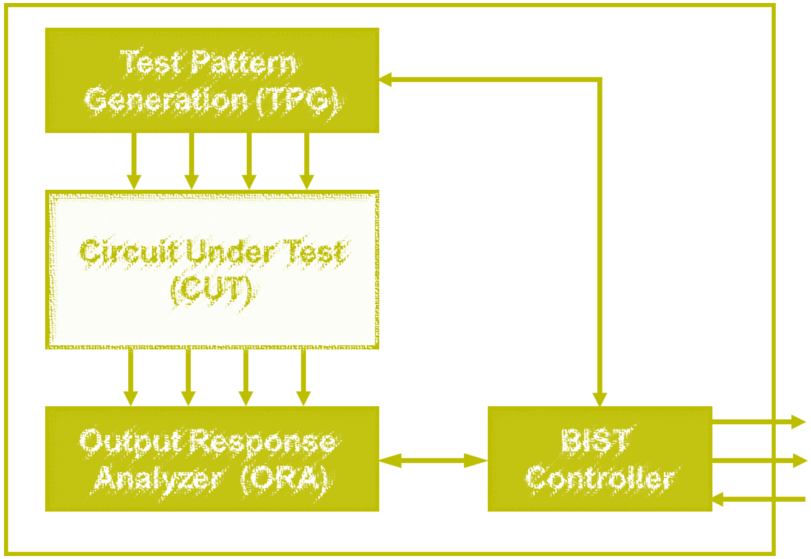
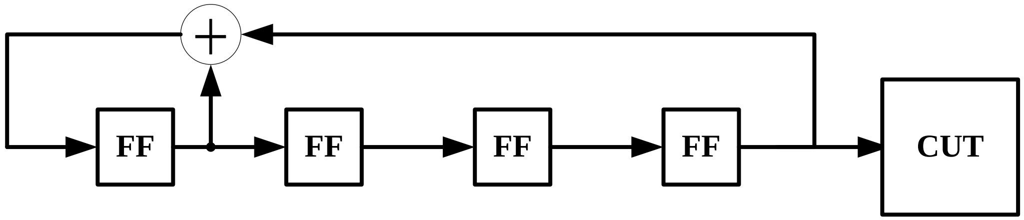
BIST Part 2

- **Output Response Analysis**
  - Simple ORA
  - **LFSR-based ORA**
    - **Serial : compress one bit at a time**
      - **CRC Theory**
      - **PAL analysis**
      - **How to design LFSR as ORA?**
    - Parallel : compress multiple bits at a time
- BIST Architecture
- Issues with BIST
- Conclusions

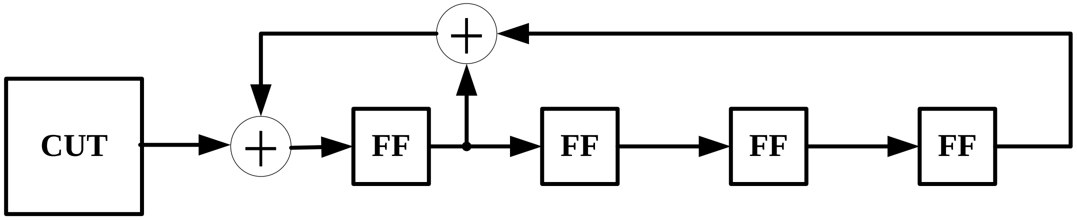


LFSR (Review)

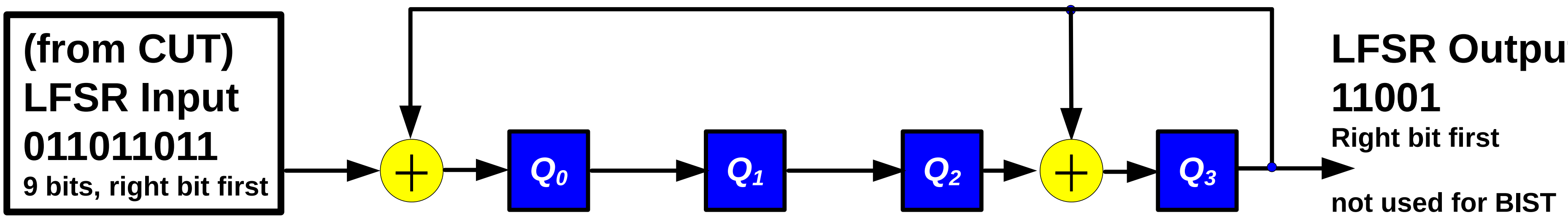
- LFSR consist of FF and feedback XOR
- Two applications of LFSR:
  - 1. LFSR without external input
    - Used for TPG



