CSCI-GA.1170-001 Midterm Slot 2

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TOTAL POINTS

79 / 99

QUESTION 1

1 Problem 1: Asymptotics 10 / 12

(a)

✓ **- 1 pts** *(iii) - incorrect - $$\Theta(4^n)$$*

✓ **- 1 pts** *(vii) - incorrect - $$\Theta(\sqrt[3]{n}\log^2 n)$$*

QUESTION 2

2 Problem 2: Recurrence Relations 18 / 19

Recursion Tree

✓ **- 2 pts** *4 Blanks Incorrect*

Domain Range

✓ **+ 1 pts** *Correct Domain Step*

✓ **+ 1 pts** *Correct Range Step*

Master Theorem - Problem 2

✓ **- 0.5 pts** *Incorrect $$f_{magic}$$*

✓ **- 0.5 pts** *Incorrect solution*

QUESTION 3

3 Problem 3: Priority Queues 12 / 11

Part (d)

✓ **+ 1 pts** *Only leaves need to be searched*

QUESTION 4

4 Problem 4: Sorting and Order Statistics 8 / 17

e)

✓ **- 1 pts** *Incorrect Number of Passes /Not Attempted - $$\sqrt n$$*

✓ **- 1 pts** *Incorrect Total Time / Not Attempted - $$\Theta(n\sqrt n)$$*

f)

✓ **- 1 pts** *Incorrect Num of Passes / Not Attempted - $$\log_{\sqrt n}(n^{\sqrt n})=2\sqrt n$$*

✓ **- 4 pts** *(h) Incorrect / Not attempted*

✓ **- 2 pts** *(i) Incorrect / Not Attempted*

QUESTION 5

5 Problem 5: Trees 22 / 27

✓ **- 2 pts** *incorrect/missing solution to f*

✓ **- 4 pts** *incorrect/missing solution to g*

+ 1 *Point adjustment*

QUESTION 6

6 Problem 6: Lower Bound, Divide and Conquer 9 / 13

Part a

✓ **- 1 pts** ***5**: $$A[mid] \le A[mid + 1]$$ or $$A[mid] \le A[mid - 1]$$*

✓ **- 0.5 pts** *Missing or Incorrect Invocation Call*

Part b

✓ **- 1.5 pts** *Incorrect no of nodes in decision tree*

✓ **- 1 pts** *Incorrect labels for decision tree*

Part c

✓ **- 0 pts** *Correct*

1.

a) $\log\left(\dfrac{144 \cdot n^6 - 15}{72 \cdot n^6 + 15}\right)$

$\simeq \dfrac{2 n^6}{n^6} \simeq 1$

$\theta(1)$

(b) $\left(\frac{1}{3}\right)^{3n} + \left(\frac{1}{3}\right)^3 \cdot \sqrt[3]{209 \cdot n}$

$\simeq \left(\frac{1}{3}\right)^{3n} + \sqrt[3]{n}$

$\simeq \theta(\sqrt[3]{n})$

(c) $8^{(10n+5)/15}$

$\simeq 8^n \simeq \theta(8^n)$

(d) $3^{\log_3 n} + \sqrt[3]{n}$

$= n^{\log_3 3} + \sqrt[3]{n}$ (using property of log.)

$\simeq n + \sqrt[3]{n} \simeq \theta(n)$

(e) $n^{0.3} + 294(\log_5 n)^{991}$

$n^{3/10} + (\log_5 n)^{991}$

$\simeq \theta\, n^{3/10}$

(f) $\dfrac{n^4}{1150} + 1150 n^3$

$\simeq n^4 \simeq \theta\, n^4$

(g) $210 \cdot \sqrt[3]{n}\,(\log n)^2$

$\simeq \sqrt[3]{n}\; 2(\log n)$

$\simeq \sqrt[3]{n} \log n = \theta\, \sqrt[3]{n} \log n$

(h) $\sqrt[3]{42n^3 + 92n^9}$.

$\simeq \sqrt[3]{n^3 + n^9}$.

$\simeq n + n^3$.

$\simeq \theta(n^3)$.

