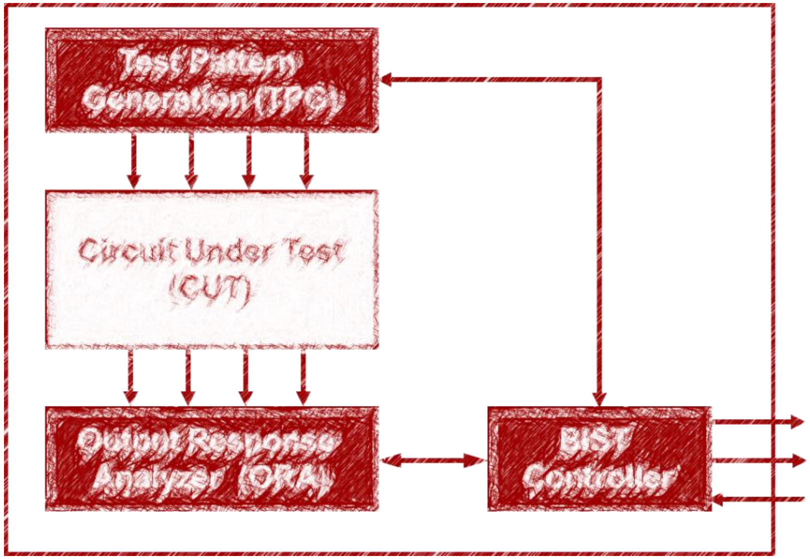
BIST Part1 - TPG

- Introduction
- Test Pattern Generation
  - Deterministic: ROM, Algorithm, Counter
  - Pseudo Random
    - Linear Feedback Shift Register, LFSR (1977)
    - Cellular Automata, CA (1984)



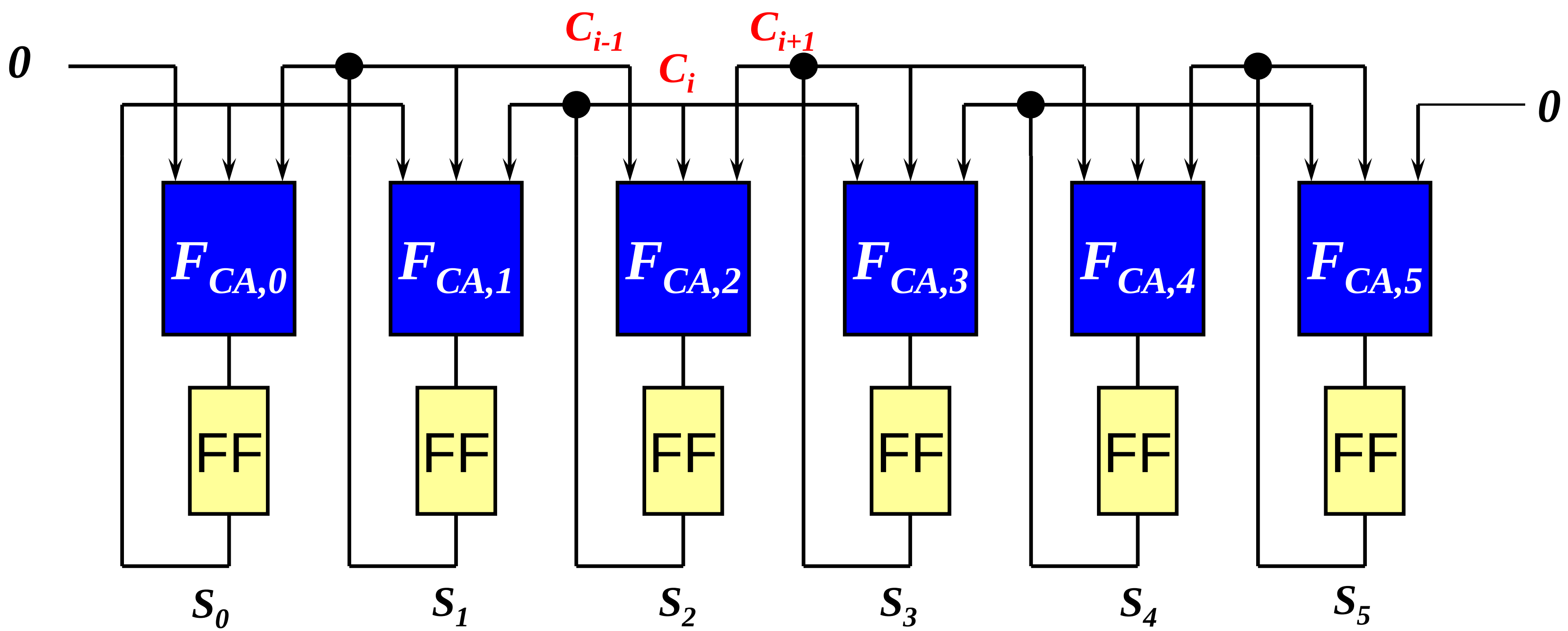
Cellular Automata [Wolfram 84]

