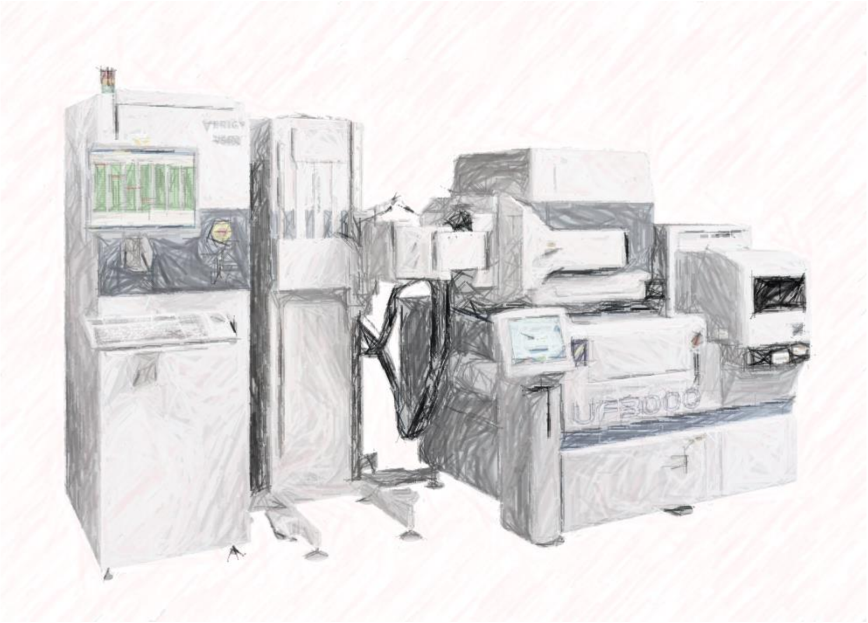
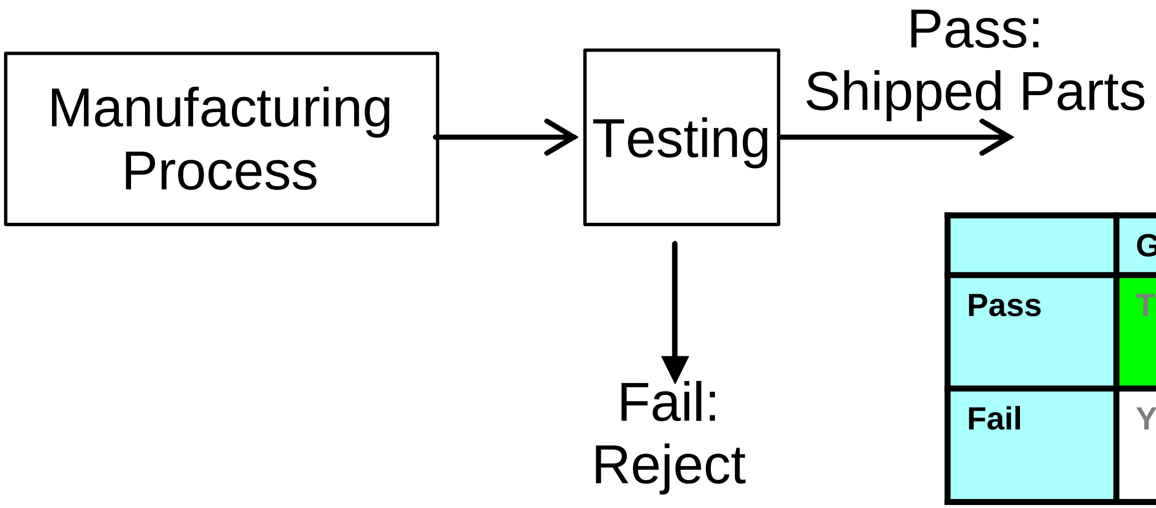
Introduction

- What Is Testing
- Types of Testing
- <u>Test Quality</u>
  - <u>Defect per Million (DPM)</u>
  - Models to Predict DPM
  - Models to Predict Yield
- Test Economics
- Issues in Testing
- Conclusion



How to Measure Test Quality?