(b) order of asymtotic growth

$\theta(1)$, $\sqrt[3]{n}$, 8^n, n, $n^{3/10}$, n^3, $\sqrt[3]{n}\log n$, n^3.

a, e, b, g, d, h, f, c.

1 Problem 1: Asymptotics **10 / 12**

(a)

✓ **- 1 pts** *(iii) - incorrect -* $\Theta(4^n)$

✓ **- 1 pts** *(vii) - incorrect -* $\Theta(\sqrt[3]{n}\log^2 n)$

2(a)

1. $T(n) = 36 \cdot T(n/6) + 991 n \log(n!)$

$a = 36$

$b = 6$

$f(n) = \theta(n^2 \log n)$

$f_{magic} = n^2$

Case 2.

$T(n) = \theta(n^2 \log n)$

2. $T(n) = 216\, T\left(\frac{n}{36}\right) + \sqrt{n} \cdot (\log n)^{\log \log n}$

$a = 216$

$b = 36$

$f(n) = \theta \sqrt{n}$

$f_{magic} = n^3$

Case 1.

$T(n) = \theta(n^3)$

3. $9T(n/3) + 3^{\log_3 n} \cdot n$

$a = 9$

$b = 3$

$f(n) = \theta\, n^2$

$f_{magic} = n^2$

Case 2.

$T(n) = \theta(n^2 \log n)$

2. C

2. (h)

$$T(n) = 9T\left(\frac{n}{3}\right) + 2n \;; \quad T(1) = 11.$$

Level.	Size of Problem	Number of Problem.	Non recursive cost for One Prob	Total
0	n	1	$2n.$	$2n$
:	$\frac{n}{3^i}$	9^i	$2\left(\frac{n}{3^i}\right)$	$3^i\, 2n.$
	$\frac{n}{3^{d-1}}$	9^{d-1}	$2\left(\frac{n}{3^{d-1}}\right)$	$3^{d-1}\, 2n.$
	$\frac{n}{3^d}$	9^d	$2\left(\frac{n}{3^d}\right)$	$3^d\, 2n.$

$$d = \log_3 n.$$

$$T n = 3^d 2n + \sum_{i=0}^{d-1} 3^i\, 2n.$$

$$T(n) = \Theta(n^2)$$

2. c.

$T(n) = 9T\left(\frac{n}{3}\right) + 2n. : \quad T(1) = 5$

Let $= n = 3^k$.

$T(3^k) = 9T(3^{k-1}) + 2(3^k)$

Let $S(k) = T(3^k)$ $\begin{cases} k = 0 \\ S(0) = 5 \end{cases}$

$S(k) = 9(S(k-1)) + 2(3)^k$

divide by 9.

$P(k) = \frac{2}{3^k} + P(k-1)$

$= \frac{2}{3^k} + \frac{2}{3^{k-1}} + P(k-2)$

$= \frac{2}{3^k} + \frac{2}{3^{k-1}} \cdots P(1) + P(0)$

$= 2\left[\frac{1}{3^k} \cdots \frac{1}{3}\right] + 5$

$= 2\left[\frac{(1/3)^k - 1}{1/3 - 1}\right] 5$

$\frac{1}{3}$ - from the above Sum of G P.

2 Problem 2: Recurrence Relations 18 / 19

Recursion Tree

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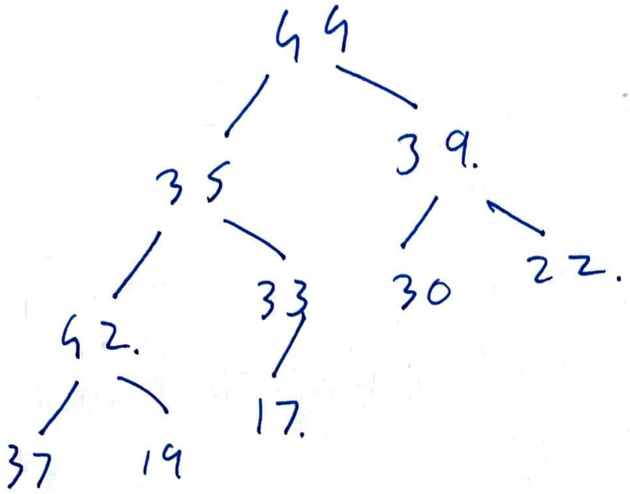
Master Theorem - Problem 2

