Design of a trijunction of Majorana nanowires

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by

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Background

1.1. Introduction

MBS appear as the zero-energy modes of a hybrid quasi-one dimensional system that combines a strong-spin orbit semiconductor with proximity induced superconductivity. Semiconducting nanowires and two-dimensional electron gases (2DEG) are candidates for creating such devices, yet no evidence of such excitations has been found. Nevertheless, the existence of MBS would allow us to design new qubits that are resilient to noise in contrast to current devices.

While coupling a single MBS pair can be done using a quantum dot, selective coupling of multiple pairs remains a challenge. In the presence of multiple pairs, coupling MBS from different fermions induces a non-trivial evolution of the ground state that supports quantum gate operation. Coupling of a pair of MBS in a S-N-S junction has been extensively studied, and the fractional Josephson effect has been found as a signature of MBS present in such system. On the other hand, coupling multiple MBS pairs remains a challenge given the constrains on the nanowires alignment and separation.

The simplest system where multiple MBS can couple non-trivially is in a trijunction geometry. *In this thesis we propose a semiconducting cavity connected to three Majorana nanowires that allows for an all-electric controlled interaction between all pairs of MBS.*

Initially, the role of geometry is investigated by simulating several cavity geometries and extracting the MBS coupling in the strong coupling regime. It is found that different cavity levels mediate differently the coupling of different MBS pairs. We found that there is an angle for a triangular cavity that induces a maximum coupling between the far MBS pairs. *Several cavity geometries are analysed, and a triangular cavity with varying angle is found to have the largest coupling for all pairs.*

Finally, a realistic model is studied via electrostatic simulations of the triangular cavity with optimal configuration defined on a 2DEG. The non-local nature of the gates makes the nanowire positions crucial in order to recover the effects found for the purely geometric case. The role of each set of gates and the range of voltages used to operate the device are discussed. *The electrostatics effects of the gate-defined triangular cavity are analysed and the operational point is described.*

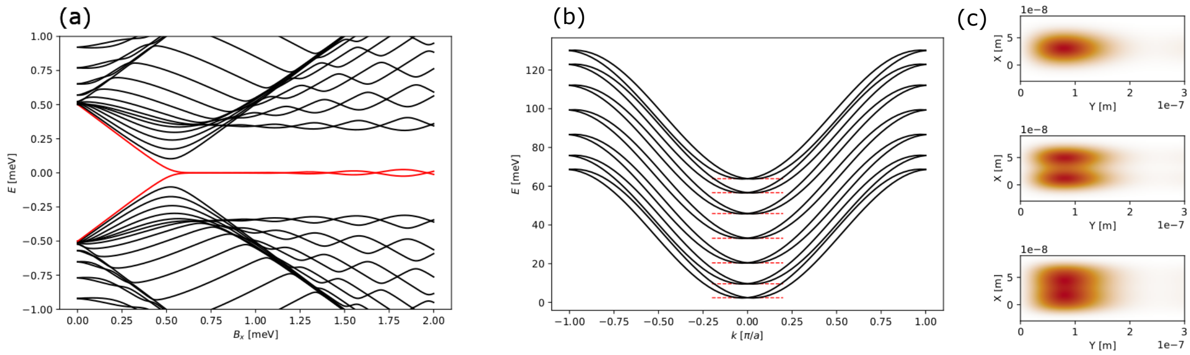

Figure 1.1: Simulations of a Majorana nanowire as described in Eq. (1.4). The following parameters will be used in all simulations: $\Delta = 0.5$[meV], $\alpha = 0.3$[eV A], and $t = \hbar^2/2m^*$ where $m^* = 0.023m_e$. Each simulated nanowire has length $L = 130 * a$ and width $W = 7 * a$ where $a = 10$[nm] is the lattice constant size. (a) Topological phase transition as a function of Zeeman field B_x. Majorana zero-energy state (red) sticks to zero after crossing the critical field. (b) Transverse bands along the translational invariant direction with $\Delta = 0$. The chemical potential is tunned to the bottom of each band (red dashed lines) to create Majoranas. (c) Majorana wavefunctions for the lowest three bands.

