Q. 1.

a. $f(n) = (n^n)^3$; $g(n) = n^{(n^3)}$

$f_n = n^{3n}$. $g_n = n^{n^3}$

Taking log both sides.

$f(n) = 3n \log n$. $g(n) = n^3 \log n$.

$$\lim_{n \to 0} \frac{f_n}{g_n} = \frac{3 \cancel{n} \log n}{n^3 \log n} = \frac{3}{n^2} = \underline{\underline{0}}.$$

Since $\lim_{n \to 0} \frac{f_n}{g_n} = 0$. (We can Conclude)

Answer.

$\boxed{① \ f(n) = o\, g(n) \qquad 2.\ f(n) = O\, g(n) \ \text{(Little \& Big Oh.)}}$

b) $f(n) = n!$; $g(n) = (n+1)!$

$f(n) = 1 \times 2 \times 3 \ldots \times (n-1) n$; $g(n) = 1 \times 2 \times 3 \times \ldots \times (n-1) n (n+1)$

$f(n) \approx n^n$. ; $g(n) \approx n^{n+1}$

$$\lim_{n \to 0} \frac{f(n)}{g(n)} = \frac{n^n}{n^{n+1}} = n^{(n-n-1)} = \frac{1}{n} = 0.$$

Answer.

We Can Conclude the following.

$\boxed{① \ f(n) = o\, g(n) \ \& \ ② \ f(n) = O\, g(n) \ \text{(Little \& big Oh.)}}$

c. $f(n) = n^{0.99} + 15(\log_2 n)^{100}$; $g(n) = 4^{\log_{16} n^2}$

$f(n) = n^{0.99} + 15(\log_2 n)^{100}$; $g(n) = n^{2 \cdot \log_{16} 4} = n^{2 \times 1/2} = n$.

$g(n) = n$.

$\boxed{\text{Assymtotically equal}} = f(n)\ \Theta\ g(n)$.

Alternatively we can take log both sides

$f(n) = 0.99 \underbrace{\log n} + 1500 \log\log n$; $g(n) = \underbrace{\log n}$;

We $\therefore$ Conclude that
$$\boxed{\begin{array}{l} (1)\ f(n) = \Theta\ g(n) \\ (2)\ f(n) = O\ g(n) \\ (3)\ f(n) = \Omega\ g(n) \end{array}}$$
$\leftarrow$ Answer.

4. $f(n) = n^{0.000001}$; $g(n) = (\log n)^{1000001}$.

Taking logs both the sides

$f(n) = 0.000001 \underline{\log n}$; $g(n) = 1000001 \underline{\log \log n}$

$f(n) \geq g(n)$.

We can Conclude
$$\boxed{\begin{array}{l} (1)\ f(n) = \Omega\ g(n) \\ (2)\ f(n) = \omega\ g(n). \end{array}}$$
$\leftarrow$ Answer.

5. $f(n) = n^{5/\log_2 n}$; $g(n) = 10000$

Take logs both sides

$f(n)\ \frac{5}{\cancel{\log_2 n}} \times \cancel{\log n}$; $g(n) = \log 10000$.

$f(n) = 5$; $g(n) = \log 10000$.

$\therefore$ Assymtotically equal.

$$f(n) \approx g(n)$$

$$\boxed{\begin{array}{l} ① \ f(n) \ \Theta \ g(n) \\ ② \ f(n) \ O \ g(n) \\ ③ \ f(n) \ \Omega . g(n). \end{array}} \longrightarrow \text{Answer.}$$

Problem 1-2.

1 $\dfrac{n^2 + 2}{1 + n^3 . 2^{-n}} \approx \dfrac{n^2}{n^3 \, 2^{-n}} \approx \dfrac{2^n}{n} \approx \Theta(2^n)$ ⟵ Answer.

2. $\log(\sqrt{\log(\sqrt[3]{n})}) \cdot \log (37n^3 + 45) = \log_2 n^{1/3} \log_2 n^3$

$\approx \frac{1}{3} \times 3 \times \log n \times \log n = (\log n)^2 \approx \Theta(\log n)^2$ ⟵ Answer.

3. $\log(n!) + 10^{205} n \approx \log (n^n) + n \approx n \log n + n$

$\approx \Theta(n \log n)$ ⟵ Answer.

4. $\dfrac{6^n - 1000}{2^n + 1} \approx \dfrac{6^n}{2^n} \approx 3^n \approx \Theta(3^n)$ ⟵ Answer.

5. $n^{2/\log n} + 1000 \approx n^{2 \cdot 1/\log n} \approx$ [illegible]

$\approx \Theta(e)$ ⟵ Answer.

6. $2^{n + \log n} \approx 2^n \cdot 2^{\log_2 n} \approx 2^n \cdot n^{\log_2 2}$

2^n

$\approx 2^n \times n \approx \Theta \, n \, 2^n$ ⟵ Answer.

7. $2^n + 10(\log n)^{\log n} \simeq 2^n = \Theta(2^n) \leftarrow$ Answer.

8. $2^{3n} + 4^n \simeq 2^{3n} + 2^{(n+1)} \simeq 2^{3n} + 2^n$

$\simeq \Theta(2^{3n}) \leftarrow$ Answer.

b) Functions for asymptotic growth. (least to maximum.)

~~$e, (\log n)^2, n \log n, 2^n, 2^n, n2^n, 3^n,$~~

$\underline{e, (\log n)^2, n \log n, 2^n, 2^n, n2^n, 2^{3n}, 3^n}$