- F_{CA} is a linear function of Its two immediate neighbors and itself

$$\boxed{F_{CA,i} = C_{i-1}S_{i-1} \oplus C_iS_i \oplus C_{i+1}S_{i+1}}$$

C_i=1 means connection exists

- ***Null Boundary Condition*** = both ends tied to zero
- ***Degree*** = number of FF = 6



CA Has No Long Global Wires

Name of CA Cell

- CA cell is named by F_{CA} function

K map of F_{CA}

S_{i+1} \ $S_i S_{i-1}$	00	01	11	10
0	A_0	A_1	A_3	A_2
1	A_4	A_5	A_7	A_6

$$Name = \sum_{i=0}^{7} A_i 2^i$$

- Example: $F_{CA,i} = S_{i-1} \oplus S_i$

S_{i+1} \ $S_i S_{i-1}$	00	01	11	10
0	0	1	0	1
1	0	1	0	1

Name = $2^6+2^5+2^2+2^1$ = 102

This is called a ***rule-102 cell***

Quiz

Q1: Given a CA cell $F_{CA,i} = S_{i-1} \oplus S_{i+1}$. What is rule number?

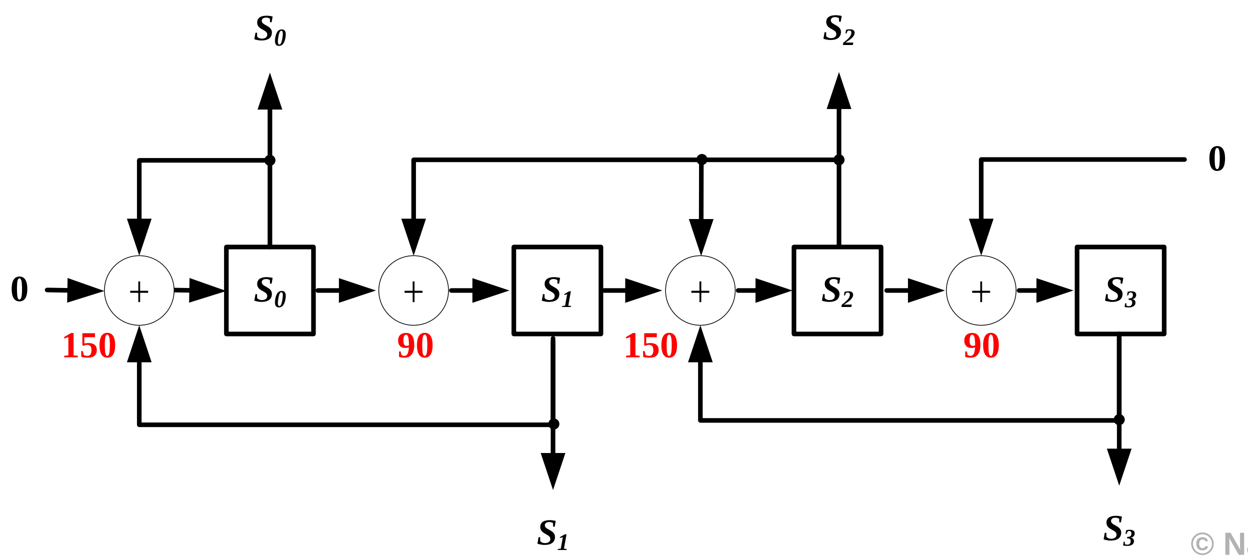
ANS:

K map of F_{CA}

S_{i+1} \ S_iS_{i-1}	00	01	11	10
0	A_0	A_1	A_3	A_2
1	A_4	A_5	A_7	A_6

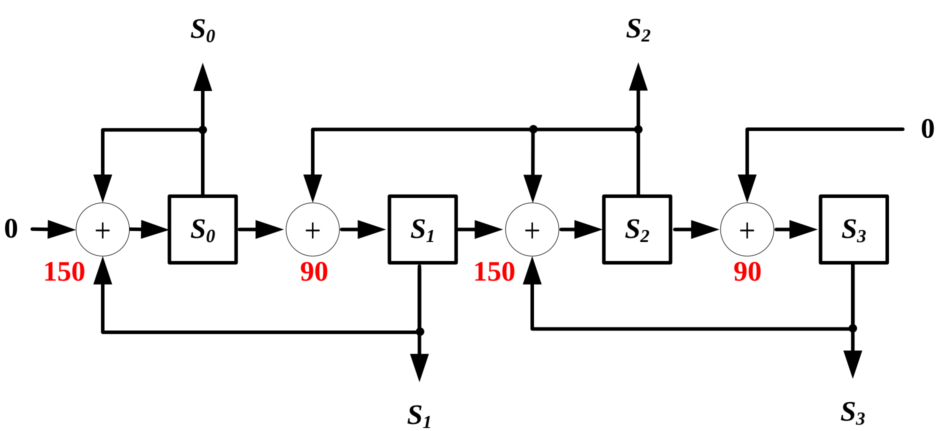
Linear Hybrid CA, LHCA

- Rule-90 CA cell
  - $F_{CA,i} = S_{i-1} \oplus S_{i+1}$
- Rule-150 CA cell
  - $F_{CA,i} = S_{i-1} \oplus S_i \oplus S_{i+1}$
- *90/150 LHCA* with null boundary condition
  - Consist of only rule-90 and rule-150 cells
  - Very popular CA, math model well studied
- Example: 4-degree 90/150 LHCA



State Transition

- Given **seed = 1000**
- CA generates **pseudo random patterns**
  - **m-sequence** of phase difference
  - **Periodic**
- Why this CA generates m-sequence?
  - to be proved later



state	S_0	S_1	S_2	S_3
0	1	0	0	0
1	1	1	0	0
2	0	1	1	0
3	1	1	0	1
4	0	1	0	0
5	1	0	1	0
6	1	0	1	1
7	1	0	0	1
8	1	1	1	0
9	0	0	0	1
10	0	0	1	0
11	0	1	1	1
12	1	1	1	1
13	0	0	1	1
14	0	1	0	1
15				

Phase diff. 11 9 1

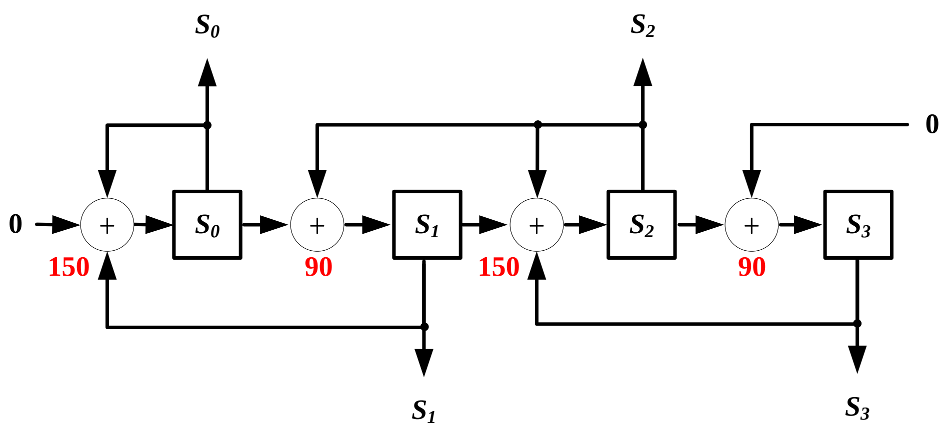
Quiz

Q1: What is state 15?

ANS:

Q2: What is cycle length of this CA?

