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Fundamental Algorithms - Mid term

Q1.] Asymptotics

a) a) $\log(n^9 + 37) \Rightarrow O(\log n)$ [Ignoring constants]

b) $\dfrac{n^2+2}{1+n^4 \cdot 2^{-n}} \Rightarrow \dfrac{2^n n^2}{2^n+n^4} + \dfrac{2^n \cdot 2}{2^n+n^4} \Rightarrow O(n^2)$

c) $(\log\sqrt{n}) \cdot \log(37n^2+45) \Rightarrow O((\log n)^2)$

d) $3^n + (\log n)^{6\log n} \Rightarrow O(3^n)$

e) $10^{207} n + \log n! \Rightarrow O(n \log n)$ [$\because \log n! = n \log n$]

f) $\dfrac{3^n - 1000}{2^n+1} \Rightarrow \dfrac{3^n}{2^n+1} - \dfrac{1000}{2^n+1} \Rightarrow O(1.5^n)$

g) $8^{(n+1)/3} = 8^{n/3} \cdot 8 \Rightarrow O(8^n)$

h) $4^{2n} + 1000 = (4^2)^n + 1000 = O(16^n)$

b) Ranking functions in increasing order of growth.

$O(\log n) < O((\log n)^2) < O(n \log n) < O(n^2) < O(1.5^n) < O(3^n) < O(8^n) < O(16^n)$

Q2.) Recurrences

a) 1. $a = 1$, $b = 32/31$, $f(n) = O(\log n)$, $g(n) = n^{\log_b a} = n^{\log_{32/31} 1} = n^0 = 1$

Even though $O(\log n)$ is greater than $O(1)$, it is not polynomially greater. ~~If~~ It falls between case 2 and case 3.

$\therefore$ Master Theorem is not applicable for $T(n) = T(31n/32) + \log n$

2. $a = 4$, $b = 2$, $f(n) = n^2$, $g(n) = n^{\log_b a} = n^{\log_2 2^2} = n^2$

This is case 2

$\therefore$ $\cancel{T(n) = \Theta(\log f(n)) = \Theta(\log n^2) = \Theta(\log n)}$

$T(n) = \Theta(f(n) \log n)$

$= \Theta(n^2 \log n)$ for $T(n) = 4T(n/2) + n^2$

3. $a = 9$, $b = 8$, $f(n) = \Theta(\log n!) = \Theta(n \log n)$

$g(n) = \Theta(n^{\log_8 9}) = n^{\log_8 9} \approx n^{1+\epsilon}$ where $0 < \epsilon < 1$

$\therefore$ $f(n) = O(g(n)) = O(n^{\log_b a - \epsilon})$

This is case 1

$\therefore$ $T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_8 9})$ for $T(n) = 9T(n/8) + 700 \log(n!)$

4. $T(n) = 8T(n-1) + 35$, $T(0) = 0$

Dividing by 8^n

$$\frac{T(n)}{8^n} = \frac{8T(n-1)}{8^n} + \frac{35}{8^n}$$

$$\Rightarrow \frac{T(n)}{8^n} = \frac{T(n-1)}{8^{n-1}} + \frac{35}{8^n}$$

Q2.] Recurrences

b) $T(n) = \Theta(1)$, $n \leq 5$

$= \frac{2}{3} T(n) + \frac{1}{3} T(n/2)$, $n > 5$

Expected Value :

1. $E(T(n)) = 7 + \frac{2}{3}(1) + \frac{1}{3}(0)$

$= 23/3$

2. When $n = 0$:

$T(n) = \cancel{\frac{1}{3} T(n) +} \frac{1}{3} T(n/2)$

3. When $n = 1$:

$T(n) = \frac{2}{3} T(n)$

4. $\therefore T(n) = \frac{2}{3} T(n) + \frac{1}{3} T(n/2) + \Theta(1)$

$\Rightarrow \frac{1}{3} T(n) = \frac{1}{3} T(n/2) + \Theta(1)$

$\Rightarrow T(n) = 1 T\left(\frac{n}{2}\right) + \Theta(1)$

$a = 1, b = 2, f(n) = \Theta(1)$

5. $g(n) = n^{\log_b a} = n^{\log_2 1} = n^0 = \Theta(1)$

$\therefore$ This is case 2 of ~~mod~~ master theorem.

