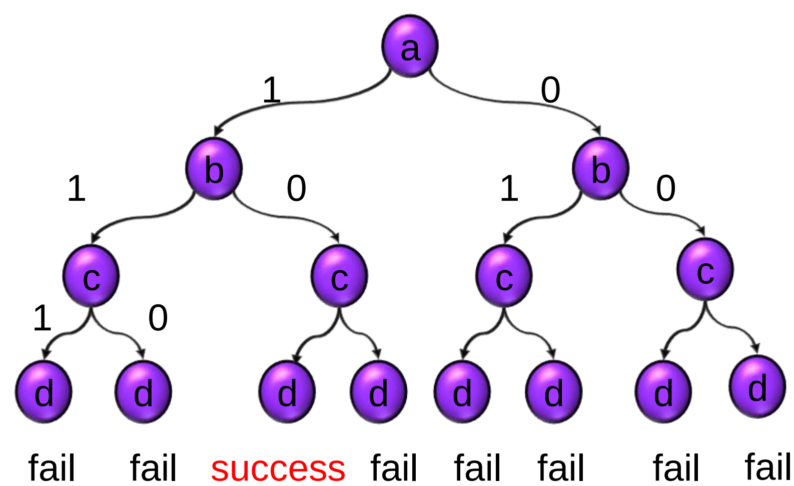
Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
  - Boolean difference*
  - Path sensitization**
  - D-Algorithm (1965)**
  - PODEM (1981)**
  - FAN(1985) **
  - SAT-based (1992)*
- Acceleration Techniques
- Concluding Remarks

*Boolean-based methods
**path-based methods



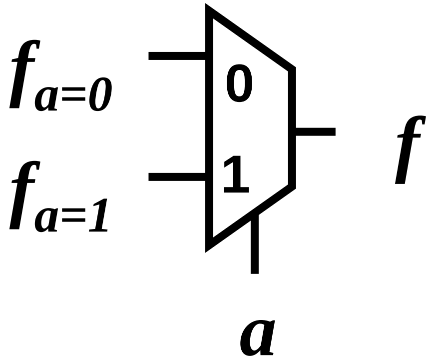
Decomposition

- **Consider a circuit that realizes the function** $f(a, b, c, \ldots)$
- **Fixed** a **to one:**
  - ***positive cofactor of f with respect to a***

$$f_{a=1} = f(a=1,\ b, c, \ldots)$$

- **Fixed** a **to zero:**
  - ***negative cofactor of f with respect to a***

$$f_{a=0} = f(a=0,\ b, c, \ldots)$$

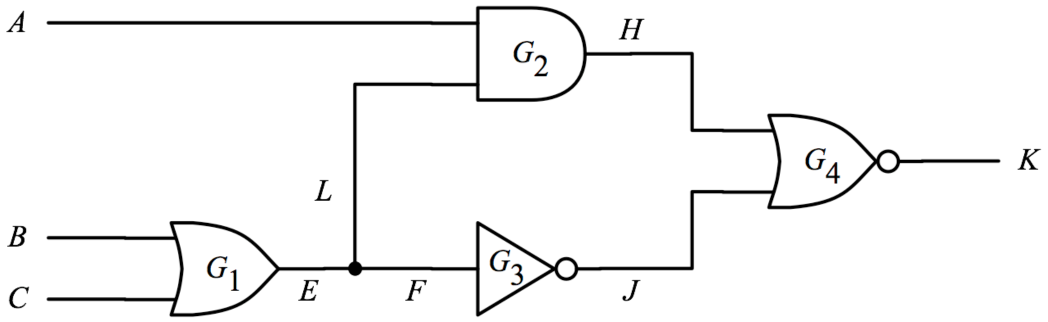


- ***Shannon's Expansion*** **w.r.t. input** a **[Shannon 1948]**

$$\boxed{f = a\ f_{a=1} + a'\ f_{a=0}}$$

Quiz

Q: Shannon's Expansion for output *K* w.r.t. input B
A:



Boolean Difference (1)