✓ **- 0.5 pts** *Incorrect $$f_{magic}$$*

✓ **- 0.5 pts** *Incorrect solution*

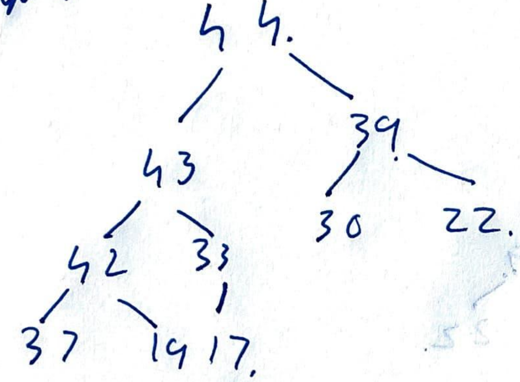
Q3)

(a).

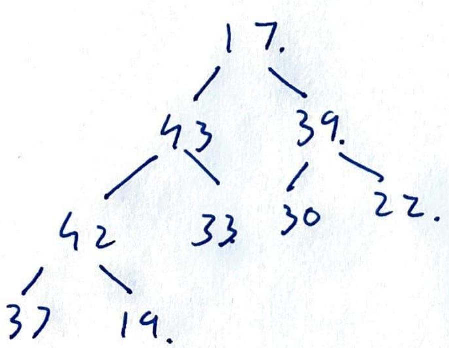


This is not a max heap as 35 can't be parent of 42.

The max heap is :

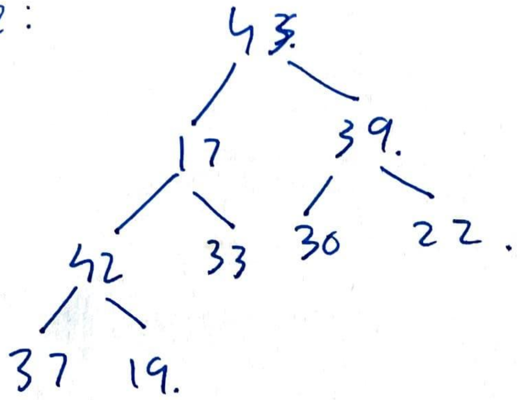


(b) Step 1.

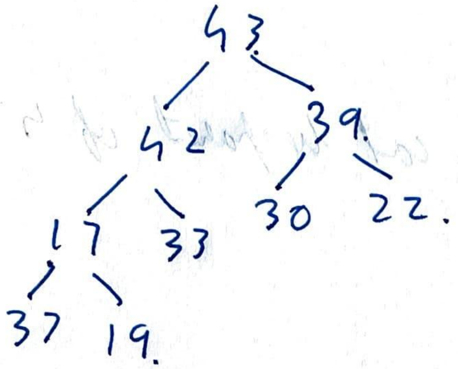


... algorithm is iterating only over the indices

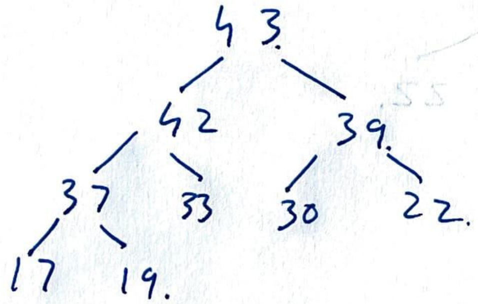
Step 2:



Step 3.



Step 4.



(f)

(2) $\left[\frac{n}{2}\right] + 1$.

(3) n.

(4) $max > A[i]$

(5) $max = A[i]$

(6) Max.

(d) Since our algorithm is iterating only over the indices from $\frac{n}{2}+1$ to n which are the leaf nodes of heap. So the iteration will be $n - \left\lfloor \frac{n}{2} \right\rfloor = \frac{n}{2}$ which will be the worst number of comparison.

So atmost it make $\frac{n}{2}$ comparison.

