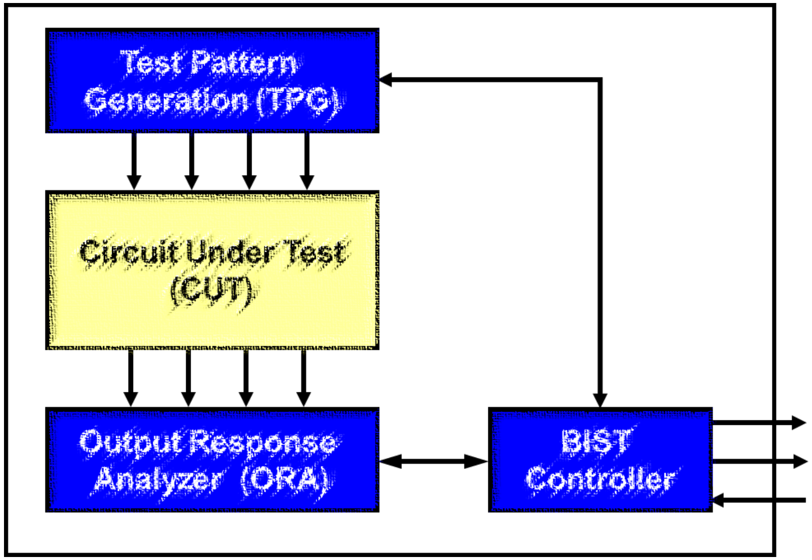
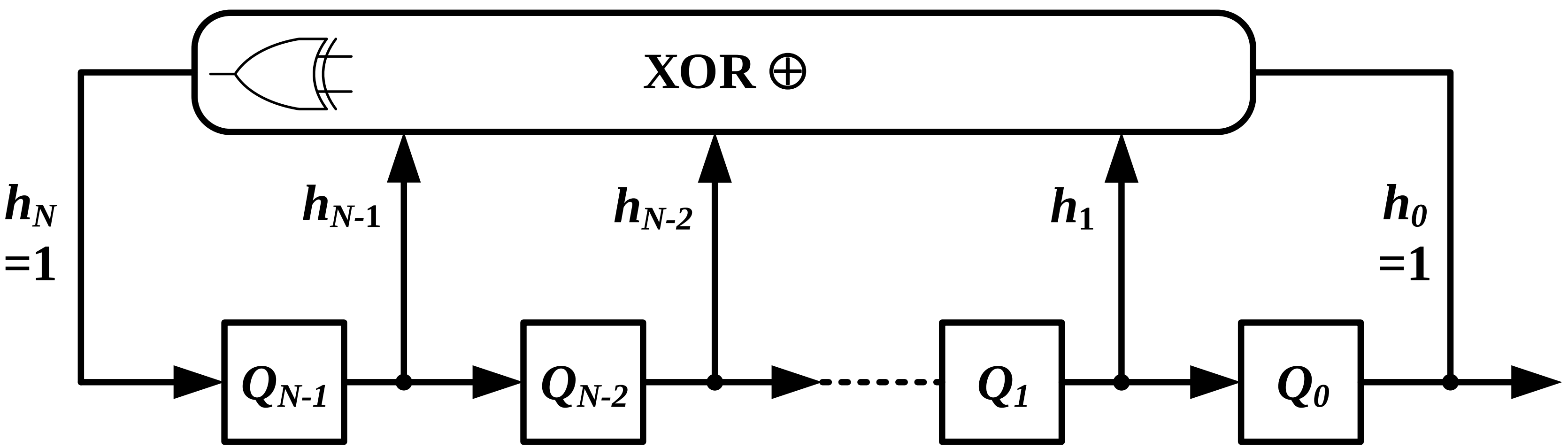
BIST Part1 - TPG

- Introduction
- Test Pattern Generation (TPG)
  - Deterministic: ROM, Algorithm, Counter
  - Pseudo Random
    - Linear Feedback Shift Register, LFSR (1977)
      - Two types of LFSR
      - Design of LFSR
    - Cellular Automata, CA (1984)