- To detect *a* stuck-at zero fault
  - Good output (*f*) and faulty output ($f_{a=0}$) are different: $f \oplus f_{a=0} = 1$
  - After Shannon Expansion: $[af_{a=1} + a' f_{a=0}] \oplus f_{a=0}$

$$= a[f_{a=1} \oplus f_{a=0}] + a'[f_{a=0} \oplus f_{a=0}]$$
$$= a[f_{a=1} \oplus f_{a=0}] + 0$$
$$= 1$$

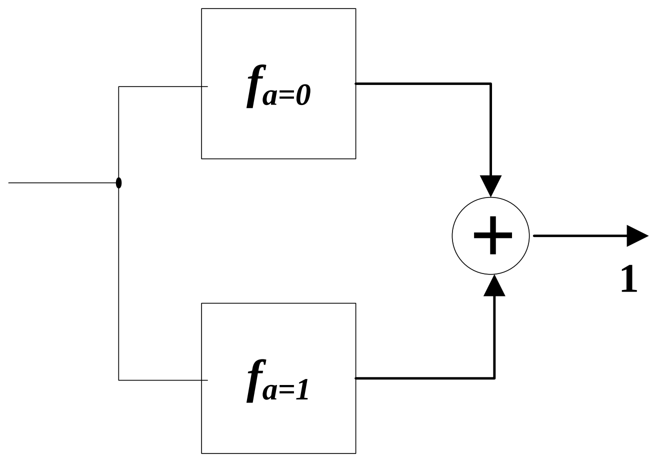
  - Thus: $\boxed{a[f_{a=0} \oplus f_{a=1}] = 1}$
- To detect *a* stuck-at one fault
  - Good output (*f*) and faulty output ($f_{a=0}$) are different: $f \oplus f_{a=1} = 1$
  - After Shannon Expansion: $[af_{a=1} + a' f_{a=0}] \oplus f_{a=1} = 1$
  - Thus: $\boxed{a'[f_{a=0} \oplus f_{a=1}] = 1}$
- ***Boolean difference* of *f* w.r.t. *a***

$$\boxed{\frac{df}{da} = [f_{a=0} \oplus f_{a=1}]}$$

Boolean Difference (2)

- Boolean Difference = 1 means *$f_{a=0}$ and $f_{a=1}$ are different*
  - which means: *a* is sensitized to output *f*

$$\boxed{\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 1}$$



- Example: AND gate, when b=1, *f* = A

$$f = ab$$

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 0 \oplus b = b$$

- Example: OR gate, when b=0, *f* = A

$$f = a + b$$

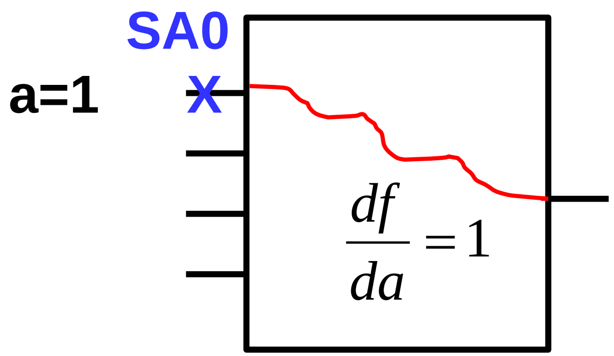
$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = b \oplus 1 = b'$$

BD=1: sensitization condition

Boolean Difference (3)

- All patterns to detect a stuck-at-0 fault
  - a = 1 : ***fault excitation***
  - BD = 1 : ***sensitization*** (aka ***fault effect propagation***)