1.2. Majorana bound states

MBS emerge as the non-local degenerate ground state of a topological superconductor. Under the appropriate conditions, a spinless one-dimensional p-wave superconductor contains two zero-energy excitations that are exponentially localised at the edges of the system. Together, these two zero-energy modes encode a single fermionic mode that can be empty or occupied,

$$f = \frac{\gamma_L + i\gamma_R}{\sqrt{2}}, \quad f^\dagger = \frac{\gamma_L - i\gamma_R}{\sqrt{2}}, \tag{1.1}$$

where $\gamma_i = \gamma_i^\dagger$ are Majorana operators that $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$.

Since a pair of spatially separated MBS encode a single fermionic mode, its quantum state is protected against local errors by particle-hole symmetry. In a sufficiently large nanowire, MBS are completely decoupled from each other, and noise sources will interact with each of them individually. The interaction with a single MBS is proportional to a single Majorana operator, i.e. $\gamma \sim f + f^\dagger$, and thus it will change the parity of the system. In a superconductor, however, electrons can only enter or leave as Cooper pairs, which means that the parity is conserved. Therefore, individual MBS are immune to local noise sources.

The ground state can only be controlled by non-local operations that involve pairs of MBS. Given parity conservation, only even powers of Majorana operators are allowed in the Hamiltonian. The simplest allowed term describes the coupling of a pair of MBS, and it is given by

$$H_{pair} = iE_{LR}\gamma_L\gamma_R = E_{LR}(1 - 2f^\dagger f). \tag{1.2}$$

Here, E_{LR} is the tunnelling coupling between the two MBS, and it is usually cancelled in sufficiently long nanowires.

In the presence of multiple MBS pairs, each parity subspace can be used as a computational subspace where quantum information is protected. *By controlling the coupling between different MBS pairs, one can control the ground state evolution*, that is,

$$|\Psi\rangle \to U(t)|\Psi\rangle, \quad U(t) = \exp\big(iH_{pair}t\big). \tag{1.3}$$

1.3. Experimental platforms

MBS can be realised in quasi one-dimensional systems defined on two-dimensional electron gases (2DEGs), or semiconducting nanowires, with strong spin-orbit and in proximity to a superconductor. The Hamiltonian that realises a Majorana nanowire is,

$$\mathcal{H} = \sum_k \Psi_k^\dagger H(k) \Psi_k, \quad H(k) = \left[\frac{|\mathbf{k}|^2}{2m^*} - \mu + \alpha(k_x\sigma_y - k_y\sigma_x) \right] \tau_z + B_x\sigma_x + \Delta\tau_x. \tag{1.4}$$

Here, $\Psi_k^\dagger = (f_{k\uparrow}^\dagger, f_{k\downarrow}^\dagger, f_{k\uparrow} f_{k\downarrow})^T$ are the Nambu spinors in k space, μ is the chemical potential, $\mathbf{k}$ is the 2D wave-vector, α is the spin orbit interaction, B_x is the Zeeman field, Δ is the superconducting gap, and σ and τ are Pauli matrices for the spin and particle-hole basis.

MBS appear as zero energy excitations of this Hamiltonian when $B_x^2 \geq \sqrt{\mu^2 + \Delta^2}$. However, *realising such material combination is difficult, and MBS transport signatures are not unambiguous.* On the one hand, these interaction destroy each other mutually as is the case of superconductivity and magnetic fields. On the other hand, MBS signatures can be reproduced by states localised in material defects or impurities. Therefore, highly tunable devices with low impurities and disorder are required to unambiguously detect MBS.

MBS can appear in different nanowire sub-bands. In a quasi-one dimensional systems, there is a translational invariant direction, and a direction with finite width W. The energy of each mode has a contribution from both, and it is given by,

$$E_n(k) = \frac{\hbar^2}{2m^*} \left(k^2 + \frac{\pi^2 n^2}{W^2} \right). \tag{1.5}$$

Here, m^* is the effective mass and the spin orbit splitting is not considered. Independent MBS with different momentum profiles can be formed at each transverse mode when the chemical potential is at the bottom of the corresponding band. Multiple channel become relevant in the presence of disorder. It couples differently to each momentum sub band, which will induce band mixing as has been suggested in experiments.

1.3.1. Two dimensional electron gases

In a clean system, electrons travel ballistically, and their motion is directly determined by the shape of the system boundaries. *2DEGs allow for arbitrary geometries to be defined in the same layer using different electrostatic gates.* Furthermore, it been shown that geometric dependence can be used to enhance the property of Majorana devices. On the other hand, in semiconducting nanowires networks MBS is limited to narrow transverse channels. Therefore, 2DEGs are an interesting platform to study the role of geometry in MBS coupling with gate defined shapes.