6. $\therefore T(n) = \Theta(n^{\log_b a} \lg n)$

$T(n) = \Theta(\lg n)$

c) 1. Expected Value $= \frac{2}{3} T(n) + \frac{1}{3} BLA\left(T\left(\frac{n}{2}\right)\right)$

$= \frac{2}{3} T(n) + \frac{1}{3} BLA(0) + 5$

$= \frac{2}{3}(1) + \frac{1}{3}(0) + 5$

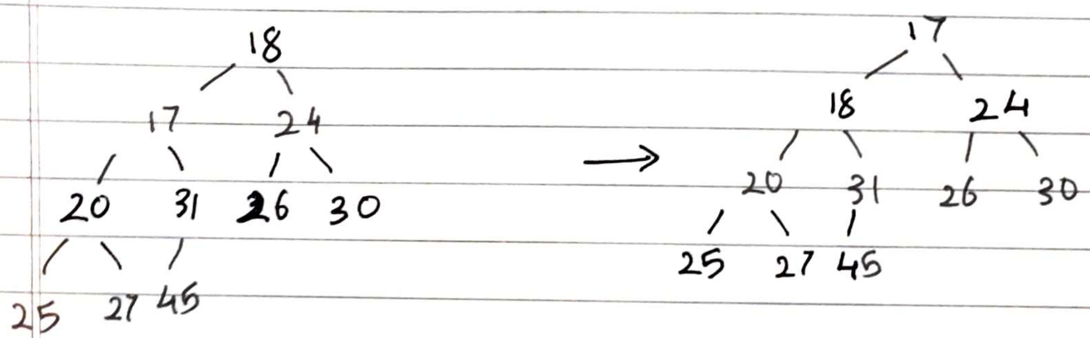
$= 17/3$

2. When $n = 0$, $T(n) = n + \frac{1}{3} BLA(T(n/2)) \Rightarrow S(n) = n + \frac{1}{3} T(S(n/2))$

3. When $n = 1$, $T(n) = n + \frac{2}{3} T(n) \Rightarrow S(n) = n + \frac{2}{3} ~~\$~~ S(n)$

Q3.] Priority Queues and Heaps

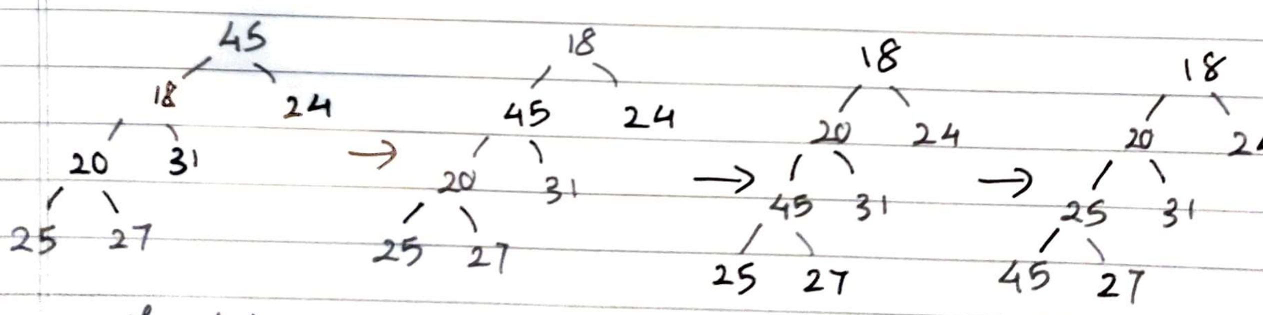
a) A = [18, 17, 24, 20, 31, 26, 30, 25, 27, 45]



A is not a min heap since the root has an element which is greater than one of its children. This violates the min heap property.