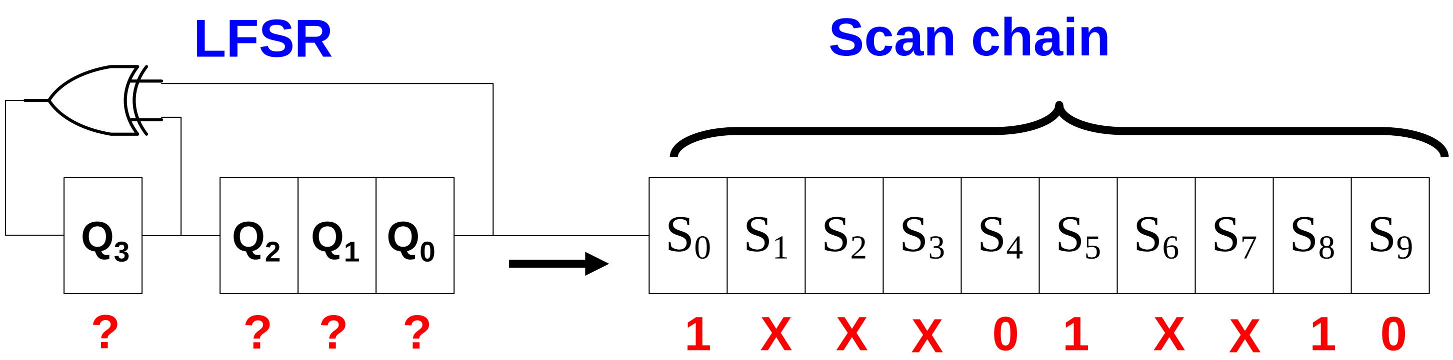
Design of LFSR

- Questions to answer
  - How to find seed?
  - What is LFSR degree?
  - What polynomial?



How to Find a Seed?

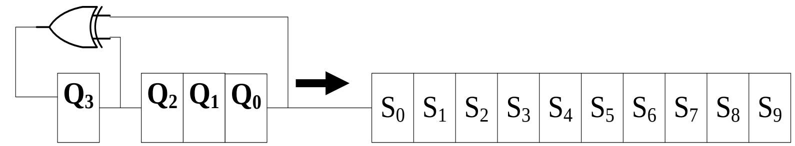
- Given LFSR and test pattern, find initial state of LFSR (seed)
- Example:
  - LFSR feeds scan chain inputs
  - *S0~S9* are test pattern (specified by ATPG)
  - Q_3~Q_0 are seeds



- Two methods:
  - Cycle-by-cycle tracing
  - System of linear equations

Cycle-by-cycle Tracing (1)

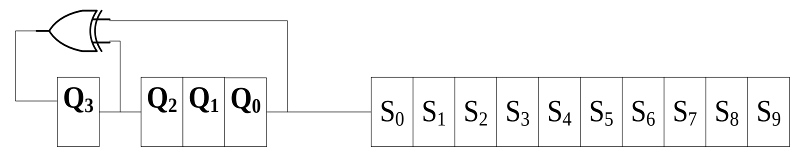
- **Test pattern desired**
  - $S_0\ S_1\ \ldots\ S_9 = 1\ X\ X\ X\ 0\ 1\ X\ X\ 1\ 0$



$$
\begin{array}{r}
9 \\ 8 \\ 7 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ \text{time } 0
\end{array}
\begin{bmatrix}
\text{last bit } S_0 & 1 \\
 & X \\
 & X \\
 & X \\
 & 0 \\
 & 1 \\
 & X \\
 & X \\
 & 1 \\
\text{First bit } S_9 \rightarrow & 0
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
 & & & 1 \\
 & & 1 & X \\
 & 1 & & X \\
1 & & & X \\
 & & & 0 \\
 & & 0 & 1 \\
 & 0 & 1 & X \\
0 & 1 & & X \\
1 & & & 1 \\
 & & 1 & 0
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
 & & & 1 \\
 & & 1 & 1 \\
 & 1 & 1 & X \\
1 & 1 & & X \\
1 & & & 0 \\
 & & 0 & 1 \\
 & 0 & 1 & 1 \\
0 & 1 & 1 & X \\
1 & 1 & & 1 \\
1 & & 1 & 0
\end{bmatrix}
\Rightarrow next\_page
$$

$Q_3\ Q_2\ Q_1\ Q_0$ **$Q_1(t)=Q_0(t+1)$** **$Q_3(t+1) = Q_3(t)+Q_0(t)$**

Cycle-by-cycle Tracing (2)



