Design of a trijunction of Majorana nanowires

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by

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Background

1.1. Introduction

MBS appear as the zero-energy modes of a hybrid quasi-one dimensional system that combines a strong-spin orbit semiconductor with proximity induced superconductivity. Semiconducting nanowires and two-dimensional electron gases (2DEG) are candidates for creating such devices, yet no evidence of such excitations has been found. Nevertheless, the existence of MBS would allow us to design new qubits that are resilient to noise in contrast to current devices.

While coupling a single MBS pair can be done using a quantum dot, selective coupling of multiple pairs remains a challenge. In the presence of multiple pairs, coupling MBS from different fermions induces a non-trivial evolution of the ground state that supports quantum gate operation. Coupling of a pair of MBS in a S-N-S junction has been extensively studied, and the fractional Josephson effect has been found as a signature of MBS present in such system. On the other hand, coupling multiple MBS pairs remains a challenge given the constrains on the nanowires alignment and separation.

The simplest system where multiple MBS can couple non-trivially is in a trijunction geometry. *In this thesis we propose a semiconducting cavity connected to three Majorana nanowires that allows for an all-electric controlled interaction between all pairs of MBS.*

Initially, the role of geometry is investigated by simulating several cavity geometries and extracting the MBS coupling in the strong coupling regime. It is found that different cavity levels mediate differently the coupling of different MBS pairs. We found that there is an angle for a triangular cavity that induces a maximum coupling between the far MBS pairs. *Several cavity geometries are analysed, and a triangular cavity with varying angle is found to have the largest coupling for all pairs.*

Finally, a realistic model is studied via electrostatic simulations of the triangular cavity with optimal configuration defined on a 2DEG. The non-local nature of the gates makes the nanowire positions crucial in order to recover the effects found for the purely geometric case. The role of each set of gates and the range of voltages used to operate the device are discussed. *The electrostatics effects of the gate-defined triangular cavity are analysed and the operational point is described.*

1.2. Majorana bound states

MBS emerge as the non-local degenerate ground state of a topological superconductor. Under the appropriate conditions, a spinless one-dimensional p-wave superconductor contains two zero-energy excitations that are exponentially localised at the edges of the system. Together, these two zero-energy modes encode a single fermionic mode that can be empty or occupied,

$$f = \frac{\gamma_L + i\gamma_R}{\sqrt{2}}, \quad f^\dagger = \frac{\gamma_L - i\gamma_R}{\sqrt{2}}, \tag{1.1}$$

where $\gamma_i = \gamma_i^\dagger$ are Majorana operators that $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$.

Since a pair of spatially separated MBS encode a single fermionic mode, its quantum state is protected against local errors by particle-hole symmetry. In a sufficiently large nanowire, MBS are completely decoupled from each other, and noise sources will interact with each of them individually. The interaction with a single MBS is proportional to a single Majorana operator, i.e. $\gamma \sim f + f^\dagger$, and thus it will change the parity of the system. In a superconductor, however, electrons can only enter or leave as Cooper pairs, which means that the parity is conserved. Therefore, individual MBS are immune to local noise sources.

The ground state can only be controlled by non-local operations that involve pairs of MBS. Given parity conservation, only even powers of Majorana operators are allowed in the Hamiltonian. The simplest allowed term describes the coupling of a pair of MBS, and it is given by

$$H_{pair} = iE_{LR}\gamma_L\gamma_R = E_{LR}(1 - 2f^\dagger f). \tag{1.2}$$

Here, E_{LR} is the tunnelling coupling between the two MBS, and it is usually cancelled in sufficiently long nanowires.

In the presence of multiple MBS pairs, each parity subspace can be used as a computational subspace where quantum information is protected. *By controlling the coupling between different MBS pairs, one can control the ground state evolution*, that is,

$$|\Psi\rangle \to U(t)\,|\Psi\rangle\,, \quad U(t) = \exp\big(iH_{pair}t\big). \tag{1.3}$$

1.3. Experimental platforms

MBS can be realised in quasi one-dimensional systems defined on two-dimensional electron gases (2DEGs), or semiconducting nanowires, with strong spin-orbit and in proximity to a superconductor. The Hamiltonian that realises a Majorana nanowire is,

$$\mathcal{H} = \sum_k \Psi_k^\dagger H(k)\Psi_k, \quad H(k) = \left[\frac{|\mathbf{k}|^2}{2m^*} - \mu + \alpha(k_x\sigma_y - k_y\sigma_x)\right]\tau_z + B_x\sigma_x + \Delta\tau_x. \tag{1.4}$$

Here, $\Psi_k^\dagger = (f_{k\uparrow}^\dagger, f_{k\downarrow}^\dagger, f_{k\uparrow} f_{k\downarrow})^T$ are the Nambu spinors in k space, μ is the chemical potential, $\mathbf{k}$ is the 2D wave-vector, α is the spin orbit interaction, B_x is the Zeeman field, Δ is the superconducting gap, and σ and τ are Pauli matrices for the spin and particle-hole basis.