3 Problem 3: Priority Queues **12 / 11**

Part (d)

✓ **+ 1 pts** *Only leaves need to be searched*

Q4.

(a) Maximum bits · be

$\log_2 (n^{\sqrt{n}}) = \sqrt{n} \log_2 n$.

(b) $\theta(n^{\sqrt{n}})$.

(c) $n(\log n)$

(d) $O(n^2)$

(e) Number of Passes of Counting Sort $= n$.

Time taken for each of these Passes.

$$= \theta(n + \log_n n^{\sqrt{n}}) = \theta(n + \sqrt{n})$$
$$= O(n)$$

∴ Total time $= n\theta(n) = \theta(n^2)$

(f) Number of Passes. $= \sqrt{n}$.

Time taken $= \theta(n)$

Total Time $= O(n\sqrt{n}) = O\ n^{3/2}$.

(g) Merge Sort

4 Problem 4: Sorting and Order Statistics **8 / 17**

e)

✓ **- 1 pts** *Incorrect Number of Passes /Not Attempted - $$\sqrt n$$*

✓ **- 1 pts** *Incorrect Total Time / Not Attempted - $$\Theta(n\sqrt n)$$*

f)

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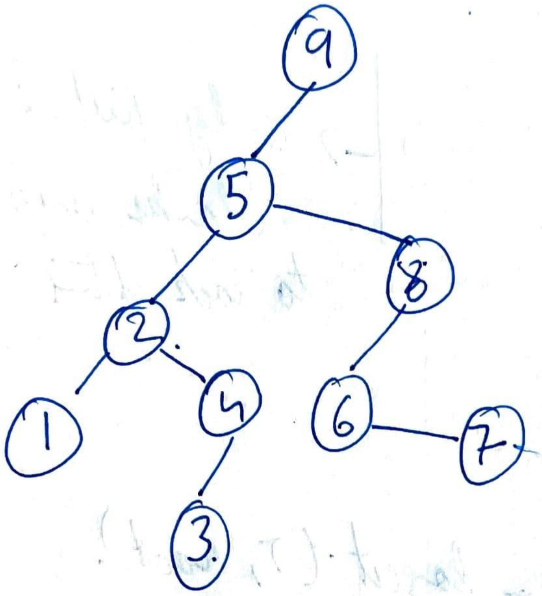
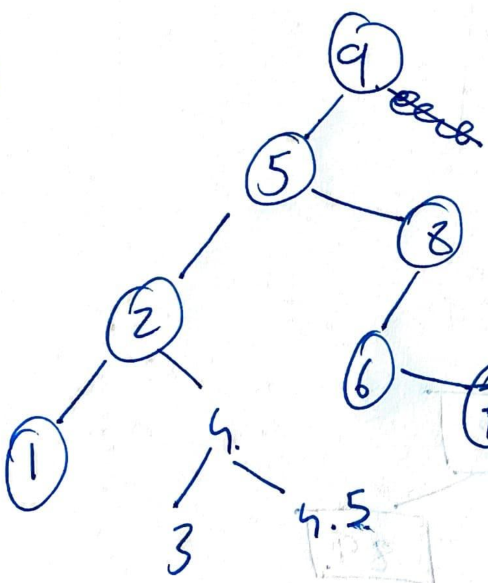
5.

(a)

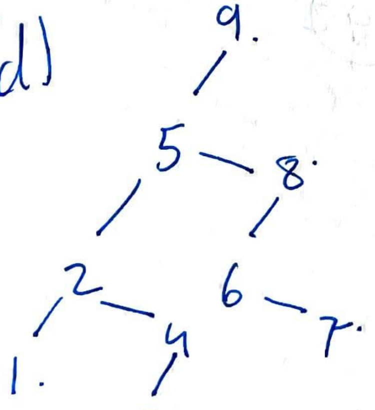


(b) 9, 5, 2, 1, 4, 3, 8, 6, 7.

(c)



(d)



yes., T. & T'' are same.

1.

(e). (1) v.dr. == v.left.dr.

(2) v.left.longest = v.longest +1.

(3) compute_longest (T, v.left).

