excel*), write the equation above as a function that accepts a vector of three input variables and returns the profit estimate y. Use this function to perform a one-at-a-time (OAT) sensitivity analysis. This is essentially a finite difference estimate, where the sensitivity of each factor x_i is quantified by $S_i = \frac{\Delta y}{\Delta x_i}$ for some small Δx_i. If the initial point is $(x_1, x_2, x_3) = (1.0,\ 1.0,\ 1.0)$, find all three sensitivity indices using $\Delta x_i = 0.01$ for each variable. Which variable appears to be the most influential?

Please submit your `.py`, `.m`, or `.r` (or any other programming language) code files along with your estimates of the three sensitivity indices. <u>Screenshots of code or output are not acceptable.</u>

S_1 = 0.4098
S_2 = 3.7526
S_3 = 0.3417

Most influential variable: X_2

Part B: Global sensitivity analysis (6 bonus assignment points)

The OAT analysis you performed in Part A is known as a local sensitivity analysis, because it is centered around a particular point. By contrast, a global sensitivity analysis samples the input variables across their entire range, which in this case is $(-\pi, \pi)$. One commonly used global method is Sobol sensitivity analysis, which aims to estimate the fraction of variance in the output that each input factor contributes.

Because the Sobol method is a bit more complicated to code, it is recommended that you download a third-party library for this part of the assignment. Being able to locate and understand other people's software is a time-saving skill, particularly when the software in question is well documented and tested. You are welcome to use any implementation of the Sobol method that you can find; here are a few:

MATLAB: A version of the Sobol method is provided in the SAFE toolbox for sensitivity analysis. See the documentation page for instructions:
https://safetoolbox.github.io/Documentation.html
The MATLAB Global Sensitivity Analysis Toolbox:
https://www.mathworks.com/matlabcentral/fileexchange/40759-global-sensitivity-analysis-toolbox

Python: An open-source sensitivity analysis library is available here: https://salib.readthedocs.io/en/latest/
Additionally, examples can be found in the examples/ folder in this GitHub repo:
https://github.com/SALib/SALib/tree/main/examples
There is also a blog post that you can read describing the steps in detail:
http://waterprogramming.wordpress.com/2013/08/05/running-sobol-sensitivity-analysis-using-salib/

R: An open-source sensitivity analysis library in R is available here:
http://cran.r-project.org/web/packages/sensitivity/index.html
It comes with a detailed PDF manual showing how to call the different functions. This library contains several different implementations of the Sobol method; any of them should do for our application.

Regardless of which library you choose, there are typically three steps to calculating the Sobol sensitivity indices. You will need to read the code and/or documentation for your library of choice to figure out how to perform these steps:

1. Create a sample of the input space. Choosing a sample size requires some guesswork, but for this problem $N = 1{,}024$ should provide accurate results.
2. Evaluate the model for all the input samples. Here you can use the function you created in Part A or create a new one if you switched languages.
3. Use the model output to calculate the sensitivity indices.

The Sobol method, depending on your version, should return both first-order and total-order sensitivity indices. First-order indices represent the sensitivity of each factor by itself, while total-order indices account for interactions with other parameters.

Report the first-order and total-order Sobol indices that you calculate in your experiment.

Reporting using code files: You do not have to submit the code for the library that you used but include any additional code that ***you*** wrote. Submit hard copies of your `.py`, `.m`, or `.r` code files (or similar files in a programming language of your choice). Screenshots of your code will not be accepted.

Reporting using a workflow description: Describe the sequence of steps that you followed to perform the sensitivity analysis calculation.

Parameter	First Order S_i	Total Order S_{Ti}
X_1	0.107	0.9176
X_2	0.7861	0.2347
X_3	0.0049	1.0031

To perform the global sensitivity analysis, I used the SALib library in Python. I did the following steps to compute it:

1. I defined the profit function in Python and set up the input parameter space in SALib with three variables (x_1, x_2, x_3), each ranging from −π to π.
2. Using SALib's saltelli.sample() function, I generated N = 1,024 samples of the three variables. This sampling method ensures even coverage of the full parameter space and creates the number of model evaluations required for Sobol analysis.
3. I passed each set of sampled inputs into the profit function and recorded the resulting profit values y for all sampled combinations. These outputs represent how profit varies across all possible combinations of height, weight, and color parameters.
4. Using the sobol.analyze() function from SALib, I calculated both the first-order and total-order Sobol indices. The first-order indices capture each variable's individual contribution to the variance in profit, while the total-order indices include both individual effects and interactions between variables.

5. The Sobol results showed that x_2 had the highest first-order and total-order indices, indicating it is the most influential factor affecting profit. The other two variables, x_1 and x_3, had smaller sensitivity indices, suggesting they have weaker effects and limited interaction contributions.

Part C: Decision implications (3 bonus assignment points)

This is a simplified example, but it represents a problem that comes up frequently: given a set of uncertain inputs and a limited budget, which of them matter enough to warrant further research? Provide a brief discussion comparing the sensitivity indices that you found in Part A (local OAT) and Part B (total-order global sensitivity). Do they provide the same ranking of importance? If so, why? If not, why not? Is there anything about the mathematical form of the function that would make the results similar or different?

What about the first-order indices from Part B—do these provide the same ranking as the local OAT sensitivity indices from Part A? Why or why not?

Briefly discuss the consequences of using OAT rather than global sensitivity analysis for this problem. Do you think this is a generalizable result? Explain.

The OAT and Sobol analyses both show that x_2 has the strongest direct effect on profit, but they differ in how they account for other variables. The OAT method only measures local effects near one point, while the Sobol analysis considers the entire input range. The Sobol total-order indices reveal that x_1 and x_3 also contribute through interactions, which OAT cannot detect. Overall, global sensitivity analysis gives a more complete picture of variable importance in nonlinear models.

References:

- I used ChatGPT to ask questions whenever I would get stuck writing code in Python. The exact prompt I used was: “What function in python is used for _________.”
- I was unfamiliar with SALib in Python, so I used the following YouTube video to get started: https://www.youtube.com/watch?v=gkR_Iz5OptU