MBS appear as zero energy excitations of this Hamiltonian when $B_x^2 \geq \sqrt{\mu^2 + \Delta^2}$. However, *realising such material combination is difficult, and MBS transport signatures are not unambiguous.* On the one hand, these interaction destroy each other mutually as is the case of superconductivity and magnetic fields. On the other hand, MBS signatures can be reproduced by states localised in material defects or impurities. Therefore, highly tunable devices with low impurities and disorder are required to unambiguously detect MBS.

MBS can appear in different nanowire sub-bands. In a quasi-one dimensional systems, there is a translational invariant direction, and a direction with finite width W. The energy of each mode has a contribution from both, and it is given by,

$$E_n(k) = \frac{\hbar^2}{2m^*}\left(k^2 + \frac{\pi^2 n^2}{W^2}\right). \tag{1.5}$$

Here, m^* is the effective mass and the spin orbit splitting is not considered. Independent MBS with different momentum profiles can be formed at each transverse mode when the chemical potential is at the bottom of the corresponding band. Multiple channel become relevant in the presence of disorder. It couples differently to each momentum sub band, which will induce band mixing as has been suggested in experiments.

1.3.1. Two dimensional electron gases

In a clean system, electrons travel ballistically, and their motion is directly determined by the shape of the system boundaries. *2DEGs allow for arbitrary geometries to be defined in the same layer using different electrostatic gates.* Furthermore, it been shown that geometric dependence can be used to enhance the property of Majorana devices. On the other hand, in semiconducting nanowires networks MBS is limited to narrow transverse channels. Therefore, 2DEGs are an interesting platform to study the role of geometry in MBS coupling with gate defined shapes.

Parallel Majorana nanowires are the basic elements for a complex Majorana device. Multiple nanowires require to be aligned in order to have a stable topological phase. Each Majorana nanowire can be defined on a 2DEG by adding a superconducting strip on top of the selected region. A a top gate is deposited next on top of the device such that depletes the surrounding 2DEG. A narrow quasi-one dimensional channel is created below the superconductor, and it is expected to find MBS at the edges.

Majorana experiments on 2DEGs have shown promising evidence for scalable and complex devices. Initial experiments[4, 7] focused on characterising the properties of semiconducting layers with a superconducting cover. Advances in material growth allowed for clean interfaces with a hard superconducting gap to develop into the nanowire region. Later experiments focused on tunnel spectroscopy of stripe-like geometries[8] where a zero bias peak (ZBP) was found. However, due to disorder and defects such ZBPs have most likely a trivial origin from Andreev states rather than MBS. Nevertheless, efforts to develop scalable devices in 2DEGs are made and new promising materials are being studied.

1.3.2. Electrostatic gates

The electrostatic potential in a 2DEG is found by solving the Poisson equation using a finite elements method on the device geometry. The potential landscape in a 2DEG can be controlled by deposition of metallic gates on a top layer with an insulating barrier in between that smooths the potential profile. The potential landscapce, $U(\mathbf{r})$, for a given geometrical configuration can be found by solving Laplace equation,

$$\nabla \cdot [\varepsilon_r(\mathbf{r})\nabla U(\mathbf{r})] = 0. \tag{1.6}$$

Here, ε_r is the relative permitivity of each layer in the material stack.

Electrostatic effects play a crucial role in designing and operating Majorana devices. Characterisation of Majorana nanowire is done via transport measurements that require tunnel coupled leads and gates. Furthermore, gates have a non-local effect on the potential landscape that differs between experimental platforms. For example, nanowires have a partial superconducting coating that allows for the electric field to penetrate and control the semiconductor and superconductor weight of the wavefunction. In 2DEGs, on the contrary, the superconducting coat fully covers it, which screens electrostatic effects.

1.4. Majorana bound states in a trijunction

There are two main approaches for MBS quantum computation: braiding and joint parity measurements. Braiding was initially proposed as moving MBS around each other in gate defined nanowire networks[1]. However, this method requires high degree of control and is highly susceptible to thermal errors[5]. On the other hand, joint parity measurements coupling multiple pairs of MBS[6] by using co-tunnelling processes between different MBS on superconducting islands. These methods do not rely on geometrical effects, and are often discussed in terms of a phenomenological Hamiltonian.

In a trijunction, demonstration of the simplest non-trivial Majorana evolution experiment can be done. In order to create a Majorana qubit, three or more MBS with precisely controlled interactions are required. The interaction between different MBS pairs is mediated by the cavity modes, which crucially depends on the nanowires positions and the cavity geometry. By controlling the coupling of each pair via a DC voltage pulse sequence, one determines the evolution of the MBS. Initial studies[3] have shown that MBS can connect via a semiconducting cavity in a fork-like geometry.

However, design and operation of a trijunction are non-trivial tasks. Simultaneous tuning of gate voltages and relative phase difference is required to optimally operate a trijunction. Selection of the MBS pair and cavity modes is realised by electrostatic gates controlling the potential on each region. Furthermore, relative phase differences between MBS modulates the coupling as in the fractional Josephson effect. The phase will be shifted by the presence of complex hopping terms and by the nanowires relative position.