2. (4) v.dr == v.right.dr.

(5) v.right.longest = v.longest +1.

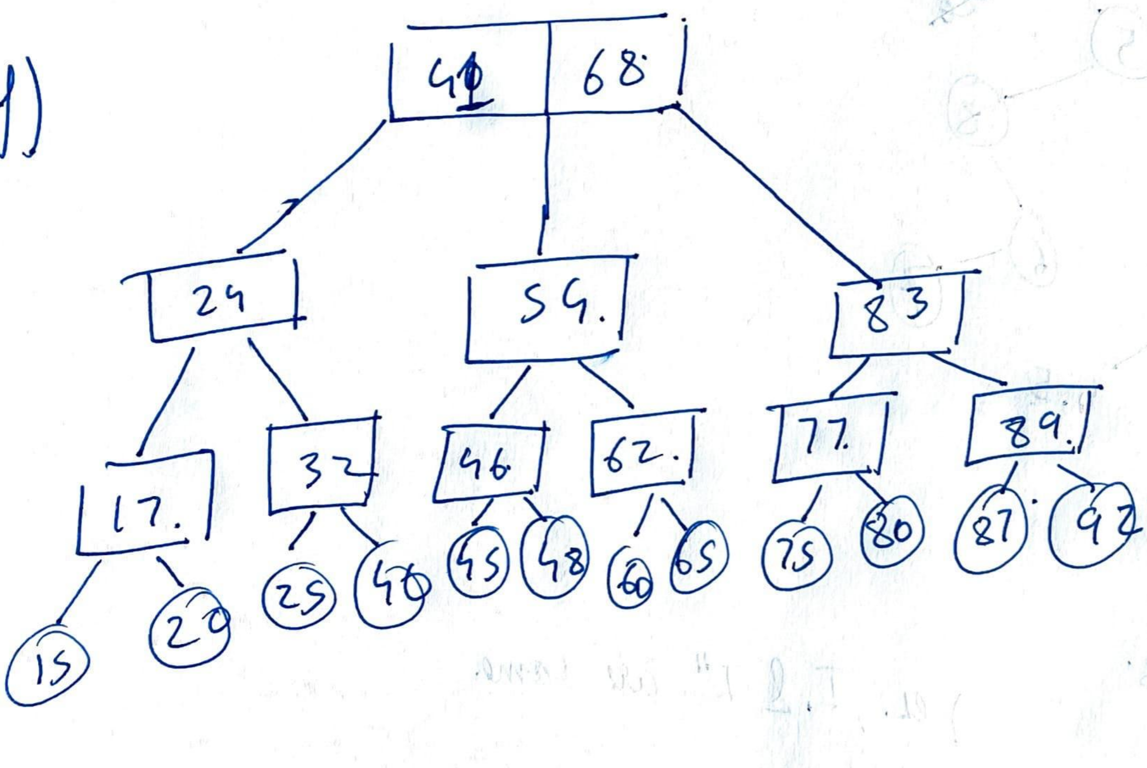
(6) compute_longest (T. v.right)

→ only fill in the blanks answers due to lack of time.

Initial/Invocation call is compute_longest (T, root).

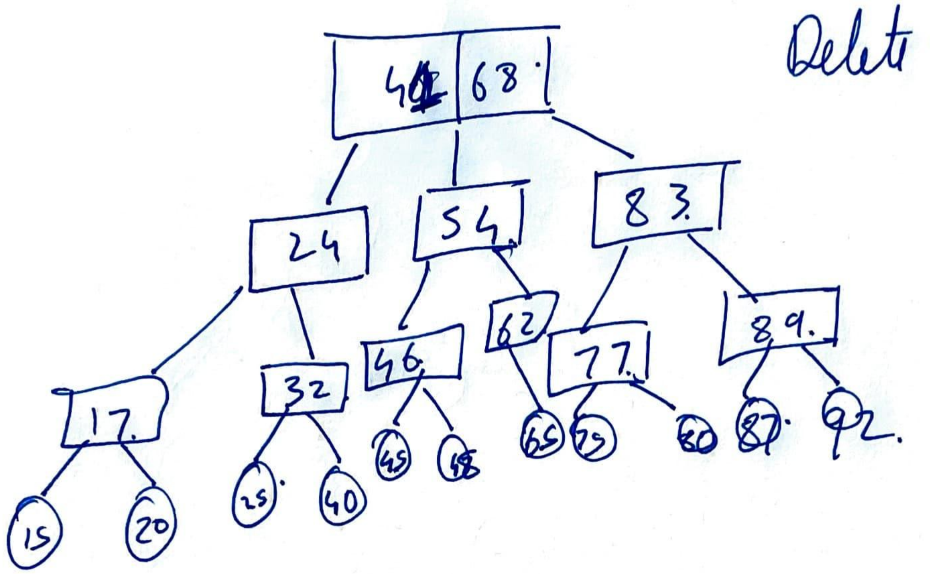
Worst case runtime = O(n) which will happen when the tree is a skewed tree.

