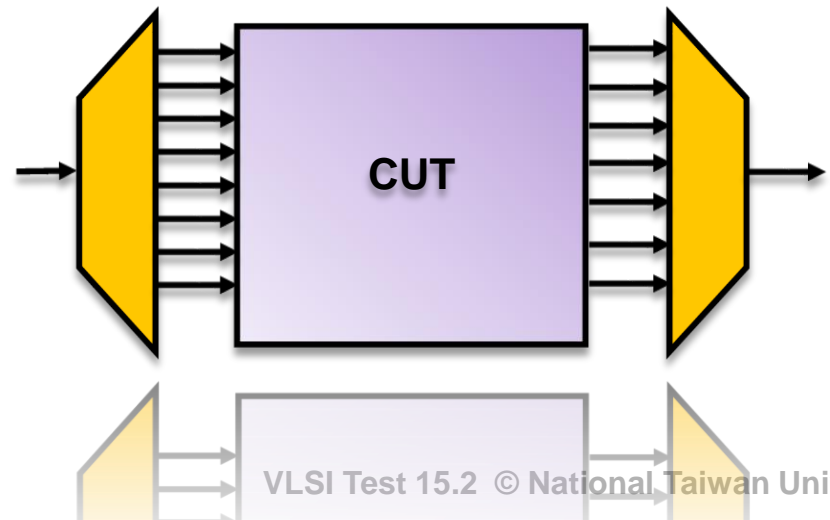


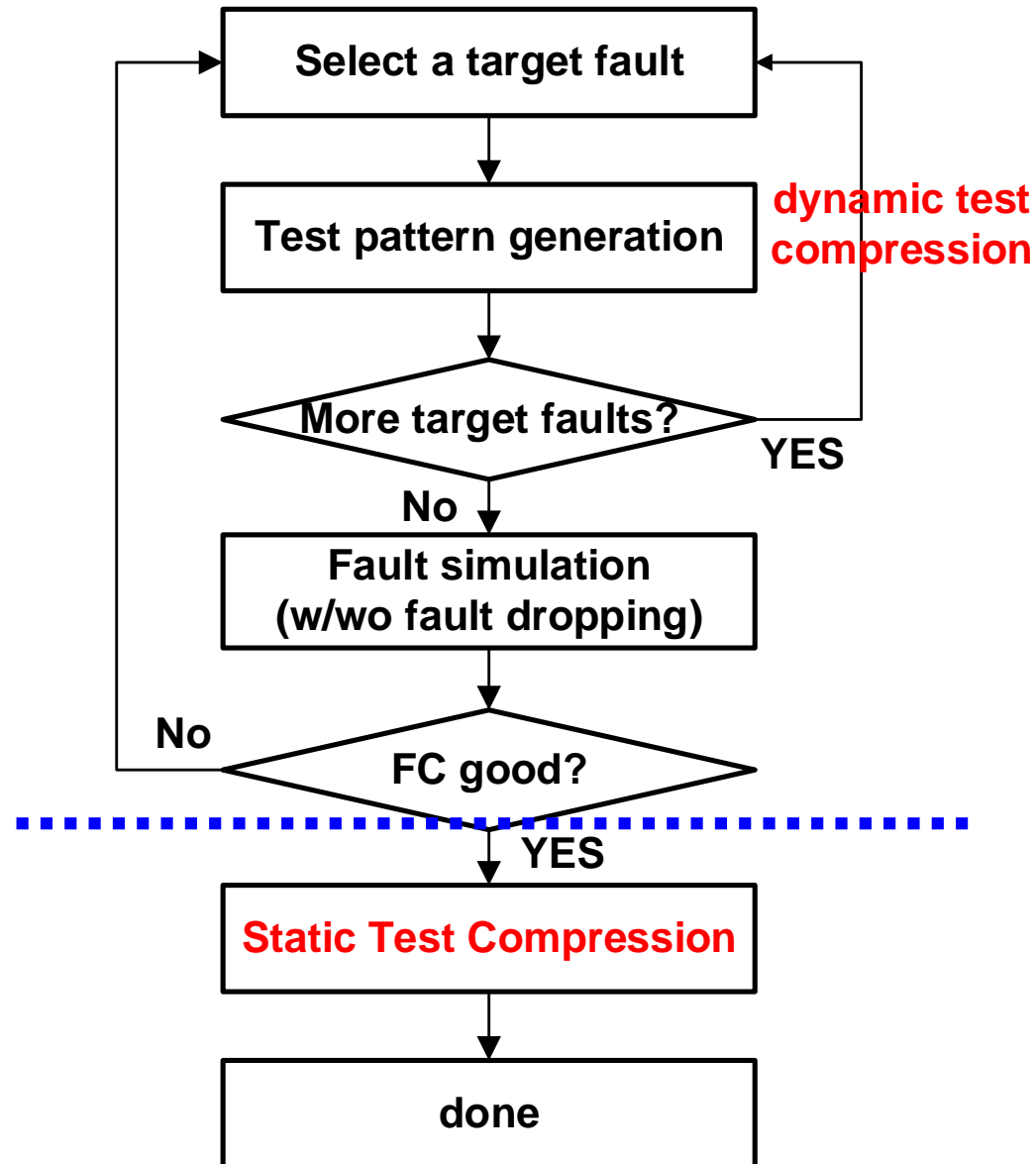
Test Compression

- Introduction
- Software Techniques
 - ♦ Dynamic Test Compression (DTC)
 - ♦ Static Test Compression (STC)
- Hardware Techniques
 - ♦ Test Stimulus Compression
 - ♦ Test Response Compression
 - ♦ Industry Practices
- Conclusion



Review: STC vs. DTC

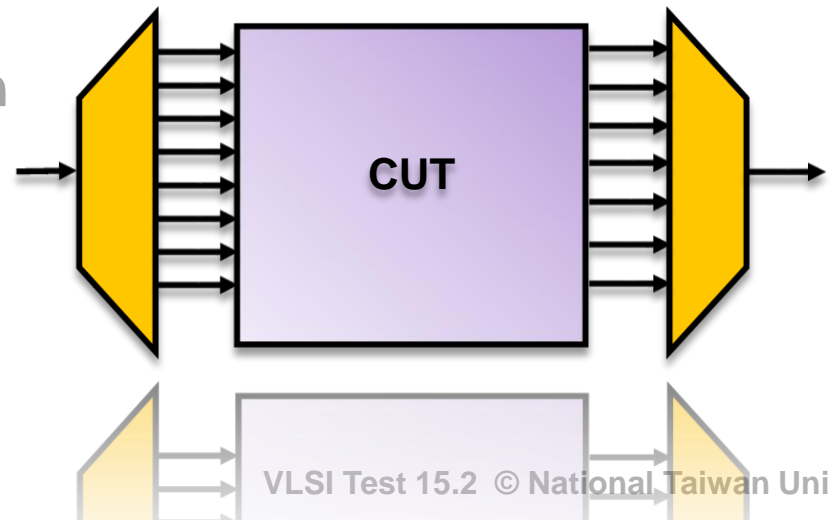
- **Dynamic test compression**
 - ♦ performed **during** TPG
 - ♦ more CPU time
 - ♦ more effective
- **Static test compression**
 - ♦ performed **after** TPG
 - ♦ less CPU time
 - ♦ less effective



Test Compression

- Introduction
- Software Techniques
 - ♦ Dynamic Test Compression
 - ♦ Static Test Compression
 - * With fault dictionary
 - * Without fault dictionary
 - Compatibility graph (X-unfilled)
 - Reverse order fault simulation (X-filled)
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3 cases



STC with Fault Dictionary

- Suppose we have a fault dictionary (**without fault dropping**)
- **Covering table**
 - ♦ Each row is a test pattern, each column is a fault
- Goal: Find minimum test set (select fewest test patterns)
 - ♦ Detect all faults
- Finding minimum test set is **minimum set covering problem**
 - ♦ NP-hard, but don't give up...

	f_1	f_2	f_3	f_4	f_5
t_1				X	
t_2		X	X		X
t_3		X		X	X
t_4	X			X	

Each row is a pattern
Each column is a fault
X= detection
(not don't care!)

Quine-McCluskey Method [McCluskey 56]

- First EDA algorithm for 2-level logic synthesis
- Fault that is detected only once is **essential fault**
 - ♦ Must select **essential patterns** that detect essential faults
- Example:
 - ♦ f_1, f_3 are essential faults; t_2, t_4 are essential patterns
 - ♦ Test set selected = $\{t_2, t_3, t_4\}$ or $\{t_1, t_2, t_4\}$, minimum test length = 3

	f_1	f_2	f_3	f_4	f_5
t_1		X		X	
t_2			X		X
t_3		X		X	X
t_4	X			X	

Each row is a pattern
Each column is a fault
X= detection

What if No Essential Faults?