  - 2. LFSR with external input
    - Used for ORA



LFSR as ORA



cycle	LFSR input	Q_0	Q_1	Q_2	Q_3	LFSR output
0	011011011	0	0	0	0	
1-3	...					
4	01101	1	0	1	1	
5	0110	0	1	0	0	1
6	011	0	0	1	0	01
7	01	1	0	0	1	001
8	0	0	1	0	1	1001
9		1	0	1	1	11001

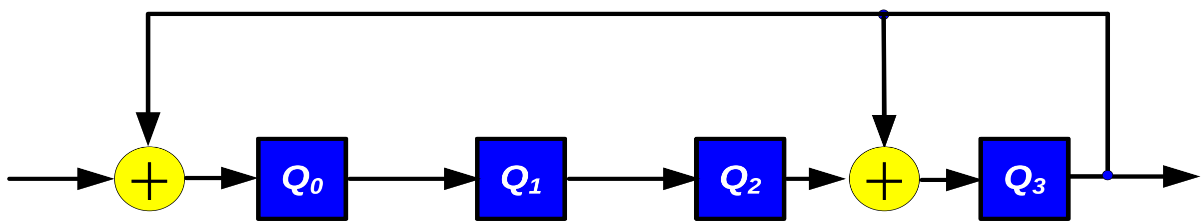
signature

not used

Quiz

Q: Suppose CUT output is ‘001001’. (right bit first)
What is the signature after 6 cycles?
ANS:

LFSR Input
(from CUT)
001001



cycle	LFSR input	Q_0	Q_1	Q_2	Q_3	LFSR output
0	001001	0	0	0	0	
1-3	…					
4						
5						
6						

Too Slow. Any Better Method?

Cyclic Redundancy Code (CRC) Theory

- **Represent bit streams by polynomials**
  - x **is dummy variable**
  - **Exponent represents delay**
  - **bits are coefficients**

$$M(x) = \sum_i b_i x^i$$

- **Example:** 011011011 → $x + x^2 + x^4 + x^5 + x^7 + x^8$
  - **Left bits = LSB, right bits =MSB**

For more details, see reference book (BA) or textbooks in *finite field*

Évariste Galois
1811-1832

Modular-2 Arithmetic

- ***Modulo-2:*** **Addition (=subtraction) is XOR, Multiplication is AND**
  - **0+0=0, 0+1=1, 1+0=1, 1+1=0**
  - **0x0=0, 0x1=0, 1x0=0, 1x1=1**
  - ***aka. Galois Field 2, GF(2)***
- **GF(2) Multiplication** **GF(2) Division**

$$(x^3+x^2+x+1)\times(x^2+x+1)$$

$$\begin{array}{r}
x^3+x^2+x+1 \\
\underline{x^2+x+1} \\
x^3+x^2+x+1 \\
x^4+x^3+x^2+x \\
\underline{x^5+x^4+x^3+x^2} \\
x^5+0+x^3+x^2+0+1
\end{array}$$

$$\begin{array}{r l}
 & x^3+x^2+x+1 \\
x^2+x^1+1 & \overline{)\,x^5+0+x^3+x^2+0+1} \\
 & \underline{x^5+x^4+x^3} \\
 & x^4+0+x^2 \\
 & \underline{x^4+x^3+x^2} \\
 & x^3+0+0 \\
 & \underline{x^3+x^2+x} \\
 & x^2+x+1 \\
 & \underline{x^2+x+1} \\
 & \underline{0}
\end{array}$$