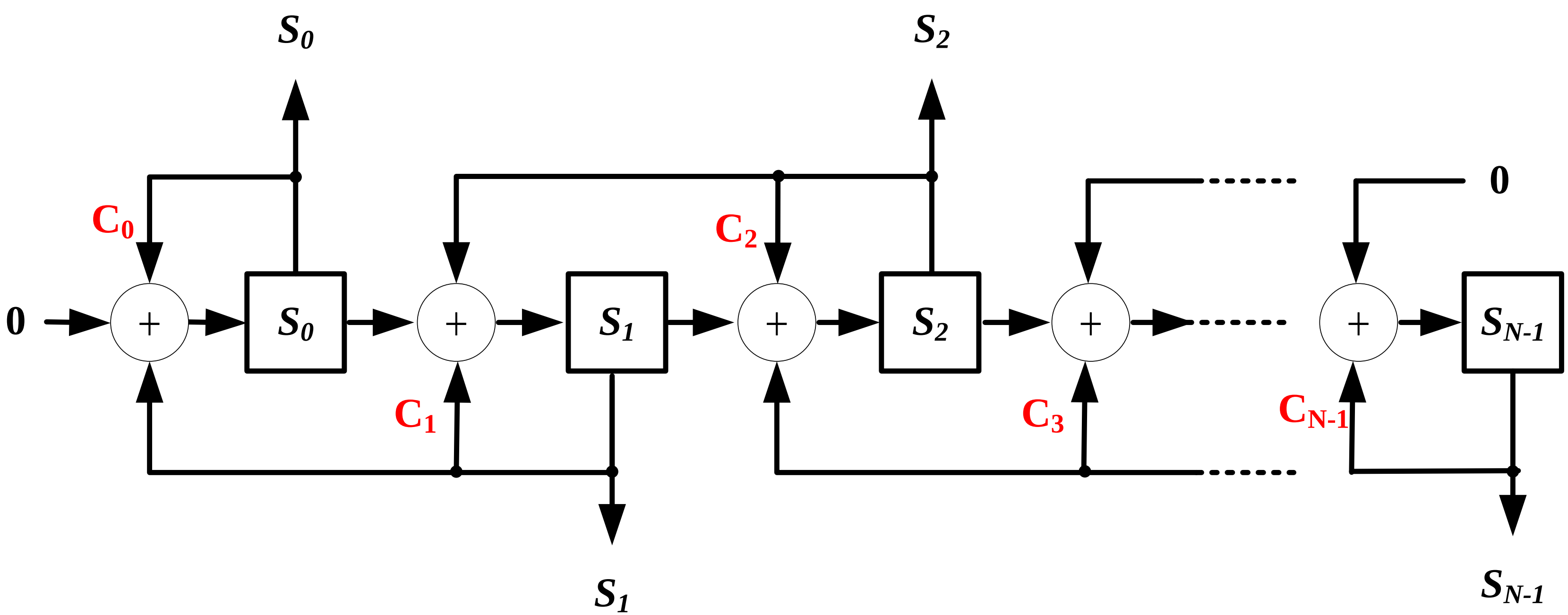
ANS:



state	S_0	S_1	S_2	S_3
0	1	0	0	0
1	1	1	0	0
2	0	1	1	0
3	1	1	0	1
4	0	1	0	0
5	1	0	1	0
6	1	0	1	1
7	1	0	0	1
8	1	1	1	0
9	0	0	0	1
10	0	0	1	0
11	0	1	1	1
12	1	1	1	1
13	0	0	1	1
14	0	1	0	1
15				

N-degree 90/150 LHCA

- C_i are coefficients
  - 1= feedback exists; rule-150 cell
  - 0 = no feedback; rule-90 cell



$$F_{CA,i} = S_{i-1} \oplus C_i S_i \oplus S_{i+1}$$

Matrix Representation (90/150 LHCA)

$$
\begin{bmatrix} S_0^{+} \\ S_1^{+} \\ \vdots \\ S_{N-3}^{+} \\ S_{N-2}^{+} \\ S_{N-1}^{+} \end{bmatrix}
=
\begin{bmatrix}
C_0 & 1 & 0 & \cdots & 0 & 0 & 0 \\
1 & C_1 & 1 & \cdots & 0 & 0 & 0 \\
\vdots & 1 & C_2 & 1 & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & C_{n-3} & 1 & 0 \\
0 & 0 & 0 & \cdots & 1 & C_{n-2} & 1 \\
0 & 0 & 0 & \cdots & 0 & 1 & C_{n-1}
\end{bmatrix}
\begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_{N-3} \\ S_{N-2} \\ S_{N-1} \end{bmatrix}
$$

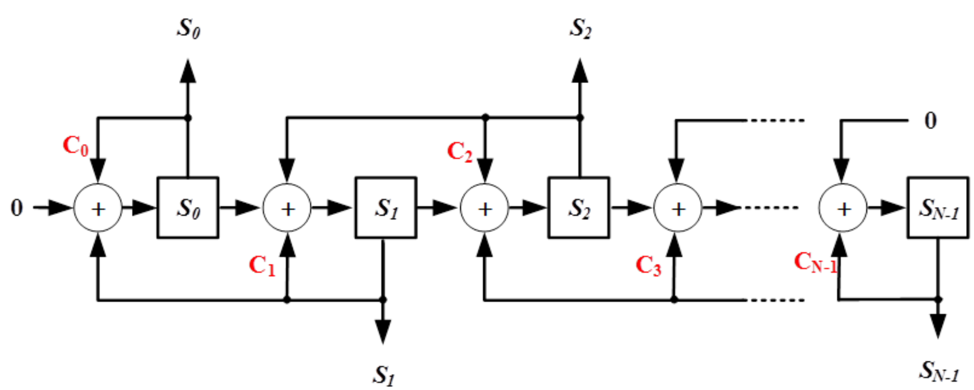
$S^{+} = C\,S$ (mod-2 addition)

