Design of a trijunction of Majorana nanowires

Juan Daniel Torres Luna

TU Delft

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by

Juan Daniel Torres Luna

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Thesis committee:	Prof. Michael Wimmer,	TU Delft, supervisor
	Prof. Anton Akhmerov	TU Delft, co-supervisor
	Dr. Chun-Xiao Liu	TU Delft, co-supervisor

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1 Background

1.1. Introduction

1. Majorana bound states (MBS) can be used to create a non-local qubit robust against local noise.
2. While coupling a single MBS pair can be done using a quantum dot, selective coupling of multiple pairs remains a challenge.
3. In this work we propose a semiconducting cavity connected to three Majorana stripes that allows for an all-electric controlled interaction between all pairs of MBS.
4. Several cavity geometries are analysed, and a triangular cavity with varying angle is found to have the largest coupling for all pairs.
5. The electrostatics effects of the gate-defined triangular cavity are analysed and the operational point is described.

1.2. Majorana bound states

1. MBS are the non-local degenerate ground state of a topological superconductor as initially proposed by Kitaev.
2. Quantum information can be encoded in the ground state since even and odd parity states are degenerate.
3. Since a pair of spatially separated MBS encodes a single fermionic mode, its quantum state is protected against local errors by particle-hole symmetry.

1.3. Experimental platforms

1. MBS can be realised in quasi one-dimensional systems defined on two-dimensional electron gases (2DEGs) or semiconducting nanowires in proximity to a superconductor.
2. Control over the chemical potential and tunnel coupling to nearby leads is mediated via electrostatic gates, but it is mostly screened inside the nanowire by the presence of the superconductor.

3. Disorder in the nanowire bulk can be detrimental for MBS since it affects its localisation and properties, such as induced gap, while disorder inside the superconductor enhances the induced gap in the nanowire.

1.3.1. Two dimensional electron gases

1. 2DEGs are a good platform for MBS based quantum computation because complex geometries can be defined in the same layer, whereas nanowires require mechanical matching.

2. Initial experiments[4, 7] focused on characterising the properties of semiconducting layers, such as large g-factor and large spin-orbit, with a superconducting layer.

3. Later experiments focused on tunnel spectroscopy of stripe-like geometries[8] where a zero bias peak (ZBP) was found, but its origin was trivial Andreev states rather than MBS.

4. 2DEGs are suitable to create gate defined shapes with interesting geometry dependent properties in the ballistic regime, yet in MBS experiments only stripes and planes have been developed so far.

1.4. Quantum operations with Majorana bound states

1. The state of the degenerate manifold of MBS can be non-trivially changed by adiabatically moving the particles around each other, i.e. non-Abelian exchange statistics.

2. Initial proposals were based on moving MBS around each other in gate defined nanowire networks[1], but this method requires high degree of control and is highly susceptible to thermal errors[5].

3. An equivalent approach that does not require to move the MBS is given by joint parity measurements[2], but it requires simultaneous measurement of different pairs of MBS.

4. Current proposals for coupling multiple pairs of Majoranas, such as the Majorana box qubit[6], couple each nanowire individually to a QD that later close in a loop.

5. To the best of our knowledge, there is a single study[3] where MBS are connected via a semiconducting cavity in a fork-like geometry, yet geometry role has proven to be fundamental in how leads states couple[9].

1.5. Quantum dot mediated coupling

1. A single nanowire coupled to a quantum dot (QD) allows to measure the parity of a pair of MBS via a measurement of the charge in the dot.

2. Two nanowires with a QD in the middle recovers the well-know Josephson junction whose spectra can be controlled by the phase difference between the two superconductors.

3. The fractional Josephson effect, 4π phase periodicity, is a consequence of the presence of a zero-energy fermionic mode made of two MBS.

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Trijunction of Majorana nanowires

2.1. Geometrical model in the strong coupling regime

1. A single ballistic cavity can be used to couple multiple pairs of MBS selectively since - in a clean 2DEG - electron transport is completely determined by the cavity's shape and the leads' positions.

2. The minimum non-trivial number of MBS that can couple via a semiconducting cavity is three.

3. The coupling energy of each pair is extracted as the value of the lowest non-zero eigenvalue with respect to the cavity chemical potential.

4. Depending on the cavity's shape, certain levels carry the largest coupling for each pair of MBS.

5. For zero-mean Gaussian noise, there is a transition where the coupling vanishes for a standard deviation comparable to the chemical potential.