Parallel Majorana nanowires are the basic elements for a complex Majorana device. Multiple nanowires require to be aligned in order to have a stable topological phase. Each Majorana nanowire can be defined on a 2DEG by adding a superconducting strip on top of the selected region. A a top gate is deposited next on top of the device such that depletes the surrounding 2DEG. A narrow quasi-one dimensional channel is created below the superconductor, and it is expected to find MBS at the edges.

Majorana experiments on 2DEGs have shown promising evidence for scalable and complex devices. Initial experiments[4, 7] focused on characterising the properties of semiconducting layers with a superconducting cover. Advances in material growth allowed for clean interfaces with a hard superconducting gap to develop into the nanowire region. Later experiments focused on tunnel spectroscopy of stripe-like geometries[8] where a zero bias peak (ZBP) was found. However, due to disorder and defects such ZBPs have most likely a trivial origin from Andreev states rather than MBS. Nevertheless, efforts to develop scalable devices in 2DEGs are made and new promising materials are being studied.

1.3.2. Electrostatic gates

The electrostatic potential in a 2DEG is found by solving the Poisson equation using a finite elements method on the device geometry. The potential landscape in a 2DEG can be controlled by deposition of metallic gates on a top layer with an insulating barrier in between that smooths the potential profile. The potential landscapce, $U(\mathbf{r})$, for a given geometrical configuration can be found by solving Laplace equation,

$$\nabla \cdot [\varepsilon_r(\mathbf{r}) \nabla U(\mathbf{r})] = 0. \tag{1.6}$$

Here, ε_r is the relative permitivity of each layer in the material stack.

Electrostatic effects play a crucial role in designing and operating Majorana devices. Characterisation of Majorana nanowire is done via transport measurements that require tunnel coupled leads and gates. Furthermore, gates have a non-local effect on the potential landscape that differs between experimental platforms. For example, nanowires have a partial superconducting coating that allows for the electric field to penetrate and control the semiconductor and superconductor weight of the wavefunction. In 2DEGs, on the contrary, the superconducting coat fully covers it, which screens electrostatic effects.

1.4. Majorana bound states in a trijunction

There are two main approaches for MBS quantum computation: braiding and joint parity measurements. Braiding was initially proposed as moving MBS around each other in gate defined nanowire networks[1]. However, this method requires high degree of control and is highly susceptible to thermal errors[5]. On the other hand, joint parity measurements coupling multiple pairs of MBS[6] by using co-tunnelling processes between different MBS on superconducting islands. These methods do not rely on geometrical effects, and are often discussed in terms of a phenomenological Hamiltonian.

In a trijunction, demonstration of the simplest non-trivial Majorana evolution experiment can be done. In order to create a Majorana qubit, three or more MBS with precisely controlled interactions are required. The interaction between different MBS pairs is mediated by the cavity modes, which crucially depends on the nanowires positions and the cavity geometry. By controlling the coupling of each pair via a DC voltage pulse sequence, one determines the evolution of the MBS. Initial studies[3] have shown that MBS can connect via a semiconducting cavity in a fork-like geometry.

However, design and operation of a trijunction are non-trivial tasks. Simultaneous tuning of gate voltages and relative phase difference is required to optimally operate a trijunction. Selection of the MBS pair and cavity modes is realised by electrostatic gates controlling the potential on each region. Furthermore, relative phase differences between MBS modulates the coupling as in the fractional Josephson effect. The phase will be shifted by the presence of complex hopping terms and by the nanowires relative position.

2

Trijunction of Majorana nanowires

In this chapter we demonstrate that the coupling of three pairs of MBS in a trijuction is determined by the geometrical details of the central semiconducting cavity. For certain geometrical configurations, a MBS pair couples resonantly with successive cavity states as in the so-called *resonant trapping*, while in other cases the coupling is mediated by individual non-overlapping levels.