- *Defect Level (DL)*
  - Fraction of bad IC passing the test (test escapes)
  - Measured by *Defective Parts per Million (DPM)*
    - < 200 DPM is acceptable for some IC
    - >1,000 DPM is very bad for IC



	Good IC	Bad IC
Pass	True PASS	Test Escapes (Defect Level)
Fail	Yield Loss	True Reject

Low DPM Means Good Test Quality

IC DPM → System DPM

- If you think DPM= 1,000 (0.1%) is good enough, consider this case

- A system has twenty IC (each 1000 DPM), what is system DPM?
  - No IC is defective for system to work correctly
    - System DPM = $1-(1-0.1\%)^{20}$ = 2% = 20,000 DPM !
  - If you sell a million system
    - You have to fix 20,000 of them!

Low IC DPM is Very Important

Quiz

Q: (Cont'd from last slide)
20 IC in a system.
Suppose we improve IC test quality to 200DPM
What is system DPM?

A:

IC DPM ↓ 5X, System DPM↓ 5X

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    - Agrawal (Bell Lab, 1982),), Poisson distribution
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Measure *Real DPM* Is difficult.
Can we predict it?

Brown & Williams Model [Williams 81]

$$DL = 1 - Y^{(1-FC)}$$

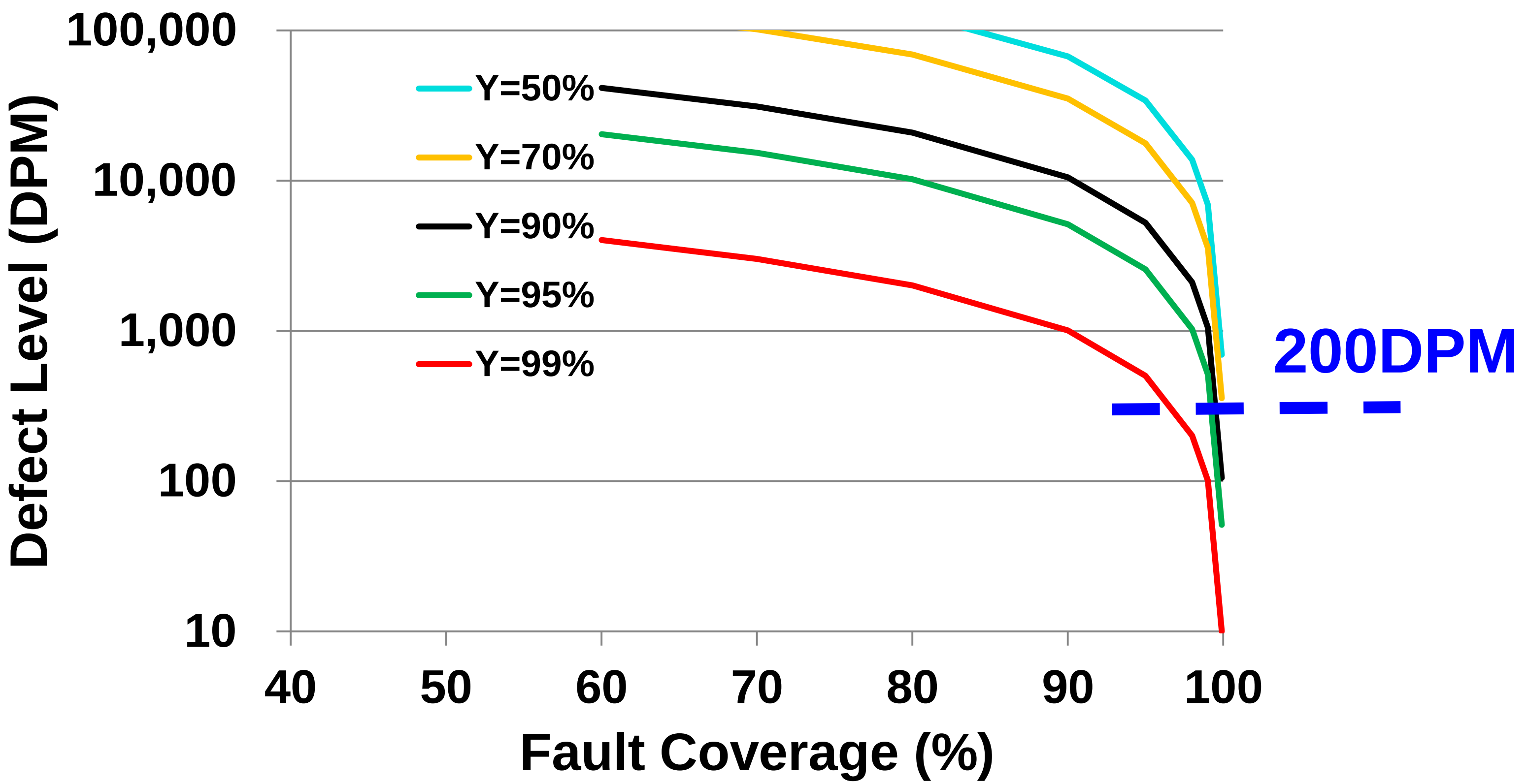
- ***Y=Yield,*** **fraction of total manufactured IC that are good**
  - **High Y means good manufacturing quality**
  - **Unknown parameter, estimated by fab. data or prediction**