Quine-McCluskey Method (cont'd)

- 1. Remove redundant **equivalent row**, keep one row is enough
 - Row t_1 is equal to row t_2 because they have X in same columns
- 2. Remove **dominated row**
 - Row t_3 dominates row t_4 because
 - (1) row t_3 has X in all columns where row t_4 has X, and
 - (2) row t_3 has at least one X where row t_4 does not have X

	f_1	f_2	f_3	f_4	f_5	
t_1	X	X	X		X	
t_2	X	X	X		X	equivalent row
t_3		X	X	X	X	
t_4			X	X	X	dominated row
t_5	X			X	X	

Quine-McCluskey Method (cont'd)

- 3. Remove **dominating column**
 - Column f_5 dominates column f_4 (f_3, f_2, f_1 also) because
 - (1) column f_5 has X in all rows where column f_4 has X, and
 - (2) column f_5 has at least one X where column f_4 does not have X
- 4. **Secondary essential**
 - After steps 1~3, t_3 is now secondary essential pattern
- Minimum test set $\{t_1, t_3\}$ or $\{t_3, t_5\}$, minimum test length =2

dominating column

	f_1	f_2	f_3	f_4	f_5
t_1	X	X	X		X
t_3		X	X	X	X
t_5	X				X

✓

(cont'd from last page)

Quiz

Q1: Which are essential faults?

Q2: Which are dominated rows? Dominating columns?

Q3: What is minimum test length?

	f_1	f_2	f_3	f_4	f_5
t_1			X	X	X
t_2		X	X		X
t_3			X		
t_4	X				
t_5		X		X	X

QM Solves Many Cases in Polynomial Time
but not all...

FFT

- Mini-set covering is NP-hard
 - ♦ Q1: Show an special case where no rule of QM can be applied
 - ♦ Q2: What are you going to do with it?

	f_1	f_2	f_3	f_4
t_1	X	X		
t_2		X	X	
t_3			X	X
t_4	X			X

Alternative Solution, 01-ILP

- Model STC as **01-Integer Linear Programming** problem

$$\text{Objective: } \min \sum_i t_i$$

$$\text{s.t. } \sum_i d_{i,j} \times t_i \geq 1, \text{ foreach fault } j$$

- ♦ $t_i=1$,if test i is selected; $t_i=0$ otherwise
- ♦ $d_{i,j}=1$ if test pattern t_i detects fault f_j

- Example: $t_1=0, t_2=t_3=t_4=1$

	f_1	f_2	f_3	f_4	f_5
t_1		X			
t_2			X		X
t_3		X		X	X
t_4	X			X	

$$\min \quad t_1 + t_2 + t_3 + t_4$$

$$\text{s.t.}$$

$$t_4 \geq 1$$

$$t_1 + t_3 \geq 1$$

$$t_2 \geq 1$$

$$t_3 + t_4 \geq 1$$

$$t_2 + t_3 \geq 1$$

It is Well-solved... What Is Wrong?

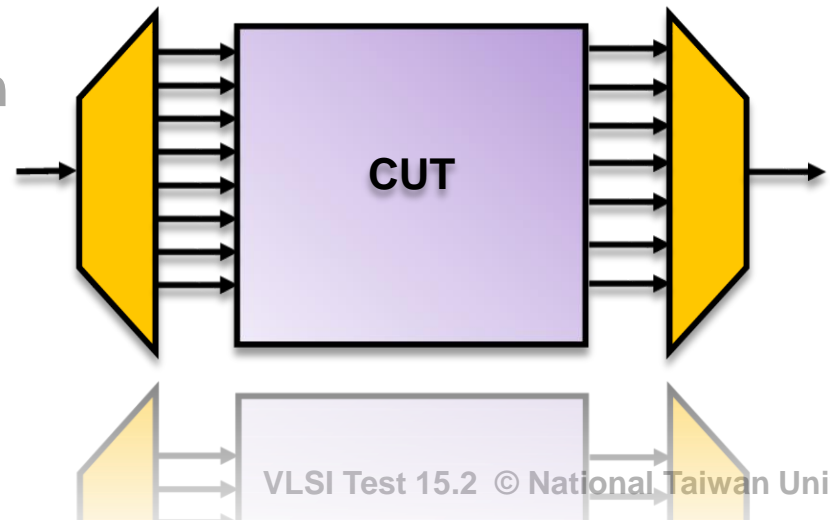
- Q: What is practical issue of STC with dictionary?
 - ◆ A: Complete fault dictionary is very large, very slow

	f_1	f_2	f_3	f_4	f_5
t_1		X	X		
t_2	X	X	X		X
t_3			X		X
t_4	X			X	

Need STC without Dictionary

Test Compression

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 - ♦ Dynamic Test Compression
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- Conclusion



STC w/o Dictionary (X-unfilled)

- Suppose that we do NOT have dictionary
 - ♦ but we have don't care bits in test cubes
- Two test cubes are **compatible** iff no conflict in specified bits
- Compatible test cubes can be **merged** into one test cube
- Example
 - ♦ t_0 and t_1 are compatible, merged to $0xx10$
 - ♦ Feasible solution:
 - * **4 patterns:** $\{t_0+t_1+t_2, t_3+t_6, t_4+t_5, t_7\}$
 - * Any better solution?