We simulate the pair coupling of three Majorana nanowires mediated by a semiconducting cavity using Kwant. For each experiment, a pair of nanowires is set to host MBS in each sub band while the other nanowire is fully depleted. We consider the strong coupling regime where there are no tunnel barriers between the cavity and the nanowires. Furthermore, the phase difference between the selected nanowires is tuned such that the coupling is at a maximum. *The coupling energy of each pair is extracted as the value of the lowest non-zero eigenvalue with respect to the cavity chemical potential.*

2.1. Quasi-one dimensional cavities

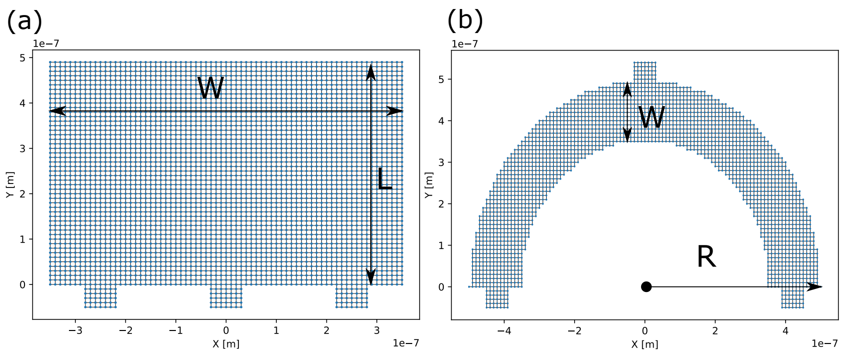

Figure 2.1: Left: Kwant system of a half-ring shaped cavity. It is defined by the radius R and the width W. Thiner rectangular segments represent the positions of the Majorana nanowires attached. Right: Lowest four eigenstates of the cavity with the nanowires fully depleted.

The simplest cavity geometry to study MBS coupling is a stripe-like geometry. The cavity eigenstates have a well-defines spatial structure, i.e. an integer number of nodes along the stripe axis. *We expect interference effects between the cavity state and MBS depending on their positions.*

We consider two quasi-one dimensional cavities: In Fig. fig:1d (a) one can observe a rectangular stripe cavity with three Majorana nanowires attached. In Fig. 2.1 (b) one can observe a half-ring stripe cavity with three Majorana nanowires attached in a fork-like geometry. *The width of each cavity is fixed, and the length and nanowires position is changed.*

Mirror symmetry along the x-axis is imposed in the device. Consequently, the phase shift is symmetric around π for the central pairs as shown in Fig. 2.2 (a). *However, the behaviour is oscillatory, non-trivial, and strongly depends on the geometry.* The left right pair, similarly, has a nontrivial behaviour with a clearer periodicity than the other two pairs. For each experiment, the phase is calibrated such that the coupling is maximum for the corresponding pair.

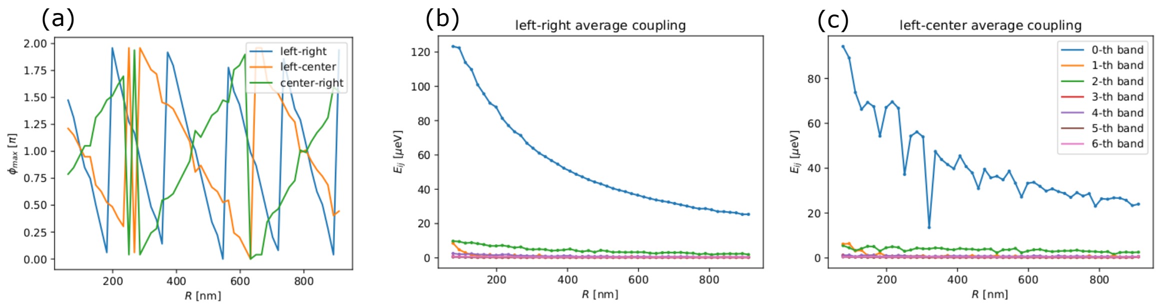

Figure 2.2: Results for half-ring cavity. (a) Phase shift ϕ_{max} vs. cavity radius R. (b) Coupling of left and right MBS pair. Colors show the band index of the nanowire. (c) Coupling of the left and central MBS pair. Remaining pair is not show since the results is the same as (b).

Initially, we study the size dependence of a half-ring shaped cavity as depicted in Fig. 2.2 (b)-(c). For a small cavity, the MBS coupling behaves as in the quantum dot regime where the level spacing is large. *As the system size increase, the coupling spectra evolves from the short-junction limit to the long-junction limit.* The number of states inside the gap increases, and the resonant peaks shrink. For very large systems, the coupling decreases exponentially until it eventually vanishes for all pairs.

The coupling of different lead modes relies on momentum conservation. If there are states with a similar momentum, then tunnel is allowed between the two structures. In this case, the coupling is dominated by the lowest Majorana mode. Therefore, in a quasi-one dimensional cavity there are few high-momentum channels that couple Majoranas in higher bands.