$$yield, Y = \frac{number\ \ of\ \ good\ \ chips}{number\ \ of\ \ total\ \ chips} \leq 100\%$$

- ***FC = Fault Coverage,*** **fraction of detected faults**
  - **High FC means good test quality**
  - **Known parameter, from *fault simulator* (see *fault simulation*)**

$$fault\ coverage, FC = \frac{number\ \ of\ \ detected\ \ faults}{number\ \ of\ \ total\ \ faults} \leq 100\%$$

DL v.s. FC



- Required DL = 200 DPM, then we need

Yield	50%	70%	90%	95%	99%
FC	99.97%	99.95%	99.8%	99.6%	98%

* DL ≈ (1-FC)(1-Y) when Y ≈ 1

poor yield
good test

⇔

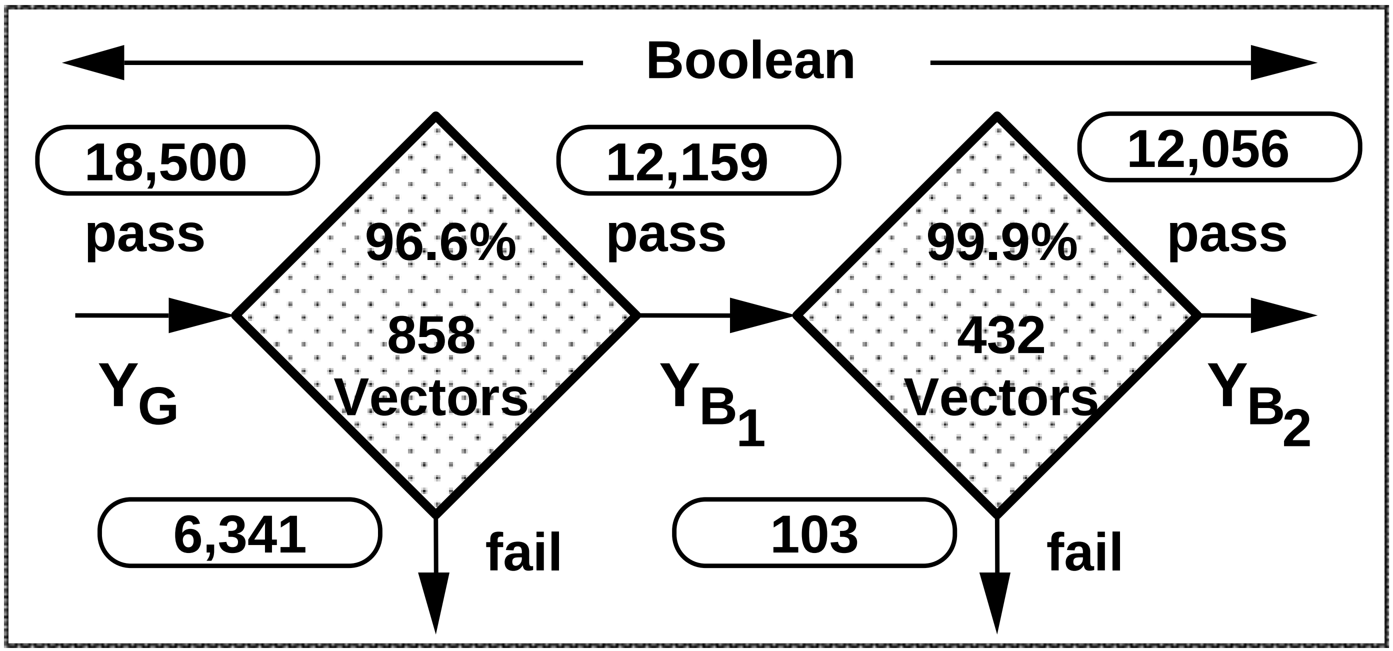
good yield
poor test

B&W Proof (simplified!)