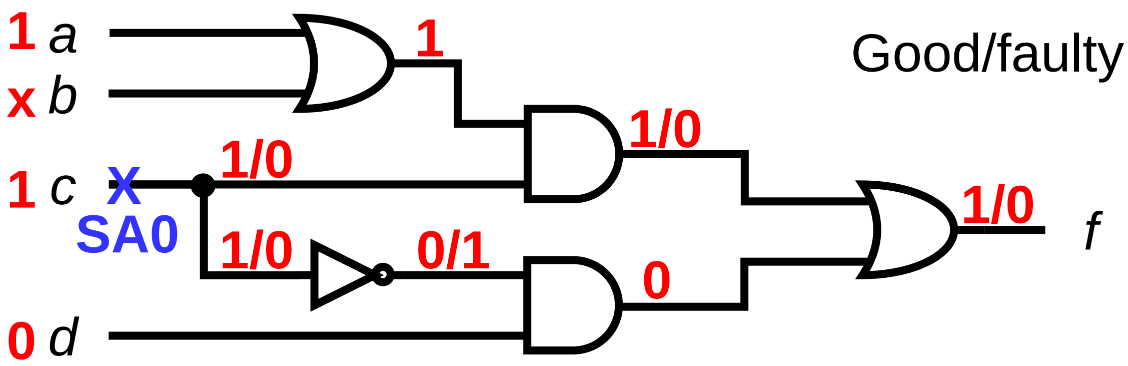
$$\boxed{a\frac{df}{da}=1}$$



- All patterns to detect a stuck-at-1 fault
  - a = 0 : fault excitation
  - BD = 1 : sensitization

$$\boxed{a'\frac{df}{da}=1}$$

Test Generation Example



- ***f = (a + b)c + c'd***
- **Set of all tests for *c stuck-at-0* is** $\boxed{c\frac{df}{dc}=1}$

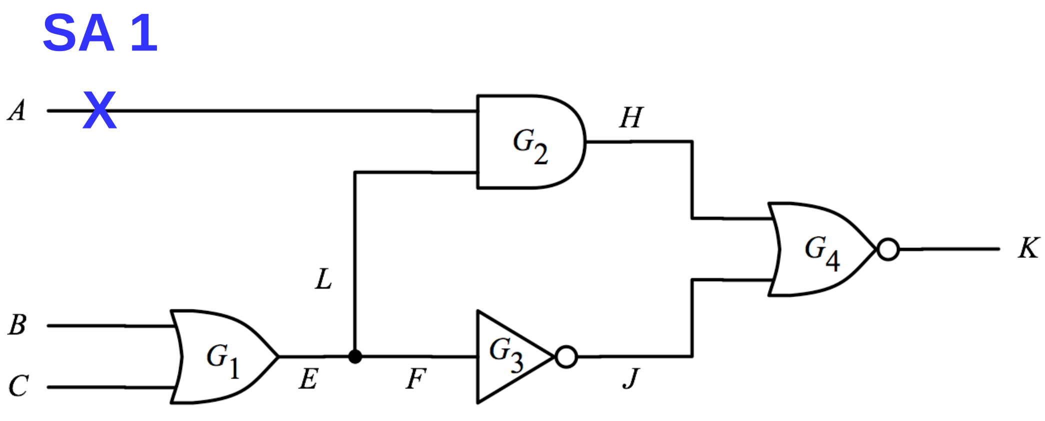
$$\frac{df}{dc} = f(a,b,0,d) \oplus f(a,b,1,d) = d \oplus (a+b) = ad' + bd' + a'b'd$$

$$c\frac{df}{dc} = acd' + bcd' + a'b'cd$$

- **Set of all tests = {1x10, x110, 0011} (x = don't care inputs)**
- **One *fully specified test pattern:* *e.g.* 1110**
- **One *partially specified test pattern (aka. test cube):* 1x10**

Quiz

(Cont'd) We already known K=A'B'C+A'B
Q1: Boolean difference dK/dA=?
Q2: Use BD to find all test patterns for A stuck-at one fault.

