CTAT and Bandgap Voltage Reference Derivation

1. CTAT Voltage: Base-Emitter Voltage of a BJT

The base-emitter voltage (V_{BE}) of a BJT biased with a constant collector current I_C decreases with temperature. It is modeled as:

$$V_{BE}(T) = V_G(0) - \eta \cdot \frac{kT}{q} \ln\left(\frac{T^3}{I_C}\right)$$

Simplifying for a rough model, it is often approximated as:

$$V_{BE}(T) \approx V_{BE}(T_0) - \alpha(T - T_0)$$

where:

- $V_G(0)$ is the bandgap of silicon at 0 K, typically about 1.205 V,
- α is the negative temperature coefficient (typically $2\,\mathrm{mV\,K^{-1}}$),
- k is Boltzmann's constant,
- q is the elementary charge,
- T is the absolute temperature,
- T_0 is the nominal reference temperature (e.g., 300 K).

Thus, $V_{BE}(T)$ exhibits a **CTAT** behavior — *Complementary To Absolute Temperature.*

2. PTAT Voltage: Thermal Voltage Difference Between Two BJTs

If two BJTs operate at different current densities J_1 and J_2, the difference in their V_{BE} voltages is:

$$\Delta V_{BE} = V_{BE1} - V_{BE2} = \frac{kT}{q} \ln\left(\frac{J_1}{J_2}\right)$$

Since $\Delta V_{BE} \propto T$, this is a **PTAT** (Proportional To Absolute Temperature) voltage.

3. Bandgap Voltage Reference: Combining CTAT and PTAT

To achieve a temperature-independent voltage reference:

$$V_{\text{REF}} = V_{BE} + K \cdot \Delta V_{BE}$$

Where K is a scaling factor (set by resistor ratio in the circuit). This gives:

$$V_{\text{REF}} = V_{BE} + K \cdot \frac{kT}{q} \ln\left(\frac{J_1}{J_2}\right)$$

Choosing K such that the positive temperature coefficient of the PTAT term cancels the negative coefficient of the CTAT term yields:

$$\frac{dV_{\text{REF}}}{dT} \approx 0$$

4. Typical Result

The resulting bandgap voltage is:

$$V_{\text{REF}} \approx 1.2\,\text{V}$$

Which is relatively constant over temperature for first-order effects.

1 PTAT Behavior (Proportional to Absolute Temperature)

Starting with the Diode Equation

The current through a diode (or base-emitter junction of a BJT) is given by:

$$I = I_S \cdot \exp\left(\frac{V_{BE}}{V_T}\right)$$

Solving for V_{BE}, we get:

$$V_{BE} = V_T \cdot \ln\left(\frac{I}{I_S}\right)$$

Where:

- $V_T = \frac{kT}{q}$ is the thermal voltage.
- I_S is the saturation current of the diode.
- k is Boltzmann's constant.
- q is the elementary charge.

Generating PTAT Using Diode Current Ratios

If two identical diodes (or BJTs) are operated at different current densities, or if n diodes are connected in parallel, the effective current through each diode is scaled.

Let:

$$\text{Single diode: } V_1 = V_T \cdot \ln\left(\frac{I_0}{I_S}\right)$$

$$\text{n parallel diodes: } V_2 = V_T \cdot \ln\left(\frac{I_0}{nI_S}\right)$$

Now, subtract the two voltages:

$$\Delta V = V_1 - V_2 = V_T \cdot \ln(n)$$

This voltage is proportional to $V_T = \frac{kT}{q}$, which is directly proportional to absolute temperature T.

Hence, $\Delta V = \frac{kT}{q} \cdot \ln(n)$ is a *PTAT* voltage.

Temperature Derivative of PTAT Voltage

$$\Delta V = \frac{kT}{q} \cdot \ln(n)$$

Differentiate with respect to temperature T:

$$\frac{d(\Delta V)}{dT} = \frac{k}{q} \cdot \ln(n)$$

Assuming $\ln(n) = \ln(8) \approx 2.079$, and using constants:

$$\frac{k}{q} = 8.617 \times 10^{-5} \text{ V/K}$$

Then,

$$\frac{d(\Delta V)}{dT} = 8.617 \times 10^{-5} \cdot 2.079 = 1.791 \times 10^{-4} \text{ V/K} = 179.1 \text{ µV/°C}$$

If instead $\ln(n) = \ln(2) \approx 0.693$, then:

$$\frac{d(\Delta V)}{dT} = 8.617 \times 10^{-5} \cdot 0.693 \approx 59.7 \text{ µV/°C}$$

For the commonly used value where:

$$\frac{d(\Delta V)}{dT} \approx 86.25 \text{ µV/°C}$$

this corresponds to:

$$\ln(n) \approx \frac{86.25 \times 10^{-6}}{8.617 \times 10^{-5}} \approx 1.000 \quad \Rightarrow n \approx e^1 \approx 2.718$$

Conclusion:

$$\Delta V = V_T \cdot \ln(n) \quad \text{is a PTAT voltage}$$

$$\frac{d(\Delta V)}{dT} \approx 86.25\,\text{µV/°C} \quad \text{for } \ln(n) \approx 1$$

This PTAT behavior is combined with the CTAT V_{BE} to generate a temperature-independent reference voltage in Bandgap circuits.