- Assume each fault occurrence probability is q (uniform independent)
  - Total t faults in an IC
  - d out of t faults detected by this test → $FC = d/t$
- Event A = an IC is good (no fault at all)
  - Prob(A) = probability that an IC is good → $Prob(A)=(1-q)^t$
  - Y = fraction of IC that are good → $Y=Prob(A)$
- Event B = an IC pass the test
  - Prob(B) = an IC free of *d* faults → $Prob(B)=(1-q)^d$
- Event AB = an IC is good and pass test
  - Assume no overkill → $Prob(AB) = Prob(A)$
- *Quality Level (QL)* = fraction of good IC passing the test
  - $QL = Prob(A|B) = Prob(AB)/Prob(B) = (1-q)^{t-d} = (1-q)^{t(1-d/t)} = Y^{(1-FC)}$
- *Defect Level (DL)* = fraction of bad IC passing the test
  - $DL = 1-QL = 1-Y^{(1-FC)}$ QED

Motorola 6802 Experiment [McCluskey 88]

- Y = 12,056/18,500 =65.16%
- test #1 FC = 96.6%
- Theoretical DL = $1\text{-}Y^{(1\text{-}FC)}$ = 14,454 DPM
- Experimental DL = 8,471 DPM (103 over 12,159 IC passing test #1)



Quiz

$$DL = 1 - Y^{(1-FC)}$$

Q1: Use B&W model.
Y=98%, FC=70%. DL=?

A:

Q2: Your boss is not happy about DL. If we add some design for testability (DFT) circuits to improves FC but harms yield a little.
Y=97%, FC=99%. DL=?

A:

DFT Very Important for DL Improvement

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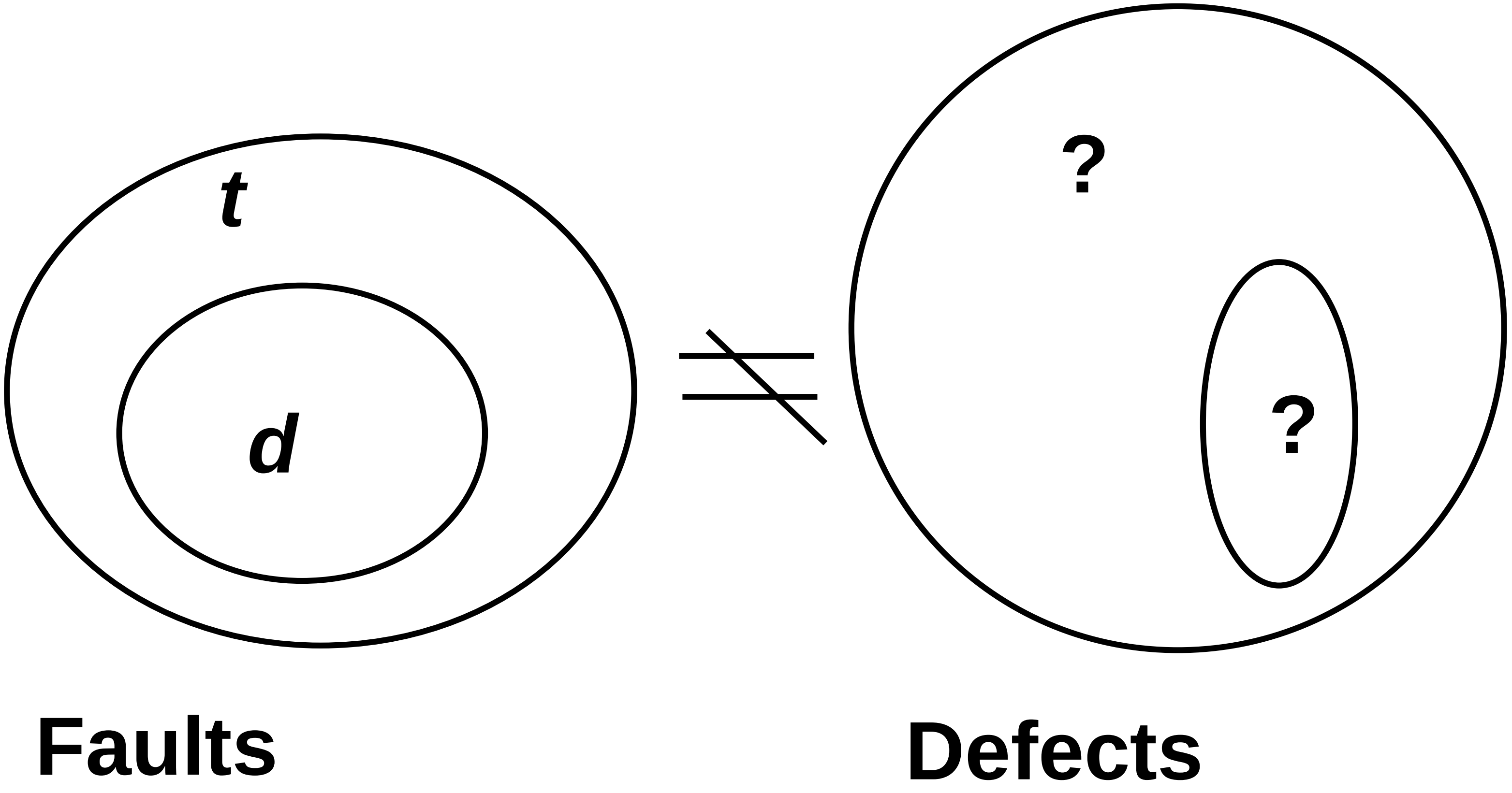
Why B&W Not Accurate?