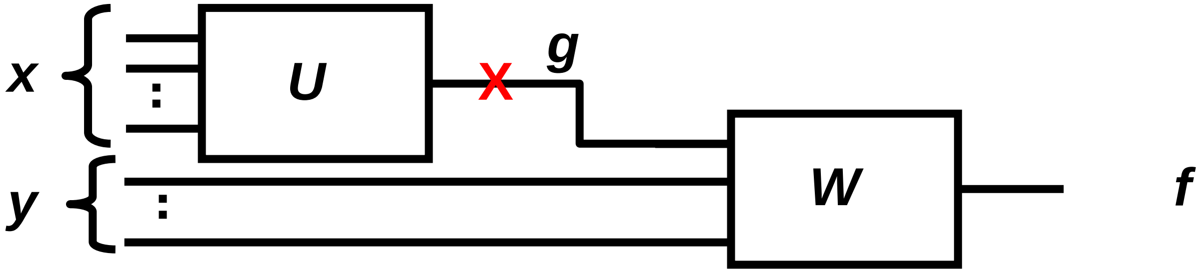


Internal Faults

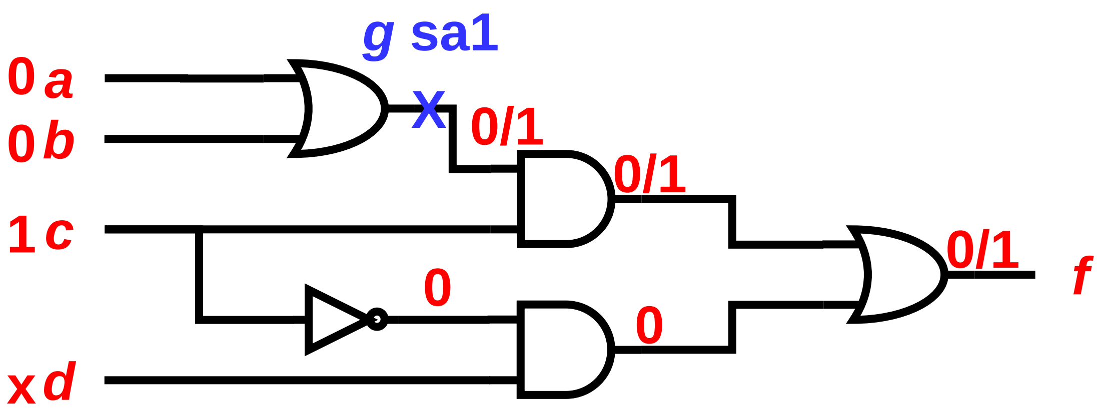
- **Same approach also used for stuck-at faults internal faults**
- **Let g be internal signal of Boolean function f**

$$f(x_1, x_2, \ldots, y_1, y_2, \ldots) = W(g, y_1, y_2, \ldots)$$

- **Test sets for faults**
  - ***g* SA0**: $\boxed{g\dfrac{dW}{dg}=1}$
  - ***g* SA1**: $\boxed{g'\dfrac{dW}{dg}=1}$



Example



$$f = (a+b)c + c'd \qquad g = a+b$$

$$W = gc + c'd$$

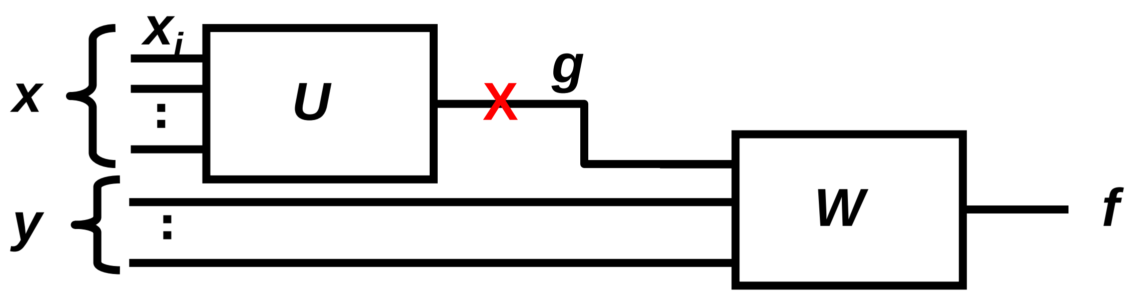
$$\frac{dW}{dg} = W_{g=0} \oplus W_{g=1} = cd' \oplus (c + c'd) = c$$

- **Tests for *g* sa0**
  (*a* + *b*)*c* = *ac* + *bc*
- **Tests for *g* sa1**
  (*a* + *b*)'*c* = *a'b'c*