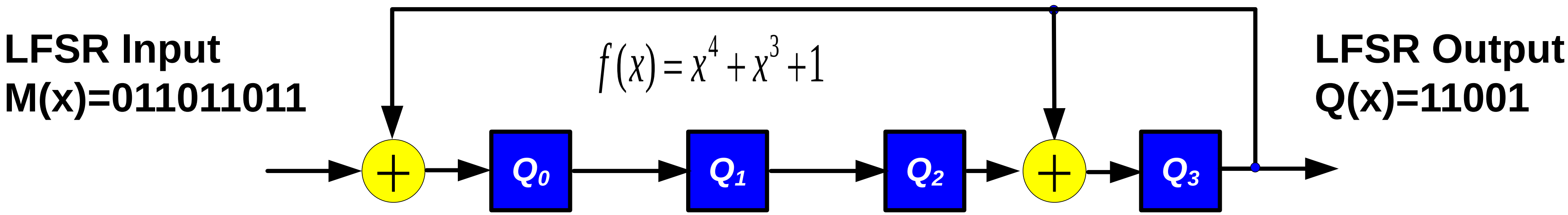
Congruent

- $M(x) \div f(x) = Q(x)$ …. $R(x)$
- If $M_1(x)$ and $M_2(x)$ have **same remainders** when divided by $f(x)$
  - $M_1(x)$ and $M_2(x)$ are ***congruent***
  - $M_1(x) \equiv M_2(x) \bmod f(x)$
- If $M(x) \equiv 0 \bmod f(x)$
  - $M(x)$ is ***divisible*** by $f(x)$

$$
\begin{array}{r}
x^3 + x^2 + x + 1 \\
x^2 + x^1 + 1 \,\big)\, \overline{x^5 + 0 + x^3 + x^2 + 0 + 1} \\
\underline{x^5 + x^4 + x^3} \qquad\qquad\quad \\
x^4 + 0 + x^2 \qquad\quad \\
\underline{x^4 + x^3 + x^2} \qquad\quad \\
x^3 + 0 + 0 \quad \\
\underline{x^3 + x^2 + x} \quad \\
x^2 + x + 1 \\
\underline{x^2 + x + 1} \\
0
\end{array}
$$

Congruent $\boxed{M_1(x) \equiv M_2(x)}$

LFSR is GF(2) Divider



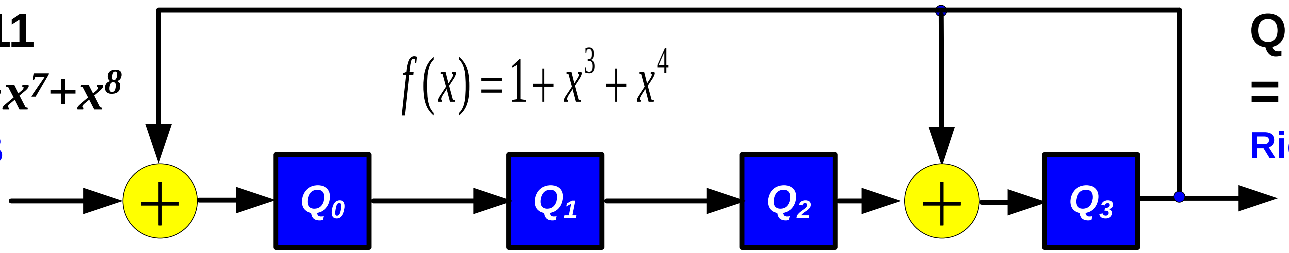
- **Assume initial LFSR content = 0000, then**
- $M(x) \div f(x) = Q(x)$ …. $R(x)$
  - **LFSR Input bit stream = dividend** $M(x)$
  - **LFSR characteristic polynomial = divisor** $f(x)$
  - **LFSR output bit stream = quotient** $Q(x)$
  - **Signature = Remainder** $R(x)$
  - $R(x) \equiv M(x)$ **mod** $f(x)$

GF(2) Divider is Simple

M(x)=011011011

= $x+x^2+x^4+x^5+x^7+x^8$

Right bit is MSB



Q(x)=11001

= $1+x+x^4$

Right bit is MSB

cycle	LFSR input	Q_0	Q_1	Q_2	Q_3	LFSR output
0	011011011	0	0	0	0	
1-3	...					
4	01101	1	0	1	1	
5	0110	0	1	0	0	1
6	011	0	0	1	0	01
7	01	1	0	0	1	001
8	0	0	1	0	1	1001
9		1	0	1	1	11001