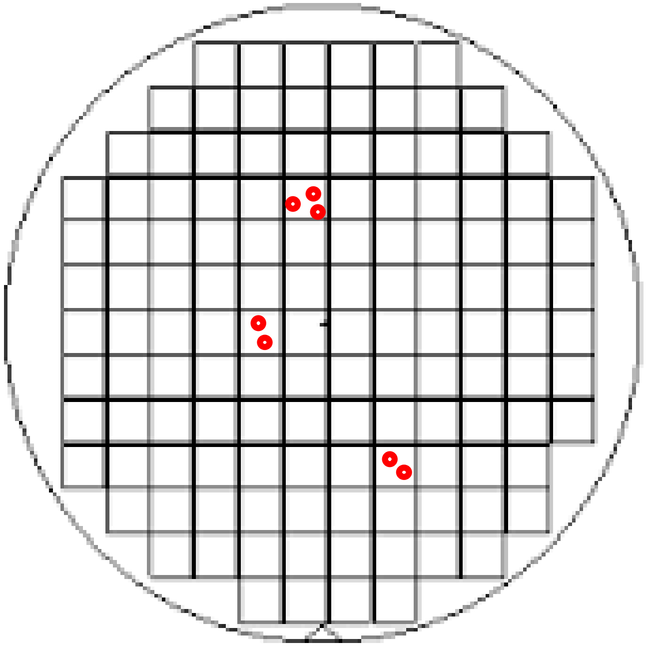
- 1. Fault coverage does not represent defect coverage
  - 100% FC ≠ 0 DPM (see *fault modeling* chapter)
- 2. Defects are clustered
  - Agrawal used Poisson distribution



Agrawal Model [Agrawal 82]

$$DL = \frac{(1-FC)(1-Y)e^{-(n-1)FC}}{Y+(1-FC)(1-Y)e^{-(n-1)FC}}$$

- Actual data show defects are clustered
  - *n* =average num. of defects on a bad die
- We need two unknown parameters
  - *n*: from experiment
  - *Y*: from fab. or prediction
- Motorola experiment revisited
  - Experiment DL = 8,471 DPM
  - B&W DL =14,454 DPM
  - Agrawal
    - *n* =1, DL = 17,849 DPM
    - *n* =2, DL = 6,869 DPM
    - experimental *n* between 1 and 2



7 defects
3 bad die
n=7/3=2.3

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    - <u>Simple</u>
    - <u>Critical Area</u>
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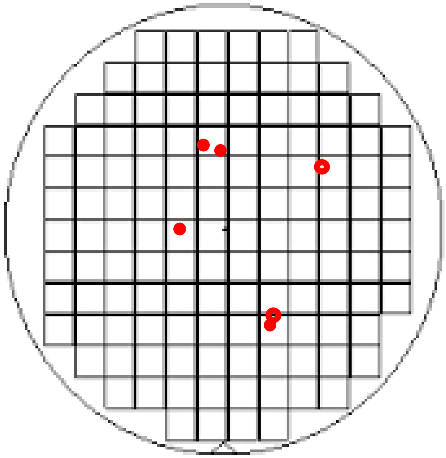
Measure *Real Yield* Is too late. Can we predict it?