(f)



(g)

Delete 60.



5 Problem 5: Trees **22 / 27**

✓ **- 2 pts** *incorrect/missing solution to f*

✓ **- 4 pts** *incorrect/missing solution to g*

+ 1 *Point adjustment*

Q6

a (1) P.

(2) A[P+1] > A[P]

(3) P.

(4) P+1)

(5) A[P] < A.[mid]

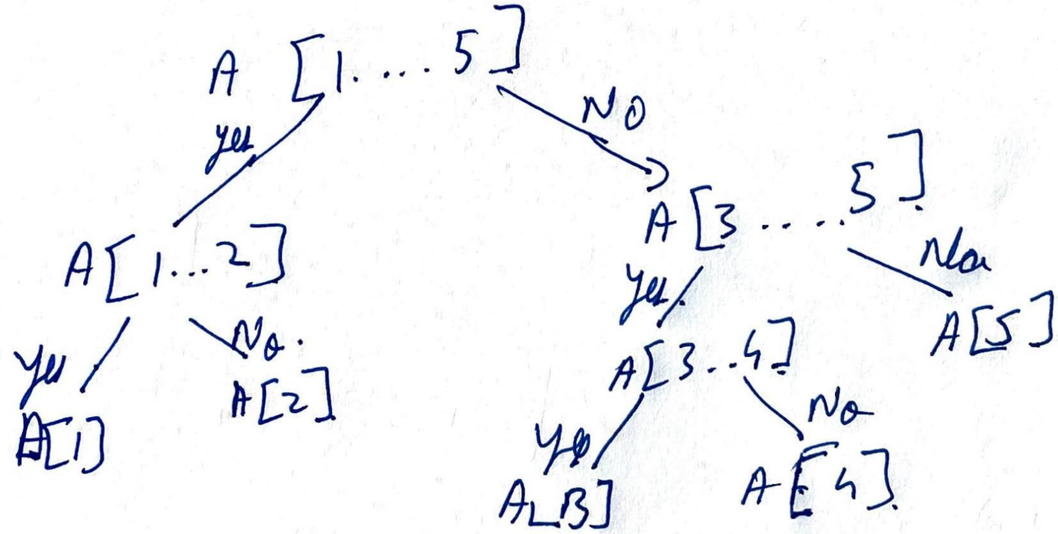
(6) Find Vallly. (A, P, mid)

(7) Find Valley (A; mid+1, r).

invocation Call -> Find Vally (A, P, mid).

Worst call Run time -> O(log n).

6c. (c)



(c) The height of the decision tree will be log n.

So it takes O(log n) time to find valley.

This is the optimal algorithm & since it ends so every vertex is traversed.

We have to make a decision to go which way.

6 Problem 6: Lower Bound, Divide and Conquer 9 / 13

Part a

✓ **- 1 pts** ***5**: $$A[mid] \le A[mid + 1]$$ or $$A[mid] \le A[mid - 1]$$*

✓ **- 0.5 pts** *Missing or Incorrect Invocation Call*

Part b

✓ **- 1.5 pts** *Incorrect no of nodes in decision tree*

✓ **- 1 pts** *Incorrect labels for decision tree*

Part c

✓ **- 0 pts** *Correct*