remainder=R(x)=$1+x^2+x^3$

quotient=Q(x)=$1+x+x^4$

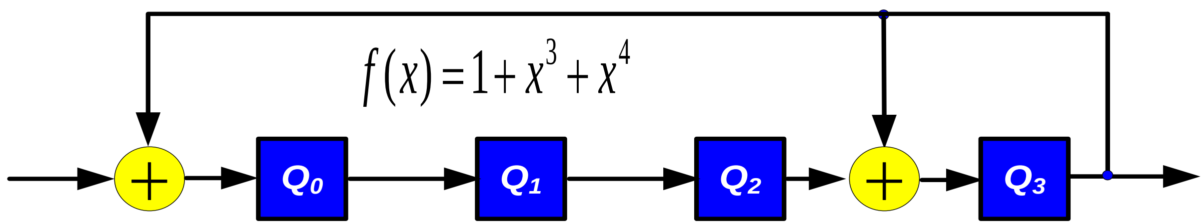
$$(x+x^2+x^4+x^5+x^7+x^8) \div (1+x^3+x^4) = 1+x+x^4 \ \ldots\ldots\ 1+x^2+x^3$$

$$M(x) \quad \div \quad f(x) \quad = \quad Q(x) \quad \ldots\ldots \quad R(x)$$

Quiz

Q: Suppose CUT output is '001001'. (right bit first)
Use GF(2) division to find quotient and remainder
ANS:

LFSR Input
(from CUT)
001001

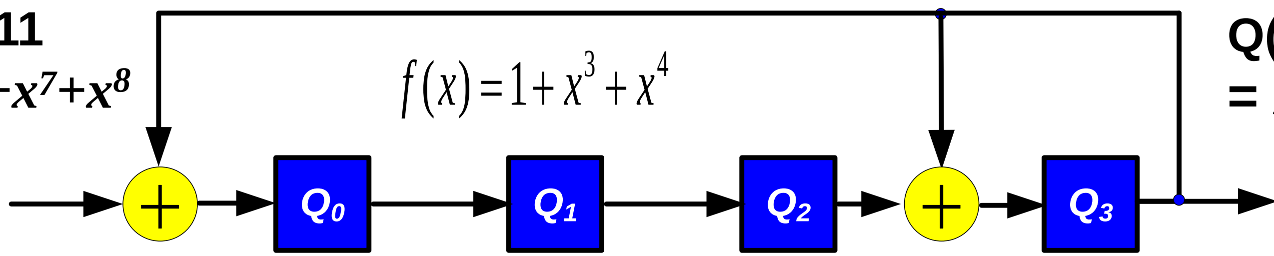


cycle	LFSR input	Q_0	Q_1	Q_2	Q_3	LFSR output
0	001001	0	0	0	0	
1-3	...					
4	00	1	0	0	1	
5	0	1	1	0	1	1
6		1	1	1	1	11

Why LFSR = Divider?

- Modular-form (Type-2) LFSR
  - shift-and-add = shift-and-subtract = mod f(x) divider

M(x)=011011011
= $x+x^2+x^4+x^5+x^7+x^8$



Q(x)=11001
= $1+x+x^4$

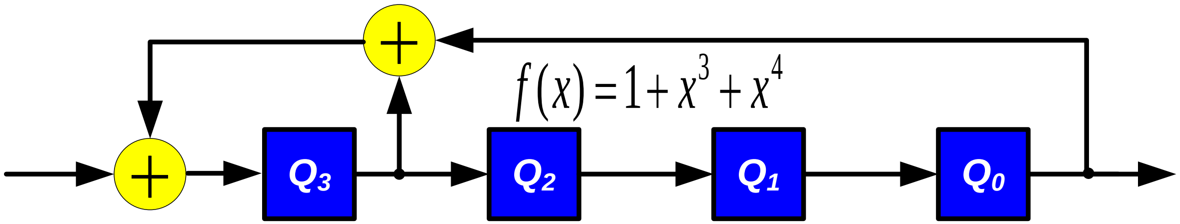
$$
\begin{array}{r l}
 & \qquad\qquad\qquad\qquad\qquad\qquad x^4 \\
1+x^3+x^4 \,\big) & x+x^2+x^4+x^5+x^7+x^8 \\
 & \qquad\quad x^4 \qquad\quad +x^7+x^8 \\
\hline
 & x+x^2+ \qquad +x^5 \\
 & \dots
\end{array}
$$

cycle	LFSR input	Q_0	Q_1	Q_2	Q_3	LFSR output
0	011011011	0	0	0	0	
1-3	…					
4	01101	1	0	1	1	
5	0110	0	1	0	0	1

So We Call It "Modular-form" LFSR

How about Standard-form LFSR?