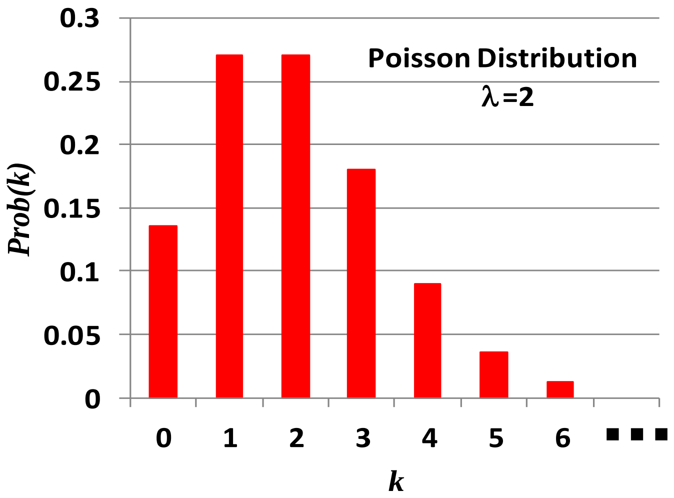
Yield Estimation (Simple)

- Number of defects *k* on a die obeys ***Poisson distribution*** with mean λ
  - ♦ *Prob(k)* = prob exactly *k* defects on a die
  - ♦ A die is good when *k*=0
  - ♦ Yield = *Prob(k=0)*



$$Prob(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$Y = Prob(k=0) = e^{-\lambda}$$



- *A* = die area, *D* = defect density
- λ = *AD* = ave. number of defects per die

$$\boxed{Y = e^{-AD} \quad \textit{Larger area, lower yield}}$$

Quiz

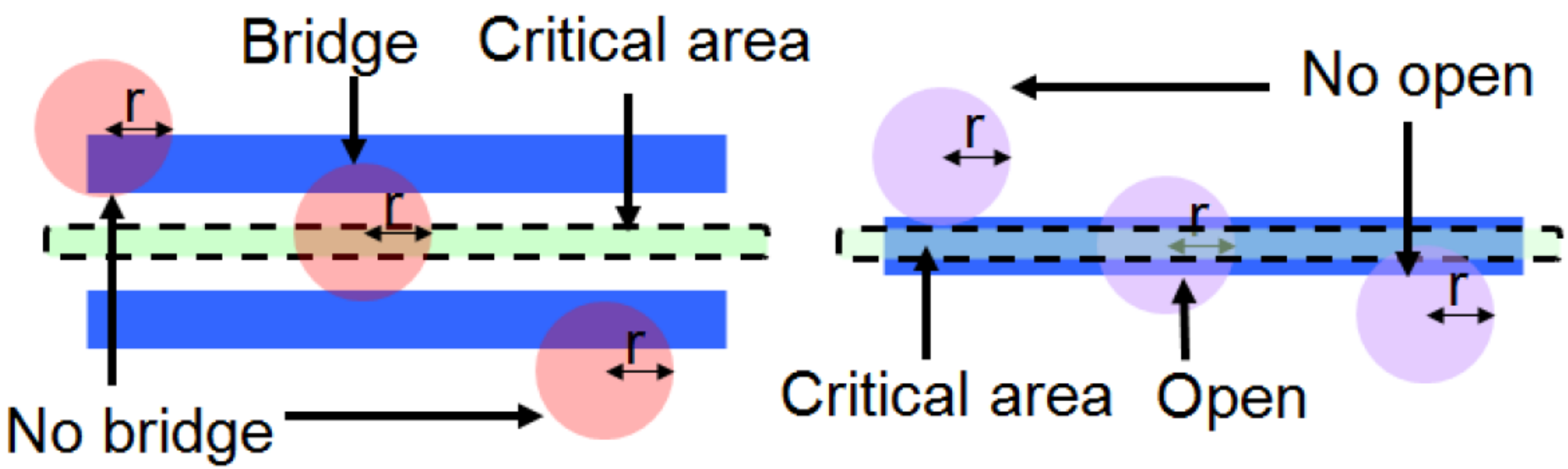
Q: A=0.8cm^2, D=0.5/cm^2. What is yield?

$$\boxed{Y = e^{-AD}}$$

A:

Critical Area , CA

- **Problem with Y^{-AD} model**
  - **Some area NOT sensitive to defect. Use total area is pessimistic**
- ***Critical area, CA(r)***
  - **Area where center of defect (radius *r*) land to create a *fault***
- **Example:**
  - **(a) bridging fault (b) open fault**

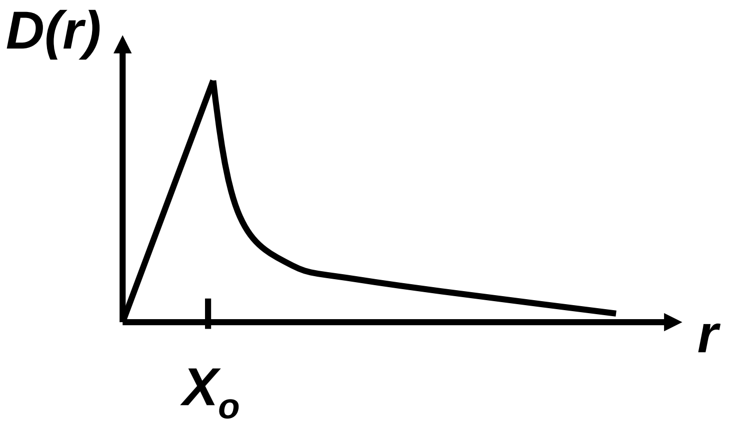


(a) (b)

Defect Size and Density Distribution, DSDD

- *D(r)* = defect density per unit defect radius *r*
- Typical DSDD curve:

$$D(r) = \begin{cases} D_0 \dfrac{2(p-1)r}{(p+1)X_o^2}, & 0 \le r < X_o \\[2ex] D_0 \dfrac{2(p-1)X_o^{p-1}}{(p+1)r^p}, & r \ge X_o \end{cases}$$



- *D(r)* is process dependent.
  - Example values:
  - $4.5\text{x}10^{-6}$ cm $\le X_o \le 5.5\text{x}10^{-6}$ cm
  - $2.0 \le p \le 3.0$
  - $0.25\ /\text{cm}^2 \le D_o \le 0.6\ /\text{cm}^2$

$$\boxed{Total\ Defect\ Density = D_0 = \int_0^{\infty} D(r)dr}$$

Yield Estimation (CA)

- **Expected number of defects per die landing in CA**

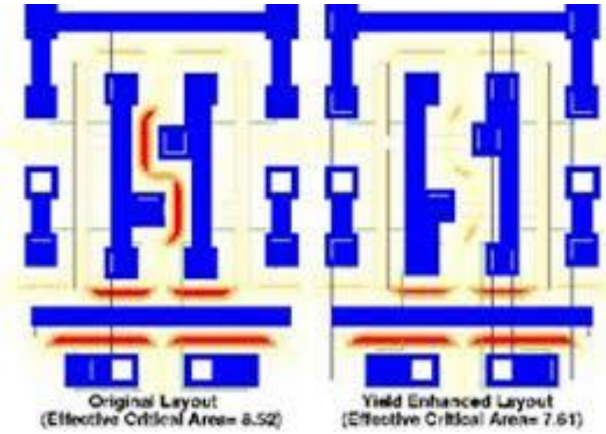
$$\lambda = \int_0^\infty CA(r) \times D\ (r)dr$$

- **Again, *Poisson distribution* with mean λ**
  - **Yield = *Prob(k=0)*, zero defect in CA**

$$\boxed{Y = e^{-\lambda} = e^{-\int_0^\infty CA(r)\times D\ (r)dr}} \Leftrightarrow \boxed{Y = e^{-AD}}$$

simple model

- ***Critical area analysis (CAA)* is useful tool to estimate yield**
  - **1. modify design for yield improvement**
  - **2. extract bridging or open faults for ATPG**