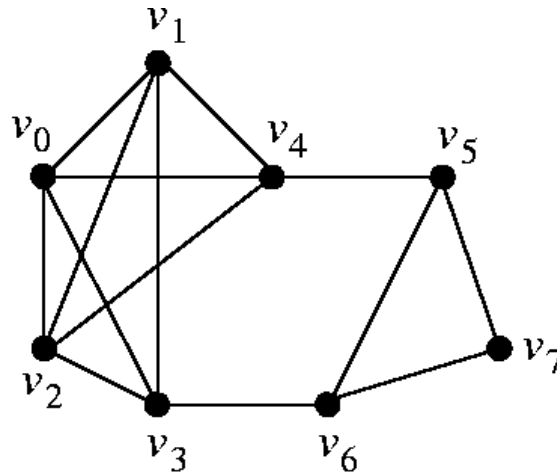
t_0	0xx10
t_1	0xx1x
t_2	0x01x
t_3	01xx0
t_4	x0xx0
t_5	1xxxx
t_6	x1x00
t_7	11xx0

x = don't cares

**Any Algorithm
to Solve This Problem?**

Compatibility Graph

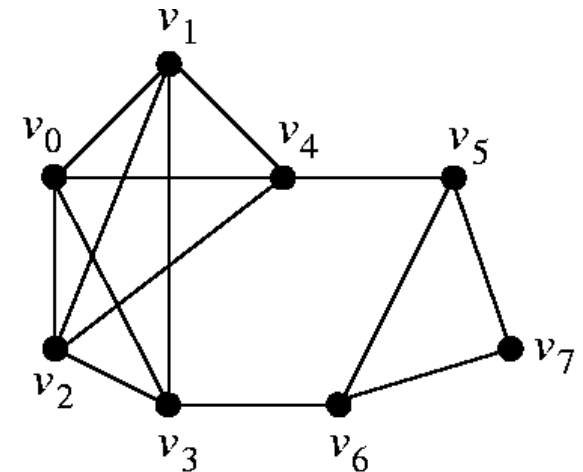
- **Compatibility graph** $G(V, E)$
 - ♦ Vertex v_i represents a test cube t_i
 - ♦ Edge e_{ij} between v_i and v_j means two test cubes are **compatible**
 - ♦ **Adjacent vertices** can be merged into one
- **Example**
 - ♦ t_0 and t_1 are adjacent, can be merged



t_0	0xx10
t_1	0xx1x
t_2	0x01x
t_3	01xx0
t_4	x0xx0
t_5	1xxxx
t_6	x1x00
t_7	11xx0

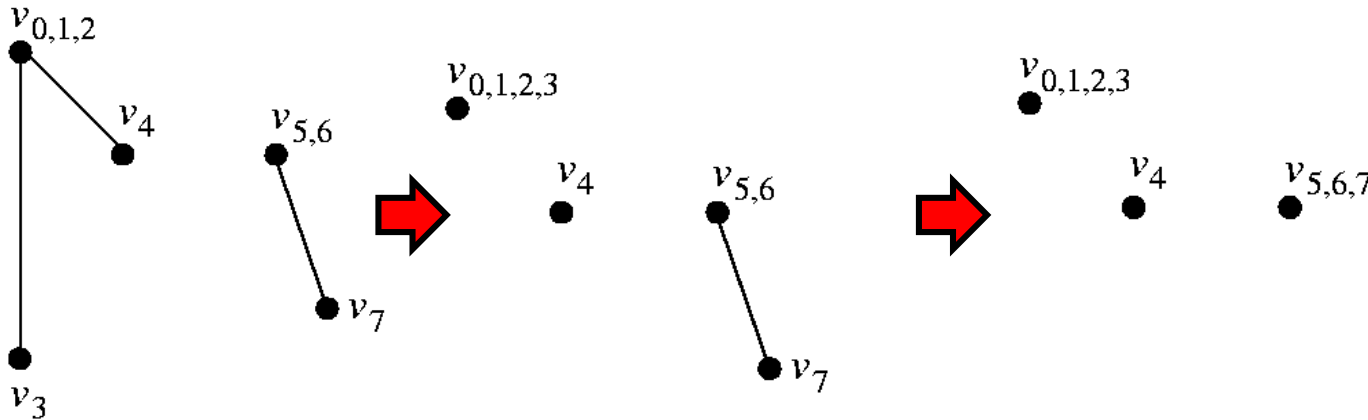
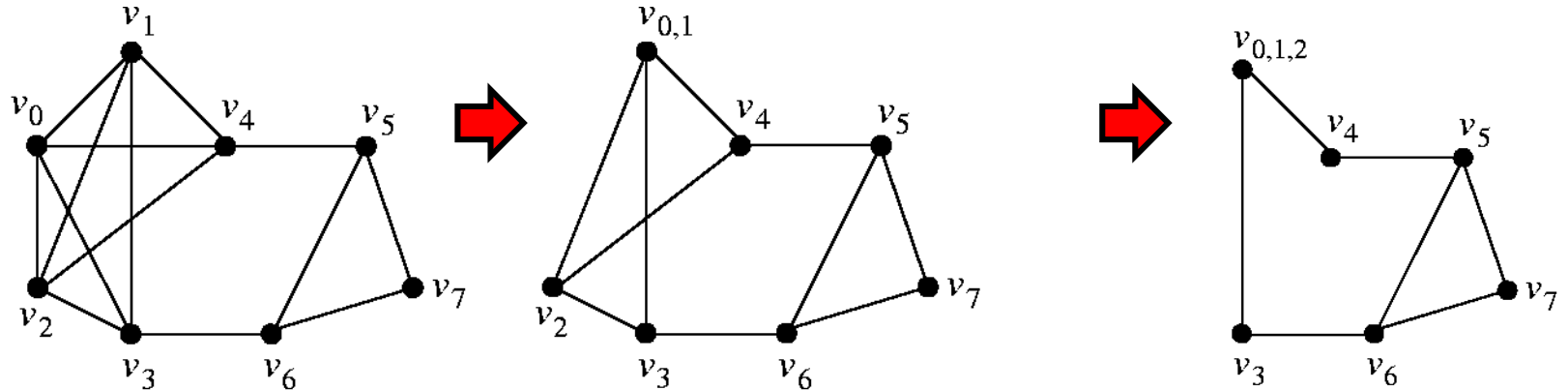
Clique