2.1.1. Resonant trapping

Let us consider the half-ring cavity depicted in Fig. .When the left and right nanowires are close to the ends of the cavity, the MBS pair couple along a sequence of overlapping resonant states. *The cavity states interfere constructively and the coupling accumulates creating a single wide peak over the resonant region.* This phenomena has been discussed in two-dimensional cavities when states along the convex border create a band of overlapping resonances with the lead states. It is known as *resonant trapping*. This phenomena is reproduced in the rectangular-stripe geometry with nanowires attached at the far ends, yet the coupling is smaller.

This phenomena depends crucially on having the lead states around all of the cavity wavefunction. For the central pairs coupling, in contrast, a band of resonances cannot

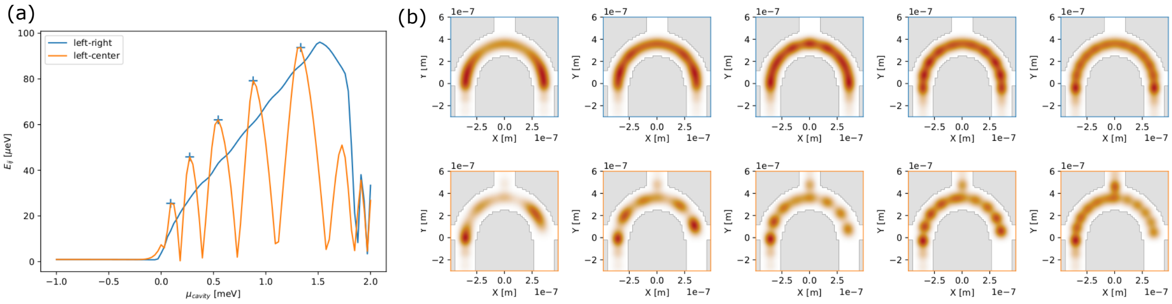

Figure 2.3: Spectra for a half-ring shaped cavity of width $W = 110$ [nm] and radius $R = 300$ [nm]. (a) Coupling of each MBS pair. Center-right pair is the same curve as left-center pair. Crosses indicate the positions at where the wavefunction (b) are taken. The color of the frame in (b) corresponds each MBS pair in (a).

form since the cavity wavefunction is not fully enclosed. When coupling the left and central MBS, the cavity region close to the right lead acts as a particle in a box with multiple individual levels that repel each other. Therefore, we observe the two spectra shown in Fig. where one of them has a resonant band while in other the band is divided in individual resonances.

2.2. Two dimensional cavities

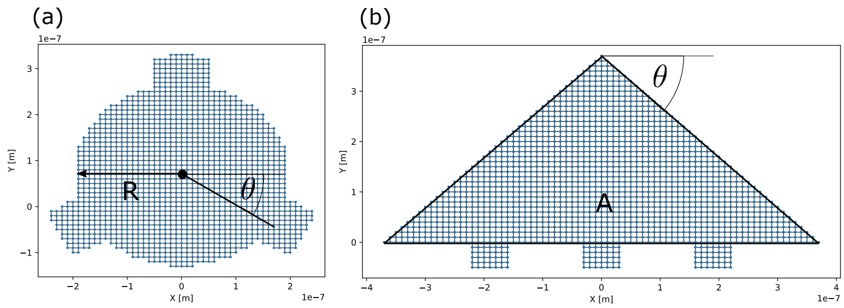

Figure 2.4: Left: Kwant system of a half-ring shaped cavity. It is defined by the radius R and the width W. Thiner rectangular segments represent the positions of the Majorana nanowires attached. Right: Lowest four eigenstates of the cavity with the nanowires fully depleted.

In a ballistic cavity, the motion of the electrons is determined by the shape of the system. From a semiclassical picture, trajectories connecting different leads contribute to the coupling of a given pair. As the system size increases, however, the MBS coupling decreases as in the long-junction limit. From the interplay of these two effect, we require *the system size to be large enough to appreciate geometrical effects, but no large enough for the coupling to vanish.*

We study the MBS coupling of all pairs in three two-dimensional cavities. In Fig. one can observe a circular cavity defined by a radius R with three nanowires symmetrically attached. The left and right nanowires are attached at an angle θ. In Fig. one can observe a rectangular cavity defined by length L and width W with nanowires in a fork-like shape. In Fig. one can observe a triangular cavity defined by an angle θ with nanowires attached in different configurations. *For the circular and rectangular cavity, the size is changed, while for the triangular cavity, the size is fixed and the angle is changed.*

The level spacing is smaller than in the quasi-one dimensional case, which means that the peaks will be narrower. As before, the phase shift depends on each geometrical configuration, and it is optimised for each coupling experiment. It is independent of the Majorana band.