LFSR Input
011011011
Right bit first



LFSR Output
11001
Right bit first

cycle	LFSR input	Q_3	Q_2	Q_1	Q_0	LFSR output
0	011011011	0	0	0	0	
1-3	...					
4	01101	1	0	0	1	
5	0110	1	1	0	0	1
6	011	1	1	1	0	01
7	01	0	1	1	1	001
8	0	0	0	1	1	1001
9		1	0	0	1	11001

1 $+x^3$ *$1+x+x^4$*

signature≠remainder quotient is correct

Std-form LFSR ≠ Divider

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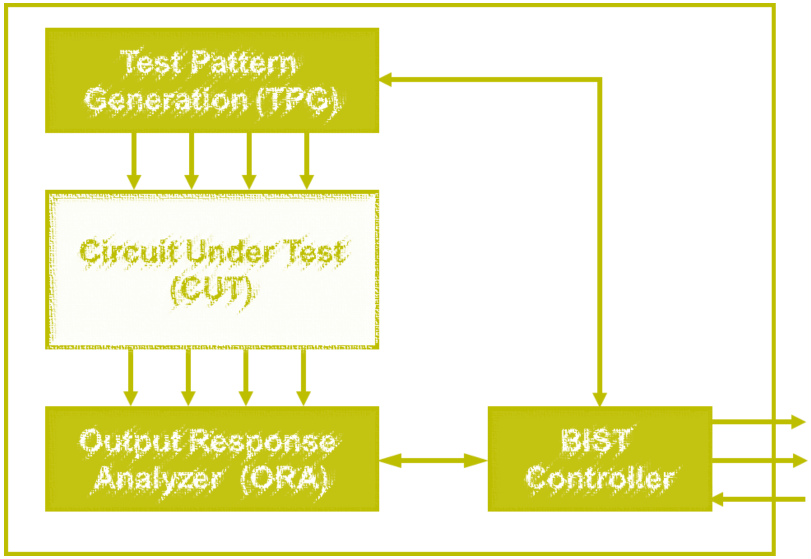
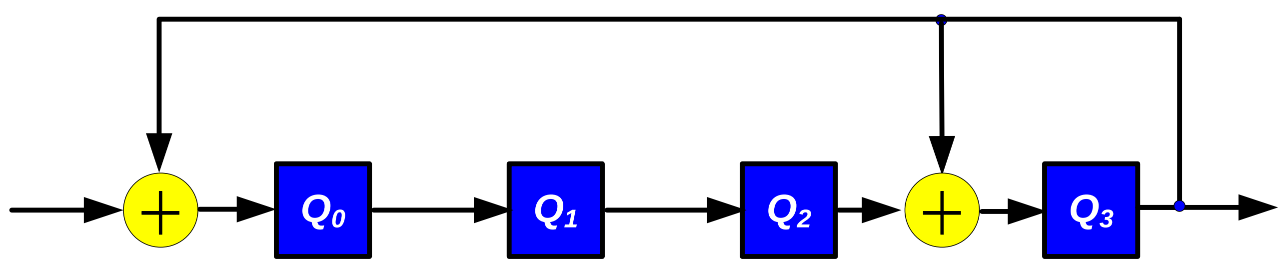


Linearity of Signature

- $[M_1(x) + M_2(x)]$ mod f(x) $\equiv$ $[M_1(x) \bmod f(x)] + [M_2(x) \bmod f(x)]$ mod f(x)
- Example:
  - $M_3(x) = M_1(x) + M_2(x)$
    - $x+x^4+x^7+x^8 = (x+x^2+x^4+x^5+x^7+x^8) + (x^2+x^5)$
  - Then signature $R_3(x) = R_2(x) + R_1(x)$
    - $(x+x^2+x^4+x^5+x^7+x^8) \div (1+x^3+x^4) = 1+x+x^4$ …… $1 \quad +x^2+x^3$
    - $(\quad x^2 \quad +x^5) \div (1+x^3+x^4) = 1+x$ …… $1+x+x^2+x^3$
    - $(x \quad +x^4 \quad +x^7+x^8) \div (1+x^3+x^4) = \quad x^4$ …… x



Signature of (Σ inputs) $\equiv$ Σ (signature of inputs)

What Is Aliasing?