This becomes a min heap by replacing 17 with 18.

b) Extract Min: 17 (Root will be extracted)



~~Final Heap~~:

(Final Heap)

c) Max-Increase-Key (A, i, key, n)
Max-Extract-Min (A, i)
Min-Heap-Insert (A, i, key)
Min-Heapify (A, n)

Q4.] <u>Sorting and Order Statistics.</u>

a) $$\sum_{j=n/3}^{2n/3-1} \text{Rank}(j)$$

FindRank(A, i)

1. Convert each element to list which holds value and index
2. RadixSort(A)
3. Loop through sorted list
4. If original index lies within given range [n/3 to 2n/3 − 1], then return Sum upto current index.

b) Running time of Counting Sort ⇒ $O(n+i)$ Where i ← # comparisons.

c) Running time of Heap Sort ⇒ $O(n \log i)$

d) Running time of Insertion Sort ⇒ $O(n^2)$ Independent of i

e) The number of passes of Counting Sort ~~is~~ will be n when Radix sort is using base n

Time for each of these passes ~~is~~ is n^n

Q5.] a) **Binary Trees**

(i) → Preorder walk : Root - Left - Right

⇒ 100, 25, 98, 28, 50, 40, 20, 91

→ Inorder walk: Left - Root - Right

⇒ 98, 25, 28, 50, 100, 20, 40, 91

→ Postorder walk: Left - Right - Root.

⇒ 98, 50, 28, 25, 20, 91, 40, 100

(ii)

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if isLeaf(v) then
    v.win-los = 1
else
    v.win_los = 0
if ~~not~~ v.left then
    ~~return v.left Compute~~
    v = v.right
    v.win-los = Compute-Win(v)
if Not v.right then
    v = v.left
    v.win_los = Compute-Win(v)
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$T(n) = 0 \quad , n = 0$

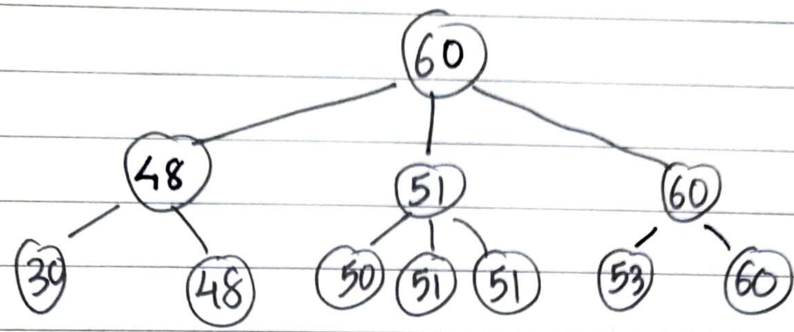
$\quad\;\; = 1 \quad , n = 1$

$T(n) = 2T(n/2) + O(1)$

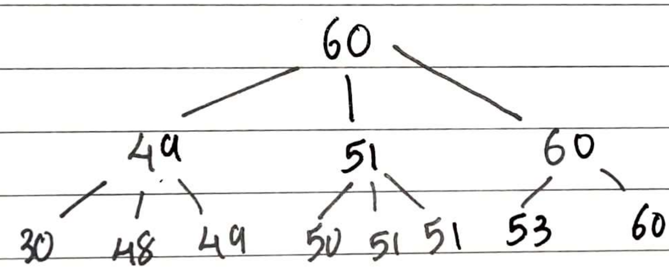
This solves to $T(n) = \cancel{O(n)}\ O(n)$ [By Master Theorem]

Q5.] b) <u>2-3 Trees</u>

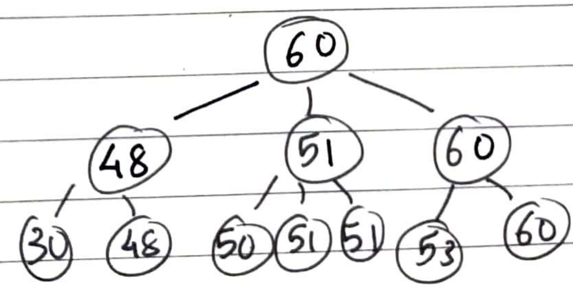
(i)



(ii) Insert 49



(iii) Delete 49



Q6.] Lower Bound and Divide and Conquer.

a) i. if low $>=$ high then
ii. return high
iii. mid = $\lfloor (low + high)/2 \rfloor$
iv. if A[mid] < y
v. return Modified-Bin-Search(A, y, mid+1, high)
vi else return Modified-Bin-Search(A, y, low, mid-1)

Invocation Call : Modified-Binary-Search(A, y, 1, n)
Worst case Running time $\Rightarrow O(\log n)$

b) $n = 6$

Compare y and A(i) — 1 2 3 — Compare y and A[k]

Compare y and A[j]