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Trijunction of Majorana nanowires

2.1. Geometrical model in the strong coupling regime

1. A single ballistic cavity can be used to couple multiple pairs of MBS selectively since - in a clean 2DEG - electron transport is completely determined by the cavity's shape and the leads' positions.

2. The minimum non-trivial number of MBS that can couple via a semiconducting cavity is three.

3. The coupling energy of each pair is extracted as the value of the lowest non-zero eigenvalue with respect to the cavity chemical potential.

4. Depending on the cavity's shape, certain levels carry the largest coupling for each pair of MBS.

5. For zero-mean Gaussian noise, there is a transition where the coupling vanishes for a standard deviation comparable to the chemical potential.

2.1.1. Phase dependence

1. The phase relation is 4π periodic, but there is a phase shift controlled by the complex part of the hopping term.

2. It is anti-symmetric with respect to the central pairs, and it does not depend on the relative distance between the nanowires.

2.1.2. Size dependence

1. The size of the system shows a transition from the low to the long junction regime of a Josephson junction with many levels inside the gap.

2. The coupling decays as the system gets bigger, but some geometries show an oscillatory pattern up to considerably large sizes.

2.2. Half-ring cavity

1. Consider a narrow strip in a half ring shape with nanowires connected in a fork-like geometry.

2. Left and right MBS couple with the lowest levels as it would be a single level.

3. Magnitude of the coupling is similar for the central MBS pairs, but each cavity level couples independently.

2.3. Rectangular cavity

1. Lowest sub band carries most of the coupling, while other bands' coupling is negligible.

2. While left and right MBS have large coupling, coupling to the central MBS depends on the side at which it is.

3. At certain distance between the nanowires, one pair dominates the coupling while the other two are canceled suggesting coupling mediated by semiclassical trajectories.

2.4. Triangular cavity

1. At certain angle of the cavity, the coupling reaches a maximum for left and right MBS coupling.

2. The coupling of the central MBS pairs is controlled by the position of the central nanowire as previously.

3. Higher sub bands couple when the nanowires are attached to the diagonal sides of the triangle.

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Gate defined triangular cavities

3.1. Gates configuration

1. The triangular cavity is defined using electrostatic gates, and the potential in the 2DEG is found as the solution to the Poisson equation.

2. It is not clear if the geometric dependence holds in a real device where the boundaries of the system are not straight, but smooth following the potential landscape.

3. In contrast to a purely geometric model, changing a single gate has a non-local effect that affects other regions of the potential, possibly inducing unexpected behaviours.

4. The MBS coupling depends on the tradeoff between tunability and shape-resolution determined crucially by the position where the nanowires attach to the cavity.

5. Consider a material stack made by an InAs 2DEG with proximity induced superconductivity, and a set of metallic gates with an oxide layer in between.

6. There are three kinds of gates in this system: plunger gates and screen gates that control the shape of the cavity, and tunnel gates that control the coupling with the nanowires.

7. Devices with three nanowires at one side are larger than those with two because of the minimum separation between tunnel barriers which is required to have well defined coupling channels for each nanowire.

3.2. Device operation

3.2.1. Nanowire channels

1. In order to have the minimum number of tunnable gates, each nanowire requires a tunnel barrier well separated from each other by fixed-voltage screen gates.

2. The operation point is below the first barrier level resonance in order to avoid interaction with spurious levels and keep a clean cavity dependence.

3. By controlling the tunnel gates height relative to the nanowire's potential, the tunnelling amplitude can be changed from the insulating regime to the strong coupling regime.

4. When the tunnel gates are far from each other, there is no crossed interaction between them, and they can be tunned symmetrically.

5. For closer tunnel gates, there's mutual interaction that modifies the barrier height, center and width, leading to a non-symmetric operational point.

3.2.2. Potential deformations

1. While the left and right MBS coupling is optimal for a triangular cavity, the coupling of the central pairs is significantly smaller due to the large system size.

2. The triangular shape of the cavity is controlled by three gates, the plunger and the screen side gates, and can be deformed in order to probe modified shapes with increased couplings.

3. The coupling of the central pairs can be significantly increases by detunning the side screen gates and effectively creating smaller triangular cavities.

4. Potential deformations are not allowed in a geometry with the central wire attached to the top triangle vertex because the screen gates determine both the cavity shape and the barrier's positions.

5. Similarly, a configuration with nanowires attached to the diagonal sides would induce an irregularities along these sides that would significantly decrease the MBS coupling.

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Conclusions

1. Triangular cavities show a maximum coupling for a certain angle for the far pair, while the coupling of the central pair can be tunned to a maximum or minimum depending on the wire's side.

2. In a gate defined cavity, the position of the nanowires plays a crucial role in the definition and tunability of the triangular shape and thus enhancing or decreasing the MBS coupling.

3. A natural extension of this work is to design an experiment where joint parity measurements can be measured via interferometry in a loop geometry, or via charge measurements with a nearby sensor.

4. Another possible extension is to include realistic noise, representing the etching process, as potential irregularities along the triangle sides.

5. In conclusion, a trijunction of MBS where the coupling of all pairs is comparable to the superconducting gap has been designed and the operation of the device has been discussed in terms of electrostatic gates.

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