- $M_{good}(x)$ is good output, $R_{good}(x)$ is gold signature
- $M_{faulty}(x)$ is faulty output, $R_{faulty}(x)$ is faulty signature
- Aliasing occurs when $R_{good}(x) = R_{faulty}(x)$

- Example:
  - $M_{good}(x)$, gold signature = $1+x^2+x^3$
    - $(x+x^2+x^4+x^5+x^7+x^8) \div (1+x^3+x^4) = 1+x+x^4$ …… $1+x^2+x^3$
  - $M_{faulty}(x)$, faulty signature = x no aliasing
    - $(x \quad +x^4 \quad +x^7+x^8) \div (1+x^3+x^4) = \quad x^4$ …… x
  - $M_{faulty2}(x)$, faulty signature = $1+x^2+x^3$ aliasing!
    - $(\quad x^2 \quad +x^7+x^8) \div (1+x^3+x^4) = 1+ \quad x^4$ …… $1+x^2+x^3$

$$\boxed{\text{Aliasing Means } R_{faulty} = R_{good}}$$

Aliasing Condition

- $M_{good}(x)$ is good output
- $M_{faulty}(x)$ is faulty output
- $M_{error}(x)$ = difference between $M_{faulty}(x)$ and $M_{good}(x)$
  - $M_{faulty}(x) = M_{good}(x) + M_{error}(x)$
- Aliasing means
  - $R_{faulty}(x) = R_{good}(x)$
  - $M_{faulty}(x) \equiv M_{good}(x) \mod f(x)$
- Aliasing condition:
  - $M_{good}(x) + M_{error}(x) \equiv M_{good}(x) \mod f(x)$
  - $M_{error}(x) \equiv 0 \mod f(x)$
  - *i.e.* $M_{error}(x)$ *divisible* by f(x) of LFSR

Aliasing when M_{error} Divisible by f

Quiz

$M_{good}(x)$, gold signature = $1+x^2+x^3$

- $(x+x^2+x^4+x^5+x^7+x^8) \div (1+x^3+x^4) = 1+x+x^4$ …… $1+x^2+x^3$

$M_{faulty2}(x)$, faulty signature = $1+x^2+x^3$ aliasing!

- $(\quad x^2 \quad +x^7+x^8) \div (1+x^3+x^4) = 1+ \quad x^4$ …… $1+x^2+x^3$

Q1: $M_{error}(x)$ = ?

ANS:

Q2: Use long division to verify that $M_{error}(x) \equiv 0$ mod $f(x)$

ANS:

PAL Estimate

- Assume $M(x)$, length m
  - $M_{faulty}(x) = M_{good}(x) + M_{error}(x)$
  - Divisor $f(x)$, degree N
  - Every bit has equal probability to flip
    - Every bit of $M_{error}(x)$ can be 1 with equal probability
- Total number of errors that can occur
  - = total number of nonzero $M_{error}(x)$ polynomials
  - $= 2^m - 1$
- Number of errors that cause aliasing
  - = number of nonzero $M_{error}(x)$ that are divisible by $f(x)$
  - $= 2^{m-N} - 1$

$$
\begin{array}{r}
11101 \\
\oplus\ 00010 \\
\hline
11111
\end{array}
\qquad m = 5
$$

$$\boxed{PAL = \frac{2^{m-N}-1}{2^m-1} \approx 2^{-N} \ (if\ m \gg N)}$$

- 5-degree LFSR PAL=1/32; 6-degree LFSR PAL=1/64
  - LFSR increases 1 bit, PAL decreases 50%

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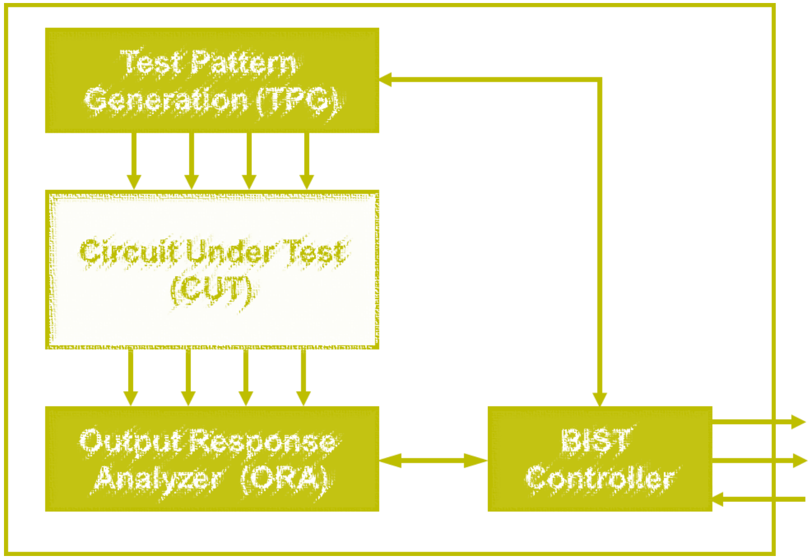