2.2.1. Circular cavity

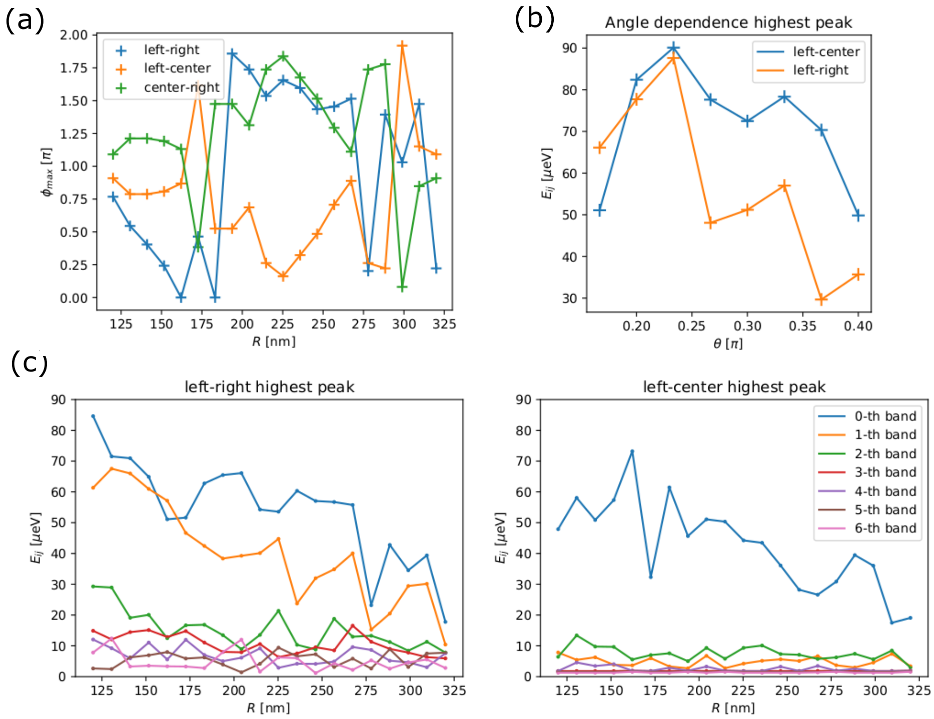

Figure 2.5: Results for circular cavity. (a) Phase shift ϕ_{max} vs. circle radius R. (b) Coupling of left and right MBS pair. Colors show the band index of the nanowire. (c) Coupling of the left and central MBS pair. Remaining pair is not show since the results is the same as (b).

Transport in a circular cavity has been extensively studied in semiconducting materials. The contribution of each closed trajectory can be identified in the conductance.

The different Majorana sub bands couple non-trivially for each pair as is shown if Fig. 2.5 (c)-(d). The momentum profile of the states inside the cavity is similar for the lowest two band when the left and right MBS couple. Consequently, it dominates over all the other bands. On the other hand, for the central pairs, the coupling is significant only for the lowest sub band. This implies that for the central MBS, the momentum distribution does not favour the coupling with the lateral MBS.

By changing the system size, one finds a transition between the short and long junction regimes. For small cavities, $R = 120$ [nm], the system behaves as a quantum dot with broad resonant peaks due to the large level spacing. However, even in small systems, the coupling of the left and right MBS is larger than the one of the central pairs. For large cavities, $R = 300$ [nm], the resonant peaks become narrow and the coupling decreases.

In order to probe different trajectories, the angle of the left and right leads is changed and the result is shown if Fig. 2.5 (b). We choose a system with $R = 200$ [nm] as an intermediate size where geometrical effects can be appreciated with a reasonable coupling. By following the largest peak, one can observe a maximum coupling develop at $\theta = 0.23\pi$. In this case , the coupling of all pairs is dominated by the same level, which develops a high and narrow peak as the angle approaches this value. As the angle increases further, there is a fast decay in the coupling for all pairs.