Summary

- Test quality measured by **Defect per Million (DPM)**
- Models for DPM
  - Brown & Williams :

$$\boxed{DL = 1 - Y^{(1-FC)}}$$

  - Agrawal

$$\boxed{DL = \frac{(1-FC)(1-Y)e^{-(n-1)FC}}{Y + (1-FC)(1-Y)e^{-(n-1)FC}}}$$

- Models for Yield
  - Simple model

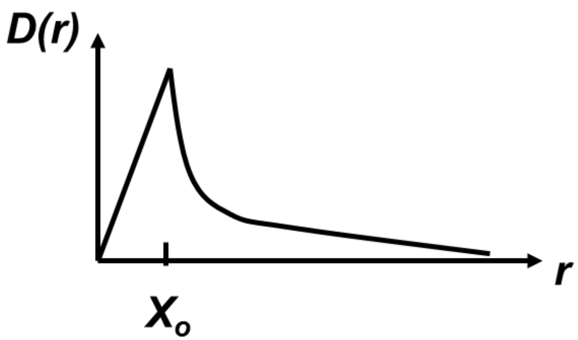
$$\boxed{Y = e^{-AD}}$$

  - Critical area

$$\boxed{Y = e^{-\int_0^{\infty} CA(r)\times D\ (r)dr}}$$

Food for Thoughts, FFT

- **Q: What is unit of *D(r)* ?**
  - ***D(r)* = defect density per unit defect radius *r***



$$Total\ Defect\ Density = D_0 = \int_0^{\infty} D\,(r)dr$$

APPENDIX#1: Agrawal Proof (1)

- **Number of defects *k* on a bad IC obeys Poisson distribution with mean *n p(k)* = probability exactly *k* faults in a bad IC *(k >0)***

$$p(k) = (1-Y)\frac{(n-1)^{k-1}e^{-(n-1)}}{(k-1)!} \quad k = 1,2,3\ldots\ (shifted\ Poisson)$$

$$p(0) = Y$$

- **Suppose total *t* defects, *d* is detected by test, so *FC = d/t***
- **$q_m(k)$ = prob. detecting exactly *m* defects when an IC has *k* defects**

$$q_m(k) = \frac{\binom{k}{m}\binom{t-k}{d-m}}{\binom{t}{d}} \quad hypergeometric\ distribution$$

- **$q_0(k)$ = prob. of passing a bad IC, having exactly *k* defects**

$$q_0(k) = \frac{\binom{t-k}{d}}{\binom{t}{d}} \approx (1-\frac{d}{t})^k = (1-FC)^k$$

Agrawal Proof (2)

- **TE = Prob. bad IC passing the test**

$$TE = \sum_{k=1}^{t} q_0(k)\,p(k) = \sum_{k=1}^{t} (1-FC)^k (1-Y)\frac{(n-1)^{k-1}}{(k-1)!}e^{-(n-1)}$$

$$= (1-Y)\sum_{k=1}^{t}(1-FC)\ \frac{[(1-FC)(n-1)]^{k-1}}{(k-1)!}e^{-(n-1)(1-FC)}e^{-(n-1)FC}$$

$$= (1-Y)(1-FC)e^{-(n-1)FC}\sum_{k=1}^{t}\frac{[(1-FC)(n-1)]^{k-1}}{(k-1)!}e^{-(n-1)(1-FC)}$$

$$\approx (1-Y)(1-FC)e^{-(n-1)FC}$$

$$\boxed{DL = \frac{TE}{Y+TE} = \frac{(1-FC)(1-Y)e^{-(n-1)FC}}{Y+(1-FC)(1-Y)e^{-(n-1)FC}}}$$

- **QED**

APPDENDIX #2: B&W Approximation

Tylor Exapnsion $$DL(Y)=\sum_{n=0}^{\infty}\frac{f^{(n)}(Y)}{n!}(Y-a)^n = f(1)+\frac{f'(1)}{1!}(Y-1)++\frac{f''(1)}{2!}(Y-1)^2+\ldots$$

when $Y\approx 1$, we expand $DL(Y)$ around a=1

$$DL(Y)=1-Y^{1-FC}$$

$$=(1-1^{1-FC})-(1-FC)\cdot 1^{-FC}(Y-1)-\frac{(-FC)(1-FC)1^{-FC-1}}{2!}(Y-1)^2-\ldots$$

$$=0+(1-FC)(1-Y)+\frac{(1-FC)FC}{2}(1-Y)^2+\ldots$$

$$\approx(1-FC)(1-Y)$$

Example FC=98.5%, Y=95%

$$DL(Y)=1-Y^{1-FC}=1-0.95^{0.015}=769\text{DPM}$$

$$DL(Y)\approx(1-FC)(1-Y)=750\text{DPM}$$

$$DL(Y)\approx(1-FC)(1-Y)+\frac{FC(1-FC)}{2}(1-Y)^2=768\text{DPM}$$

DL ≈ (1-FC)(1-Y)