2.1.1. Phase dependence

1. The phase relation is 4π periodic, but there is a phase shift controlled by the complex part of the hopping term.

2. It is anti-symmetric with respect to the central pairs, and it does not depend on the relative distance between the nanowires.

2.1.2. Size dependence

1. The size of the system shows a transition from the low to the long junction regime of a Josephson junction with many levels inside the gap.

2. The coupling decays as the system gets bigger, but some geometries show an oscillatory pattern up to considerably large sizes.

2.2. Half-ring cavity

1. Consider a narrow strip in a half ring shape with nanowires connected in a fork-like geometry.
2. Left and right MBS couple with the lowest levels as it would be a single level.
3. Magnitude of the coupling is similar for the central MBS pairs, but each cavity level couples independently.

2.3. Rectangular cavity

1. Lowest sub band carries most of the coupling, while other bands' coupling is negligible.
2. While left and right MBS have large coupling, coupling to the central MBS depends on the side at which it is.
3. At certain distance between the nanowires, one pair dominates the coupling while the other two are canceled suggesting coupling mediated by semiclassical trajectories.

2.4. Triangular cavity

1. At certain angle of the cavity, the coupling reaches a maximum for left and right MBS coupling.
2. The coupling of the central MBS pairs is controlled by the position of the central nanowire as previously.
3. Higher sub bands couple when the nanowires are attached to the diagonal sides of the triangle.

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Gate defined triangular cavities

3.1. Gates configuration

1. The triangular cavity is defined using electrostatic gates, and the potential in the 2DEG is found as the solution to the Poisson equation.

2. It is not clear if the geometric dependence holds in a real device where the boundaries of the system are not straight, but smooth following the potential landscape.

3. In contrast to a purely geometric model, changing a single gate has a non-local effect that affects other regions of the potential, possibly inducing unexpected behaviours.

4. The MBS coupling depends on the tradeoff between tunability and shape-resolution determined crucially by the position where the nanowires attach to the cavity.

5. Consider a material stack made by an InAs 2DEG with proximity induced superconductivity, and a set of metallic gates with an oxide layer in between.

6. There are three kinds of gates in this system: plunger gates and screen gates that control the shape of the cavity, and tunnel gates that control the coupling with the nanowires.

7. Devices with three nanowires at one side are larger than those with two because of the minimum separation between tunnel barriers which is required to have well defined coupling channels for each nanowire.

3.2. Device operation

3.2.1. Nanowire channels

1. In order to have the minimum number of tunnable gates, each nanowire requires a tunnel barrier well separated from each other by fixed-voltage screen gates.

2. The operation point is below the first barrier level resonance in order to avoid interaction with spurious levels and keep a clean cavity dependence.

3. By controlling the tunnel gates height relative to the nanowire's potential, the tunnelling amplitude can be changed from the insulating regime to the strong coupling regime.

4. When the tunnel gates are far from each other, there is no crossed interaction between them, and they can be tunned symmetrically.

5. For closer tunnel gates, there's mutual interaction that modifies the barrier height, center and width, leading to a non-symmetric operational point.

3.2.2. Potential deformations

1. While the left and right MBS coupling is optimal for a triangular cavity, the coupling of the central pairs is significantly smaller due to the large system size.

2. The triangular shape of the cavity is controlled by three gates, the plunger and the screen side gates, and can be deformed in order to probe modified shapes with increased couplings.

3. The coupling of the central pairs can be significantly increases by detunning the side screen gates and effectively creating smaller triangular cavities.

4. Potential deformations are not allowed in a geometry with the central wire attached to the top triangle vertex because the screen gates determine both the cavity shape and the barrier's positions.

5. Similarly, a configuration with nanowires attached to the diagonal sides would induce an irregularities along these sides that would significantly decrease the MBS coupling.

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Conclusions

1. Triangular cavities show a maximum coupling for a certain angle for the far pair, while the coupling of the central pair can be tunned to a maximum or minimum depending on the wire's side.

2. In a gate defined cavity, the position of the nanowires plays a crucial role in the definition and tunability of the triangular shape and thus enhancing or decreasing the MBS coupling.

3. A natural extension of this work is to design an experiment where joint parity measurements can be measured via interferometry in a loop geometry, or via charge measurements with a nearby sensor.

4. Another possible extension is to include realistic noise, representing the etching process, as potential irregularities along the triangle sides.

5. In conclusion, a trijunction of MBS where the coupling of all pairs is comparable to the superconducting gap has been designed and the operation of the device has been discussed in terms of electrostatic gates.

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