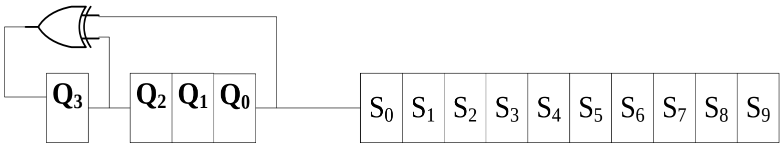
Solve $X_3=0$, $X_2=1$ Solve $X_1=1$

$$\Rightarrow \begin{bmatrix} & & & 1 \\ & & 1 & 1 \\ & 1 & 1 & X_3 \\ 1 & 1 & X_3 & X_2 \\ 1 & X_3 & X_2 & 0 \\ X_3 & X_2 & 0 & 1 \\ X_2 & 0 & 1 & 1 \\ 0 & 1 & 1 & X_1 \\ 1 & 1 & X_1 & 1 \\ 1 & X_1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} & & & 1 \\ & & 1 & 1 \\ & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & X_1 \\ 1 & 1 & X_1 & 1 \\ 1 & X_1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} & & & 1 \\ & & 1 & 1 \\ & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Seed = 1110

This is Slow! Can We Do Better?

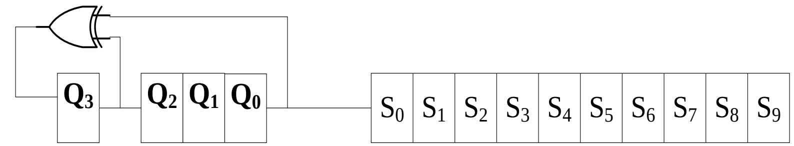
System of Linear Equations (1)



- **Initial conditions**
  - $S_9 = Q_0$
  - $S_8 = Q_1$
  - $S_7 = Q_2$
  - $S_6 = Q_3$
- $S_n = S_{n+4} \oplus S_{n+1}$
  - $S_5 = S_9 \oplus S_6 = Q_3 \oplus Q_0$
  - $S_4 = S_8 \oplus S_5 = Q_3 \oplus Q_1 \oplus Q_0$
  - $S_3 = S_7 \oplus S_4 = Q_3 \oplus Q_2 \oplus Q_1 \oplus Q_0$
  - $S_2 = S_6 \oplus S_3 = Q_2 \oplus Q_1 \oplus Q_0$
  - $S_1 = S_5 \oplus S_2 = Q_3 \oplus Q_2 \oplus Q_1$
  - $S_0 = S_4 \oplus S_1 = Q_2 \oplus Q_0$
- **4 variables, 10 equations**

$$
\begin{bmatrix}
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
Q_3 \\ Q_2 \\ Q_1 \\ Q_0
\end{bmatrix}
=
\begin{bmatrix}
S_0 = 1 \\
S_1 = X \\
S_2 = X \\
S_3 = X \\
S_4 = 0 \\
S_5 = 1 \\
S_6 = X \\
S_7 = X \\
S_8 = 1 \\
S_9 = 0
\end{bmatrix}
$$

System of Linear Equations (2)

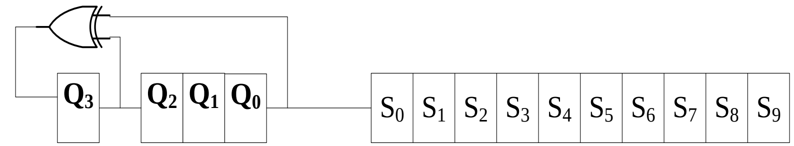


- **$S_9 = 0 = Q_0$**
- **$S_8 = 1 = Q_1$**
- **$S_5 = 1 = Q_0 \oplus Q_3 \Rightarrow Q_3 = 1$**
- **$S_4 = 0 = Q_3 \oplus Q_1 \oplus Q_0$**
  - **consistent !**
- **$S_0 = 1 = Q_2 \oplus Q_0 \Rightarrow Q_2 = 1$**
- **Solution :**
  - **Seed =[Q_3 Q_2 Q_1 Q_0]= 1110**

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_3 \\ Q_2 \\ Q_1 \\ Q_0 \end{bmatrix} = \begin{bmatrix} S_0 = 1 \\ S_1 = X \\ S_2 = X \\ S_3 = X \\ S_4 = 0 \\ S_5 = 1 \\ S_6 = X \\ S_7 = X \\ S_8 = 1 \\ S_9 = 0 \end{bmatrix}$$

Automate This Process?