2.3. Triangular cavity

Given that in previous geometries, we have been able to modulate the coupling based on geometry, we pursue a similar approach in a triangular cavity as depicted in Fig. 2.4 (b). By changing the angle of the diagonal sides of the cavity, we expect to modulate the coupling such it is enhanced for a given pair. The area of the triangle is fixed, and consequently the length of the lateral sides will change for every angle. On this setup, the relative position of the nanowires is fixed to lie at the center of the diagonal sides.

2.3.1. Angular dependence

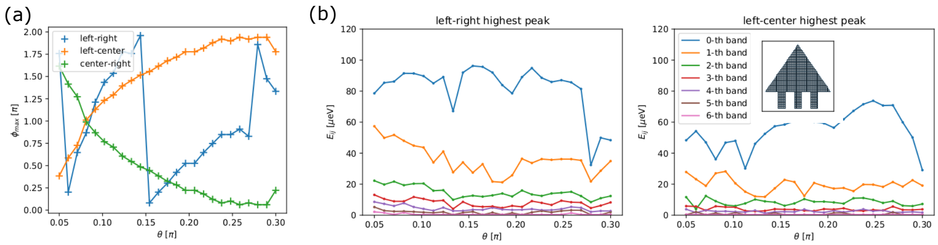

Figure 2.6: Results for circular cavity. (a) Phase shift ϕ_{max} vs. circle radius R. (b) Coupling of left and right MBS pair. Colors show the band index of the nanowire. (c) Coupling of the left and central MBS pair. Remaining pair is not show since the results is the same as (b).

The left and right MBS mean coupling remains roughly the same when the angle of the diagonal sides changes. However the coupling mechanism is different for small and large angles. For small angles, the triangular cavity resembles a cutted rectangular strip where the wavefunction weight is concentrated in the central part of the cavity. Therefore, resonant trapping couples the MBS over a range of cavity levels. As the angle increases, the highest levels of the resonant band will decouple into individual peaks. For large angles, the coupling will be mediated by individual resonances.

For the central MBS pairs, the mean coupling increases with the angle. For angles around $\theta = \pi/4$ these pairs reach a maximum coupling. As before, most of the coupling will be carried by the lowest Majorana band.

2.3.2. Lead position dependence

The position of the central lead inverts the angle dependence of the central MBS pairs coupling. As can be observed in Fig. (c), the coupling decreases with angle, yet it remains at a comparable value with respect to the case with the lead in the lower side.

Similarly, the position of the left and right leads can be taken to the opposite side. In such case, we have found that the coupling decreases by approximately 1/2. Nevertheless, the coupling of the higher momentum bands becomes much larger than in all previous cases.

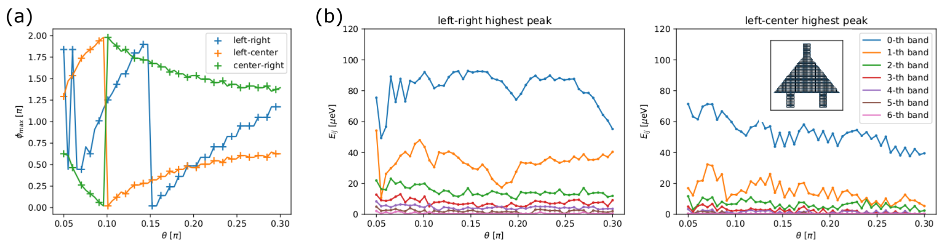

Figure 2.7: Results for circular cavity. (a) Phase shift ϕ_{max} vs. circle radius R. (b) Coupling of left and right MBS pair. Colors show the band index of the nanowire. (c) Coupling of the left and central MBS pair. Remaining pair is not show since the results is the same as (b).

2.4. Summary

Overall, all the geometries can be classified according to the mean coupling and the mean level spacing as in Fig. 2.8. The quasi-one dimensional geometries show a much larger coupling and level spacing. This is reasonable because of the system size and dimensionality. For two-dimensional systems, on the other hand, it is not trivial what the geometry effect would be. One can observe that the circular geometry has the smaller level spacing and mean coupling. The two triangular cavities are close to each other. The difference between them is induced by the position of the central lead.