- **Clique** is a subset of vertices such that
 - ♦ Each pair of vertices are connected
 - ♦ Clique is a *complete subgraph*
- **Minimum clique partition problem**
 - ♦ Partition graph into minimum number of cliques
- Example:
 - ♦ $\{v_0, v_1, v_2, v_3\}$ $\{v_5, v_6, v_7\}$ are cliques
 - ♦ minimum clique partition
 - * $\{v_0, v_1, v_2, v_3\} \{v_4\} \{v_5, v_6, v_7\}$
 - * 3 partitions
- MCP is NP-hard problem
 - ♦ Can be solved by Tseng-Siewiorek
 - * Greedy algorithm, does NOT guaranteed optimal solution



Tseng-Siewiorek Idea

- Select two **adjacent vertices** of maximum **common neighbors**
- Merge two vertices into a **supervertex**
 - ♦ e.g., merge $v_0 v_1 \Rightarrow v_{0,1}$ supervertices
- Iteratively merge vertices until no more edge



$t_{0,1,2,3}$	01010
t_4	x0xx0
$t_{5,6,7}$	11x00

Tseng-Siewiorek Algorithm (1)

```

 $k \leftarrow 0;$ 
 $G_c^k(V_c^k, E_c^k) \leftarrow G_c(V_c, E_c);$ 
while ( $E_c^k \neq \emptyset$ ) {
    find  $(v_i, v_j) \in E_c^k$  with largest set of common neighbors;
     $N \leftarrow$  set of common neighbors of  $v_i$  and  $v_j$ ;
     $s \leftarrow i \cup j$ ;
     $V_c^{k+1} \leftarrow V_c^k \cup \{v_s\} \setminus \{v_i, v_j\};$   $\setminus$  means remove
     $E_c^{k+1} \leftarrow \emptyset;$ 
    for each  $(v_m, v_n) \in E_c^k$  build new edges
        if ( $v_m \neq v_i \wedge v_m \neq v_j \wedge v_n \neq v_i \wedge v_n \neq v_j$ )
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_m, v_n)\};$ 
        for each  $v_n \in N$ 
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_n, v_s)\};$ 
     $k \leftarrow k + 1;$ 
}

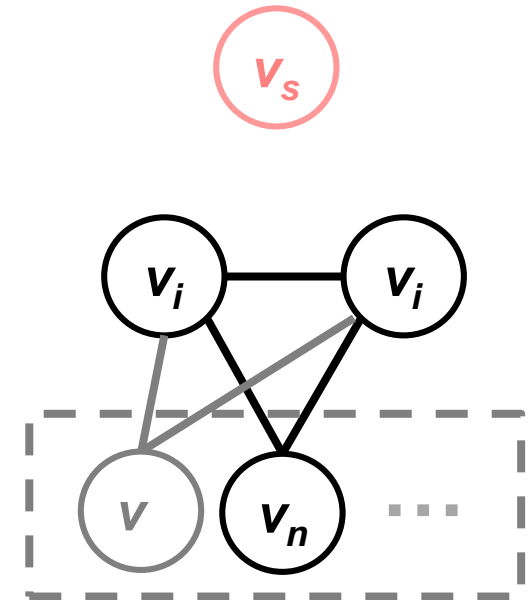
```