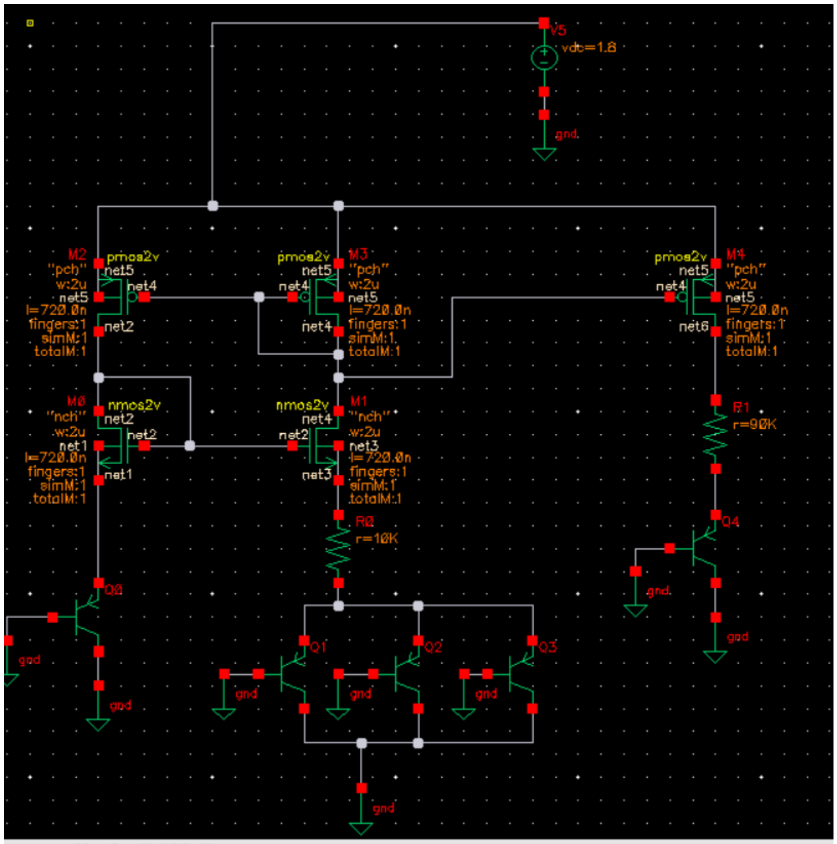

Figure 1: CIRCUIT

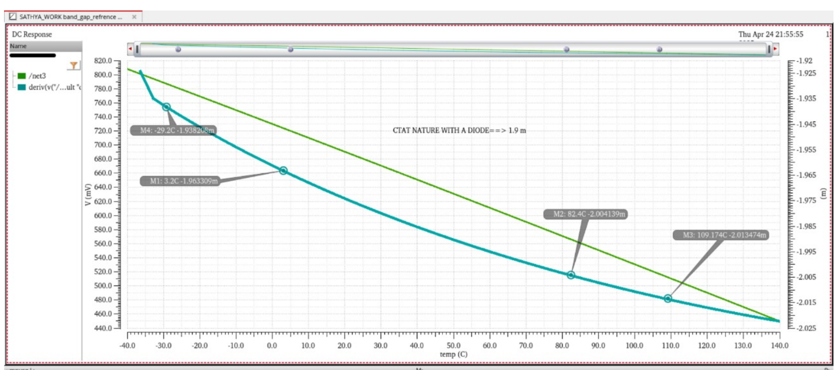

Figure 2: CTAT NATURE

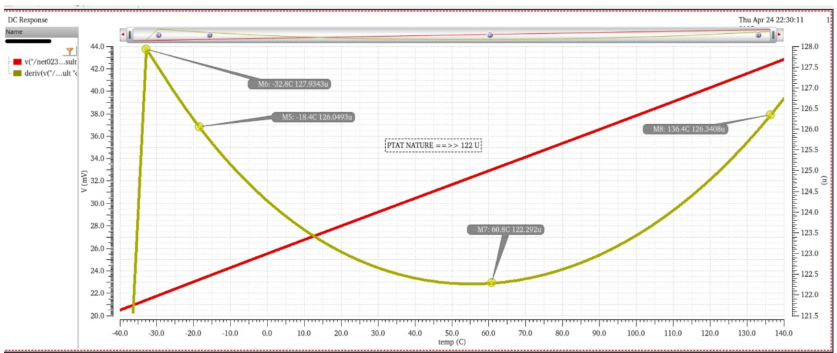

Figure 3: PTAT NATURE

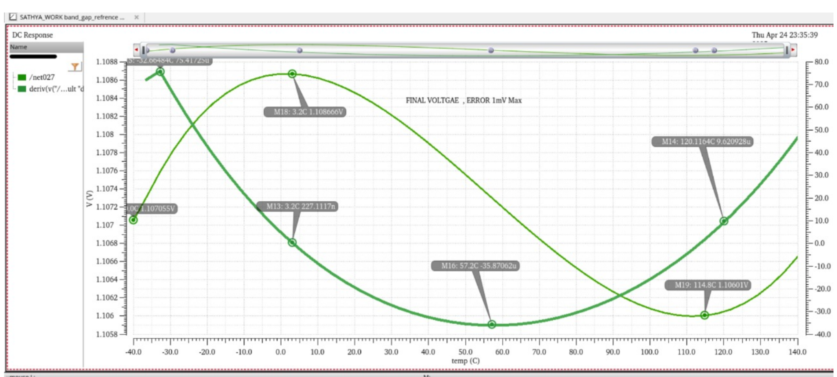

Figure 4: Final Result