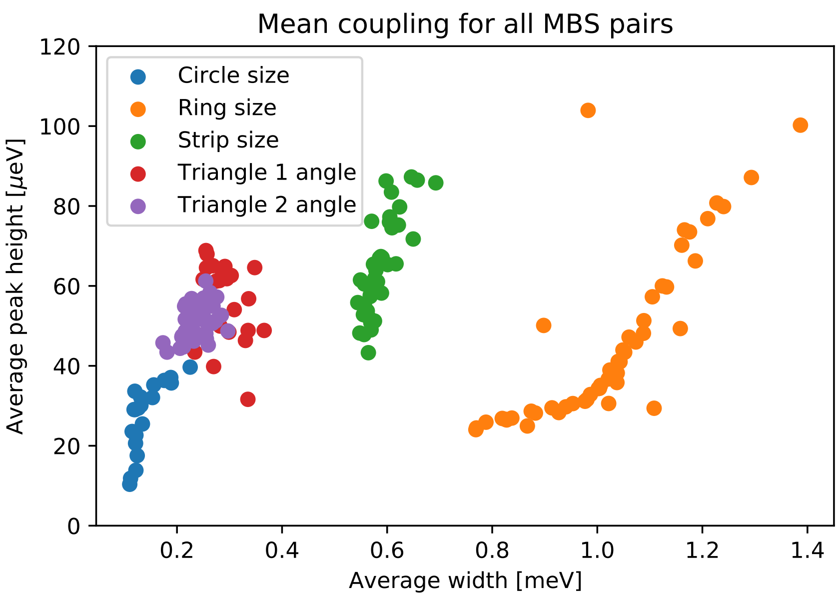

Figure 2.8

3

Gate defined triangular cavities

3.1. Gates configuration

1. The triangular cavity is defined using electrostatic gates, and the potential in the 2DEG is found as the solution to the Poisson equation.

2. It is not clear if the geometric dependence holds in a real device where the boundaries of the system are not straight, but smooth following the potential landscape.

3. In contrast to a purely geometric model, changing a single gate has a non-local effect that affects other regions of the potential, possibly inducing unexpected behaviours.

4. The MBS coupling depends on the tradeoff between tunability and shape-resolution determined crucially by the position where the nanowires attach to the cavity.

5. Consider a material stack made by an InAs 2DEG with proximity induced superconductivity, and a set of metallic gates with an oxide layer in between.

6. There are three kinds of gates in this system: plunger gates and screen gates that control the shape of the cavity, and tunnel gates that control the coupling with the nanowires.

7. Devices with three nanowires at one side are larger than those with two because of the minimum separation between tunnel barriers which is required to have well defined coupling channels for each nanowire.

3.2. Device operation

3.2.1. Nanowire channels

1. In order to have the minimum number of tunnable gates, each nanowire requires a tunnel barrier well separated from each other by fixed-voltage screen gates.

2. The operation point is below the first barrier level resonance in order to avoid interaction with spurious levels and keep a clean cavity dependence.

3. By controlling the tunnel gates height relative to the nanowire's potential, the tunnelling amplitude can be changed from the insulating regime to the strong coupling regime.

4. When the tunnel gates are far from each other, there is no crossed interaction between them, and they can be tunned symmetrically.

5. For closer tunnel gates, there's mutual interaction that modifies the barrier height, center and width, leading to a non-symmetric operational point.

3.2.2. Potential deformations

1. While the left and right MBS coupling is optimal for a triangular cavity, the coupling of the central pairs is significantly smaller due to the large system size.

2. The triangular shape of the cavity is controlled by three gates, the plunger and the screen side gates, and can be deformed in order to probe modified shapes with increased couplings.

3. The coupling of the central pairs can be significantly increases by detunning the side screen gates and effectively creating smaller triangular cavities.

4. Potential deformations are not allowed in a geometry with the central wire attached to the top triangle vertex because the screen gates determine both the cavity shape and the barrier's positions.

5. Similarly, a configuration with nanowires attached to the diagonal sides would induce an irregularities along these sides that would significantly decrease the MBS coupling.

4

Conclusions

1. Triangular cavities show a maximum coupling for a certain angle for the far pair, while the coupling of the central pair can be tunned to a maximum or minimum depending on the wire's side.

2. In a gate defined cavity, the position of the nanowires plays a crucial role in the definition and tunability of the triangular shape and thus enhancing or decreasing the MBS coupling.

3. A natural extension of this work is to design an experiment where joint parity measurements can be measured via interferometry in a loop geometry, or via charge measurements with a nearby sensor.

4. Another possible extension is to include realistic noise, representing the etching process, as potential irregularities along the triangle sides.

5. In conclusion, a trijunction of MBS where the coupling of all pairs is comparable to the superconducting gap has been designed and the operation of the device has been discussed in terms of electrostatic gates.

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