G_c^k = compatibility graph in K_{th} iteration

V_c^k = set of vertices in G_c^k

E_c^k = set of edges in G_c^k

v_s = supervertex of v_i and v_j



$N = \{\text{common neighbors}\}$

Tseng-Siewiorek Algorithm (2)

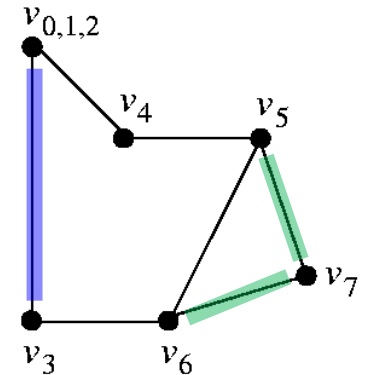
```

 $k \leftarrow 0;$ 
 $G_c^k(V_c^k, E_c^k) \leftarrow G_c(V_c, E_c);$ 
while ( $E_c^k \neq \emptyset$ ) {
    find  $(v_i, v_j) \in E_c^k$  with largest set of common neighbors;
     $N \leftarrow$  set of common neighbors of  $v_i$  and  $v_j$ ;
     $s \leftarrow i \cup j$ ;
     $V_c^{k+1} \leftarrow V_c^k \cup \{v_s\} \setminus \{v_i, v_j\};$ 
     $E_c^{k+1} \leftarrow \emptyset;$ 
    for each  $(v_m, v_n) \in E_c^k$ 
        if ( $v_m \neq v_i \wedge v_m \neq v_j \wedge v_n \neq v_i \wedge v_n \neq v_j$ )
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_m, v_n)\};$ 
    for each  $v_n \in N$ 
         $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_n, v_s)\};$ 
     $k \leftarrow k + 1;$ 
}

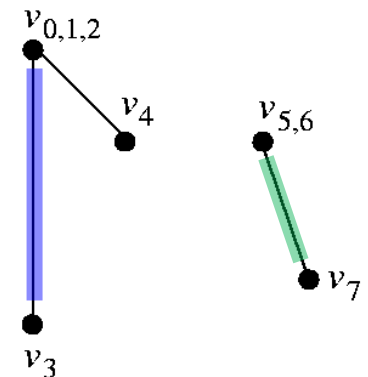
```

keep edges touch
neither v_i nor v_j
e.g. edge (v_{012} , v_3)

new edge between N and v_s
e.g. edge (v_5 , v_7)



\downarrow $v_i, v_j = v_5, v_6$



Quiz

Q1: Draw compatibility graph

Q2: What is minimum test length using T-S algorithm?

t_0	0xx10
t_1	x1x10
t_2	0x11x
t_3	00x11

FFT

- Q: What is practical problem with this method?
 - ◆ Practically, there are many don't cares in test cubes
 - ◆ X-unfilled test generation is slower and length is longer
 - * than X-filled test generation

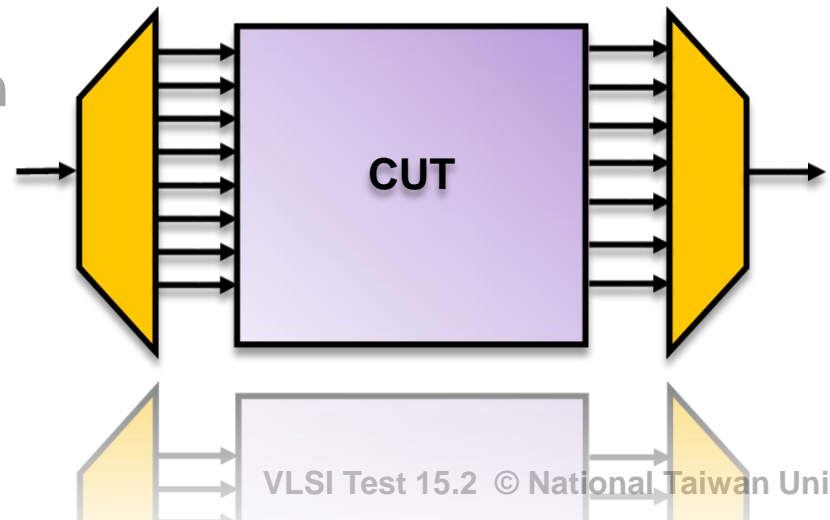
t_0	0xx10xxxxxx1
t_1	0xx1xxxxxxxxxx
t_2	0x01xxx1xxxx
t_3	01xx0xxxxxxxx
t_4	X0xx0xxxxx0x
t_5	1xxxxxxxxxxxx
t_6	x1x00xxxx1xx
t_7	11xx0xxxxxx0

t_0	011101100011
t_1	010111000000
t_2	010011010101

Need STC with X-filled

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STC with X-filled (1)