Derive Linear Equations (1)



- **Start with identity matrix *I***
  - $t = [h_3\ h_2\ h_1\ h_0]$
- **Iteratively bottom up, *k*=4~10**
  - $\text{Row}_k = t \times (\text{Row}_{k-4} \sim \text{Row}_{k-1})$
  - ***k*++**
- **Example**
  - $t = [h_3\ h_2\ h_1\ h_0] = [1\ 0\ 0\ 1]$
  - $t \times I_1 = [\ 1\ 0\ 0\ 1]$
  - $t \times I_2 = [1\ 0\ 1\ 1]$
  - **…..**
  - **Eventually, 10 rows, 4 columns**

$$
\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}
\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{I}
\rightarrow
\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\rightarrow
\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\cdots \rightarrow
\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Solve Linear Equations (1)

- ***Gauss-Jordan elimination,*** **but *Mod-2 addition***
- **Example**
  - **Append specified bits to last column (*Augment matrix*)**
  - **Remove useless rows with X**
  - **Exchange row 1 with row 2**

$$
\left[\begin{array}{cccc:c}
0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & X \\
0 & 1 & 1 & 1 & X \\
1 & 1 & 1 & 1 & X \\
1 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & X \\
0 & 1 & 0 & 0 & X \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{array}\right]
\quad
\begin{bmatrix}
0 & 1 & 0 & 1 & 1 \\
 & & & & \\
 & & & & \\
 & & & & \\
1 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
 & & & & \\
 & & & & \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\quad
\begin{bmatrix}
1 & 0 & 1 & 1 & 0 \\
 & & & & \\
 & & & & \\
 & & & & \\
0 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 \\
 & & & & \\
 & & & & \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

Solve Linear Equations (2)

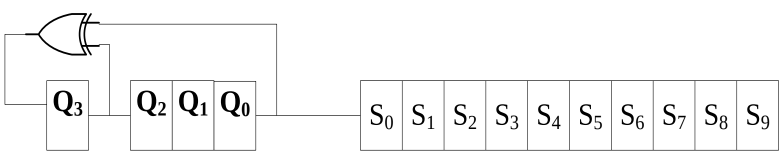
- **Pivot column #1 (add row 1 to row 3)**
- **Pivot column #2 (do nothing)**
- **Pivot column #3 (add row3 to row1; add row3 to row4)**
- **Pivot column #4 (add row5 to row1, add row 5 to row2)**
- **Solution: seed =[Q_3 Q_2 Q_1 Q_0] = [1 1 1 0]**

$$
\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
$$

Pivot elements **are circled**

Summary

- Number of ***equations*** depends on ***number of care bits***
- Number of ***variables*** = ***LFSR degree***
- If more care bits than degree,
  - May not be solvable every time
  - This time we are lucky :)

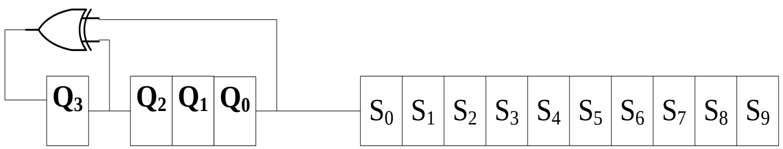


$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_3 \\ Q_2 \\ Q_1 \\ Q_0 \end{bmatrix} = \begin{bmatrix} S_0 = 1 \\ S_1 = X \\ S_2 = X \\ S_3 = X \\ S_4 = 0 \\ S_5 = 1 \\ S_6 = X \\ S_7 = X \\ S_8 = 1 \\ S_9 = 0 \end{bmatrix}$$

Can We Guarantee Solution Exists?

FFT

- **Q: What should we do if we find no solution?**
  - **1**
  - **2**
  - **3**



$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_3 \\ Q_2 \\ Q_1 \\ Q_0 \end{bmatrix} = \begin{bmatrix} S_0 = 1 \\ S_1 = X \\ S_2 = X \\ S_3 = X \\ S_4 = 0 \\ S_5 = 1 \\ S_6 = X \\ S_7 = X \\ S_8 = 1 \\ S_9 = 0 \end{bmatrix}$$