Design Guideline

- **Given a PAL, design an LFSR**
  - **1. How many stages, *N* =? (Degrees of LFSR)**
    - **$N = -\log_2 \text{PAL}$**
  - **2. Which polynomial?**
    - **Primitive polynomial**
  - **3. Test length, *m* must be greater than *N***

$$PAL = \frac{2^{m-N}-1}{2^m-1} \approx 2^{-N} \ (if \ \ m \ >> N)$$

- **Example: target PAL = 10^{-6}, test length = 1,000**
  - **$N = 20$**
  - **PAL = $2^{-20} \approx 10^{-6}$**
  - **Find a primitive polynomial of degree 20**
    - ***e.g.*** $1+x^3+x^{20}$
  - **Test length >> 20**
    - **Assumption valid**

What Polynomial?

- Study shows [Williams 88]
  - PAL of primitive polynomial converge to final steady state value
    - Faster than non-primitive polynomials
  - So it is good to use primitive polynomials

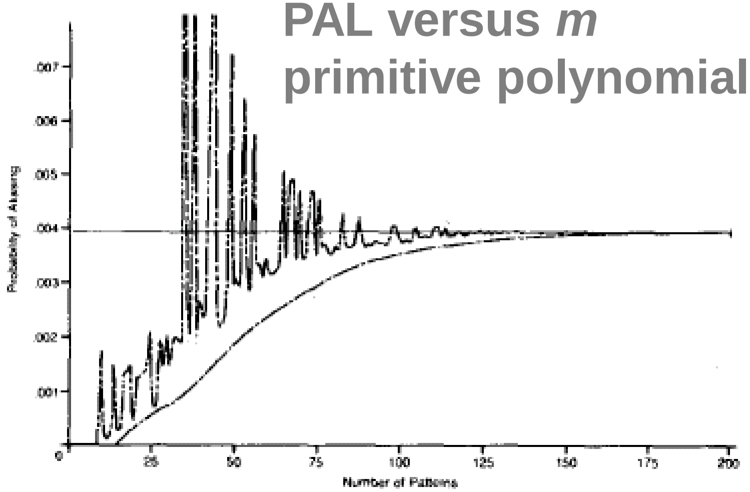

Fig. 15. Aliasing probability as a function of the test length.

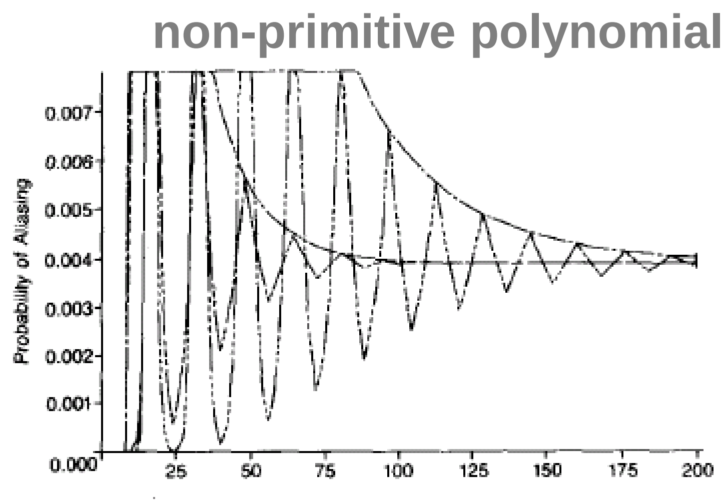

Fig. 16. Aliasing probability as a function of the test length $X^8 + 1$.

Use Primitive Polynomial

Summary

- **LFSR-based ORA**
  - Type-2 (modular form) LFSR is divider
  - Aliasing occurs when M_{error} is divisible by f
  - $PAL_{LFSR} = 2^{-N}$ very low
  - Use primitive polynomial