- **Reverse-order fault simulation**

- ♦ Fault simulate X-filled patterns in reverse order of ATPG

- * Delete redundant test patterns

- ♦ Example:

- * First simulate t_4 , and then t_3, t_2, t_1

- * Delete t_1 . Choose test set $\{t_4, t_3, t_2\}$

- ♦ Advantage: **Simple, no dictionary needed. Most popular STC**

ATPG order ↓	f_1	f_2	f_3	f_4	f_5	Fault sim. order ↑
	t_1		X			
	t_2			X	X	
	t_3		X	X	X	
	t_4	X		X		

- Q: Why ATPG generated redundant t_1 at beginning?

STC with X-filled (2)

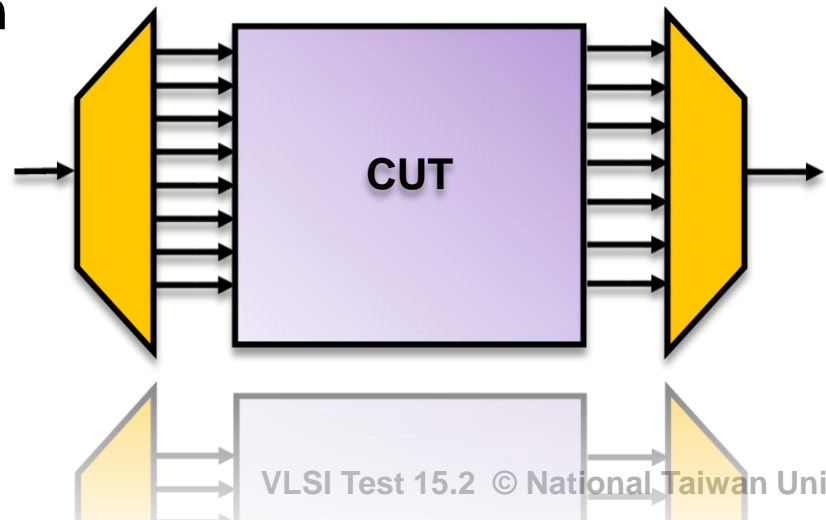
- **Random-order fault simulation**
 - ♦ Fault simulate test patterns in random order
 - ♦ Example:
 - * First simulate t_4 , and then t_2 , t_1 , t_3
 - * Choose test set $\{t_4, t_1, t_2\}$

