Design of a trijunction of Majorana nanowires

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by

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Background

1.1. Introduction

1. Majorana bound states (MBS) can be used to create a non-local qubit robust against local noise.
2. While coupling a single MBS pair can be done using a quantum dot, selective coupling of multiple pairs remains a challenge.
3. In this work we propose a semiconducting cavity connected to three Majorana stripes that allows for an all-electric controlled interaction between all pairs of MBS.
4. Several cavity geometries are analysed, and a triangular cavity with varying angle is found to have the largest coupling for all pairs.
5. The electrostatics effects of the gate-defined triangular cavity are analysed and the operational point is described.

1.2. Majorana bound states

1. MBS are the non-local degenerate ground state of a topological superconductor as initially proposed by Kitaev.
2. Quantum information can be encoded in the ground state since even and odd parity states are degenerate.
3. Since a pair of spatially separated MBS encodes a single fermionic mode, its quantum state is protected against local errors by particle-hole symmetry.

1.3. Experimental platforms

1. MBS can be realised in quasi one-dimensional systems defined on two-dimensional electron gases (2DEGs) or semiconducting nanowires in proximity to a superconductor.
2. Control over the chemical potential and tunnel coupling to nearby leads is mediated via electrostatic gates, but it is mostly screened inside the nanowire by the presence of the superconductor.

3. Disorder in the nanowire bulk can be detrimental for MBS since it affects its localisation and properties, such as induced gap, while disorder inside the superconductor enhances the induced gap in the nanowire.

1.3.1. Two dimensional electron gases

1. 2DEGs are a good platform for MBS based quantum computation because complex geometries can be defined in the same layer, whereas nanowires require mechanical matching.

2. 2DEGs are suitable to create gate defined shapes with interesting geometry dependent properties in the ballistic regime, yet in MBS experiments only stripes and planes have been developed so far.

1.4. Quantum operations with Majorana bound states

1. The state of the degenerate manifold of MBS can be non-trivially changed by adiabatically moving the particles around each other, i.e. non-Abelian exchange statistics.

2. Initial proposals were based on moving MBS around each other in gate defined nanowire networks, but this method requires high degree of control and is highly susceptible to thermal errors.

3. An equivalent approach that does not require to move the MBS is given by joint parity measurements, but it requires simultaneous measurement of different pairs of MBS.

4. Current proposals for coupling multiple pairs of Majoranas couple each nanowire individually to a QD that later close in a loop.

5. To the best of our knowledge, there is a single study where MBS are connected via a semiconducting cavity in a fork-like geometry, yet geometry role has proven to be fundamental in how leads states couple.

1.5. Quantum dot mediated coupling

1. A single nanowire coupled to a quantum dot (QD) allows to measure the parity of a pair of MBS via a measurement of the charge in the dot.

2. Two nanowires with a QD in the middle recovers the well-know Josephson junction whose spectra can be controlled by the phase difference between the two superconductors.

3. The fractional Josephson effect, 4π phase periodicity, is a consequence of the presence of a zero-energy fermionic mode made of two MBS.

2

Trijunction of Majorana nanowires

1. A single ballistic cavity can be used to couple multiple pairs of MBS selectively since - in a clean 2DEG - electron transport is completely determined by the cavity's shape and the leads' positions.

2. The minimum non-trivial number of MBS that can couple via a semiconducting cavity is three.

3. The coupling energy of each pair is extracted as the value of the lowest non-zero eigenvalue with respect to the cavity chemical potential.

4. Depending on the cavity's shape, certain levels carry the largest coupling for each pair of MBS.

5. For zero-mean Gaussian noise, there is a transition where the coupling vanishes for a standard deviation comparable to the chemical potential.

2.1. Phase dependence

1. The phase relation is 4π periodic, but there is a phase shift controlled by the complex part of the hopping term.

2. It is anti-symmetric with respect to the central pairs, and it does not depend on the relative distance between the nanowires.

2.2. Size dependence

1. The size of the system shows a transition from the low to the long junction regime of a Josephson junction with many levels inside the gap.

2. The coupling decays as the system gets bigger, but some geometries show an oscillatory pattern up to considerably large sizes.

2.3. Half-ring cavity

1. Consider a narrow strip in a half ring shape with nanowires connected in a fork-like geometry.

2. Left and right MBS couple with the lowest levels as it would be a single level.

3. Magnitude of the coupling is similar for the central MBS pairs, but each cavity level couples independently.

2.4. Rectangular cavity

1. Lowest sub band carries most of the coupling, while other bands' coupling is negligible.

2. While left and right MBS have large coupling, coupling to the central MBS depends on the side at which it is.

3. At certain distance between the nanowires, one pair dominates the coupling while the other two are canceled suggesting coupling mediated by semiclassical trajectories.

2.5. Triangular cavity

1. At certain angle of the cavity, the coupling reaches a maximum for left and right MBS coupling.

2. The coupling of the central MBS pairs is controlled by the position of the central nanowire as previously.

3. Higher sub bands couple when the nanowires are attached to the diagonal sides of the triangle.

3

Gate defined triangular cavities

In this chapter we discuss the implementation of a triangular cavity via electrostatic gates as well as the operational point where the coupling of all MBS pairs is maximum.

3.1. Gates configuration

1.