Chain Rule

- **Chain rule:**

$$\frac{df}{dx_i} = \frac{df}{dg}\frac{dg}{dx_i}$$



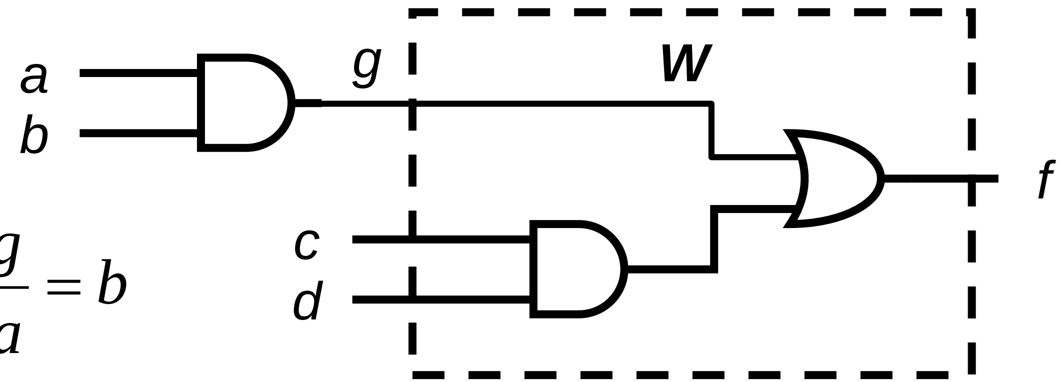
- **Example**

$$g = ab$$

$$W = g + cd$$

$$\frac{dW}{dg} = 1 \oplus cd = c' + d' \qquad \frac{dg}{da} = b$$

$$\frac{df}{da} = \frac{dW}{dg}\frac{dg}{da} = (c' + d')b$$



Problem with Boolean Difference

- **Boolean expression not always available**
  - **Especially in synthesized circuits**
- **Automatic algorithm is difficult**
- **Generating *ALL* test patterns for a fault is waste of time**
  - **Only need *ONE* test pattern for a fault**

BD is Not Useful in Practice

Summary

- **Deterministic Test Pattern Generation**
  - **BD=1 means sensitization condition**

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 1$$

  - **Test patterns to detect *a* stuck-at-0 fault**

$$a\frac{df}{da} = 1$$

  - **Test patterns to detect *g* stuck-at-0 fault**

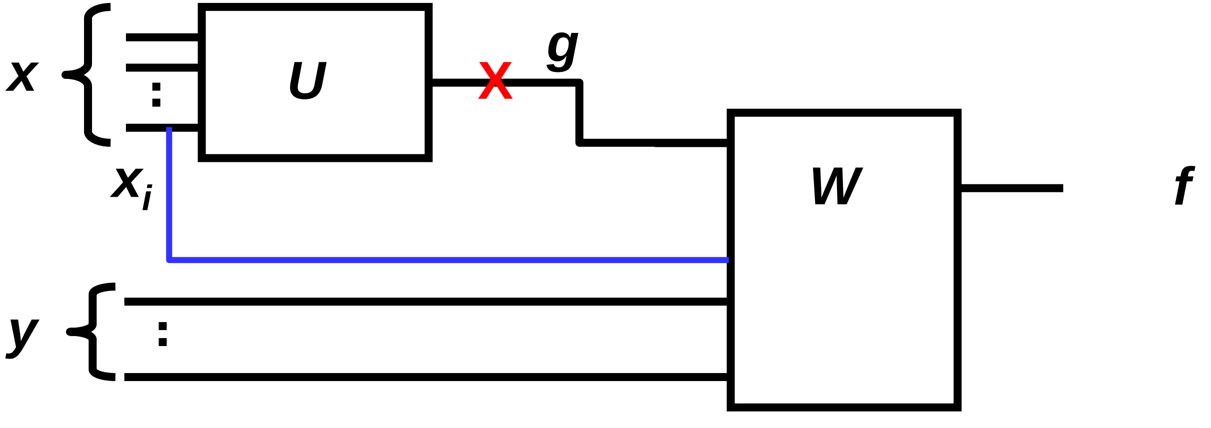
$$g\frac{dW}{dg} = 1$$

  - **Chain rule**

$$\frac{df}{dx_i} = \frac{df}{dg}\frac{dg}{dx_i}$$

FFT1

- Q: Does it still apply if fanout exists?
  - U and W share some inputs x_i
- Test sets for faults
  - *g* sa0: $\boxed{g \frac{dW}{dg} = 1}$
  - *g* sa1: $\boxed{g' \frac{dW}{dg} = 1}$



FFT2

- Q: Does it still apply if fanout exists?
  - U and W share some inputs x_i
- Chain rule:

$$\boxed{\frac{df}{dx_i} = \frac{df}{dg}\frac{dg}{dx_i}}$$