	f_1	f_2	f_3	f_4	f_5
t_1		X			
t_2			X		X
t_3		X		X	X
t_4	X			X	

Too Many Orders to Try

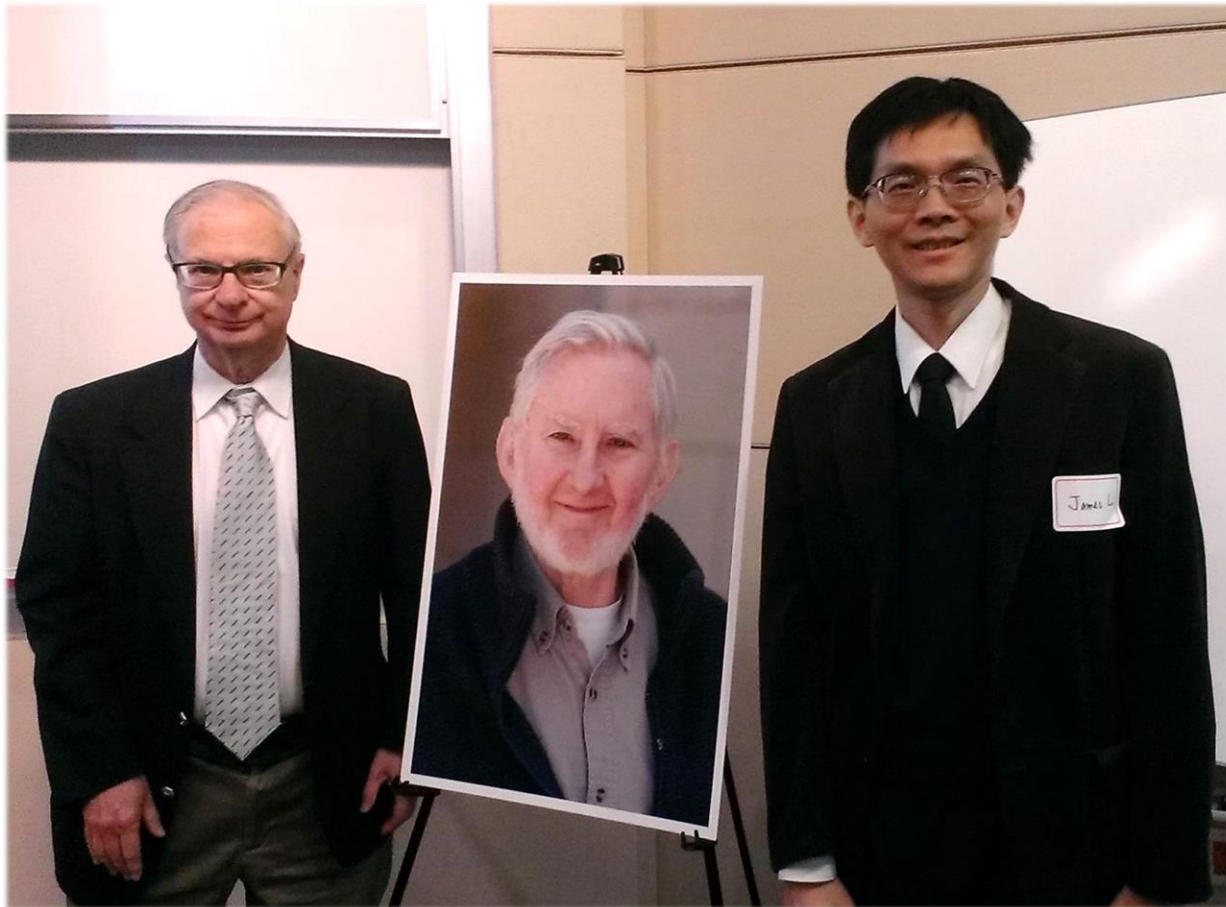
Summary

- Static test compression
 - ◆ With fault dictionary
 - * **Minimum set-covering** problem
 - **Quine-McCluskey** or **01-ILP**
 - * **Too large dictionary**
 - ◆ Without fault dictionary
 - * **Compatibility graph** (X-unfilled)
 - T-S Algorithm
 - * **Reverse order fault simulation** (X-filled)
 - Most popular solution



Three Authors Together

- Prof. Siewiorek, Prof. McCluskey, Prof. James Li
- 2016 Stanford University



FFT1

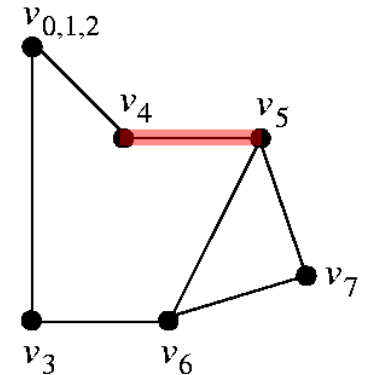
```

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     $N \leftarrow$  set of common neighbors of  $v_i$  and  $v_j$ ;
     $s \leftarrow i \cup j$ ;
     $V_c^{k+1} \leftarrow V_c^k \cup \{v_s\} \setminus \{v_i, v_j\};$ 
     $E_c^{k+1} \leftarrow \emptyset;$ 
    for each  $(v_m, v_n) \in E_c^k$ 
        if ( $v_m \neq v_i \wedge v_m \neq v_j \wedge v_n \neq v_i \wedge v_n \neq v_j$ )
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_m, v_n)\};$ 
    for each  $v_n \in N$ 
         $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_n, v_s)\};$ 
     $k \leftarrow k + 1;$ 
}

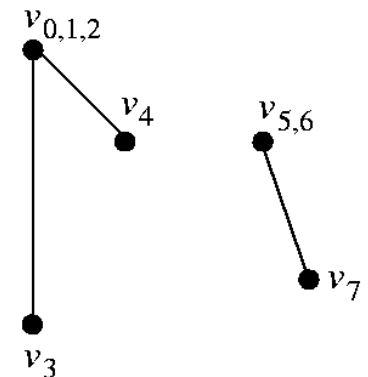
```

keep edges touch
neither v_i nor v_j
e.g. edge (v_{012} , v_3)

new edges between N and v_s
e.g. edge (v_5 , v_7)



$v_i, v_j = v_5, v_6$



Why Edge (v_4, v_5) Removed?

FFT2

- MCP is NP-hard
- Show an example when TS-algorithm fails to find an optimal solution