S = current state

S^{+} = next state

Characteristic polynomial:

$$\boxed{f(\lambda) = \det (C-\lambda I)}$$



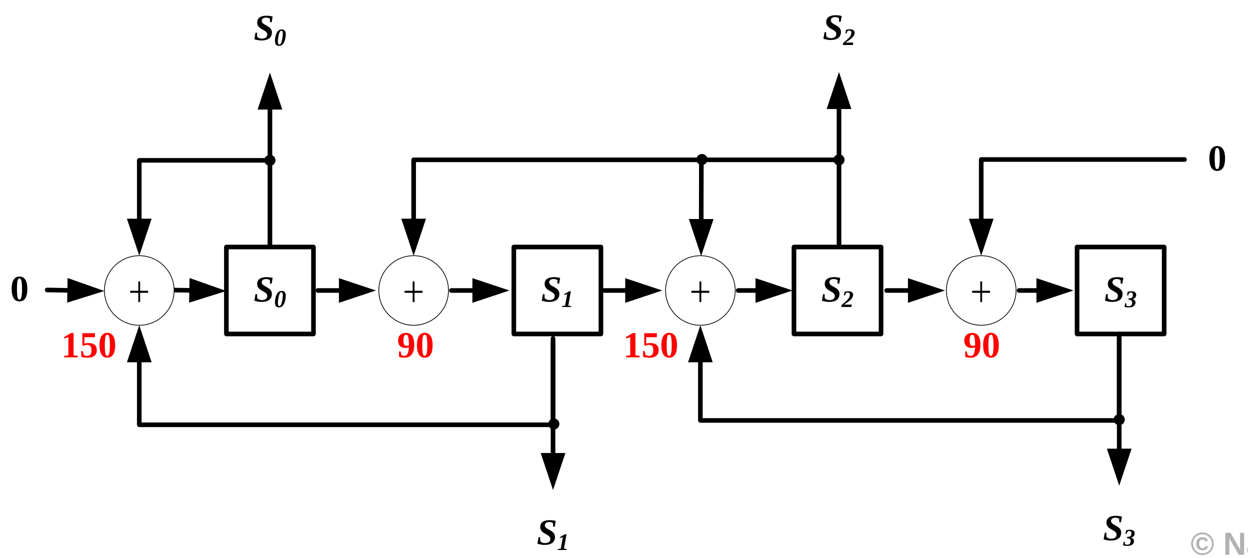
Quiz

Q1: Given this LHCA, what is matrix C ?

ANS:

Q2: What is characteristic polynomial?
NOTE : please use mod-2 addition

ANS:



Determinant

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left| \begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array} \right.$$

Determinant = *[aei+bfg+cdh]-[gec+hfa+idb]*

Determinant (column expansion)

- *Column expansion* **(with respect to the j^{th} column)**

$$\boxed{\det(A)=|A|=\sum_{i=1}^{n}a_{ij}C_{ij}=a_{1j}C_{1j}+a_{2j}C_{2j}+\cdots+a_{nj}C_{nj}}$$

- ***Cofactor* of a_{ij} = C_{ij} = $(-1)^{i+j}$ x M_{ij}**
  - **M_{ij} is determinant of A by removing i^{th} row and j^{th} column**

- **Example: j=1**

$$\det(A)=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=a_{11}C_{11}+a_{21}C_{21}+a_{31}C_{31}=a_{11}(-1)^{1+1}M_{11}+a_{21}(-1)^{2+1}M_{21}+a_{31}(-1)^{3+1}M_{31}$$

$$=a_{11}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix}-a_{21}\begin{vmatrix}a_{12} & a_{13}\\ a_{32} & a_{33}\end{vmatrix}+a_{31}\begin{vmatrix}a_{12} & a_{13}\\ a_{22} & a_{23}\end{vmatrix}$$

Characteristic Polynomial of CA

$$C = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\det(C - \lambda I) = \det \begin{pmatrix} 1-\lambda & 1 & 0 & 0 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 \\ 0 & 0 & 1 & -\lambda \end{pmatrix}$$

$$= (-1)^{1+1}(1-\lambda)\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} + (-1)^{1+2}(1)\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix}$$

$$= (1-\lambda)[\lambda^2(1-\lambda) + \lambda + \lambda] + (-1)[-\lambda(1-\lambda) - 1]$$

$$= \lambda^2(1-\lambda)^2 + 2\lambda(1-\lambda) + \lambda(1-\lambda) + 1$$

$$= \lambda^4 - 2\lambda^3 - 2\lambda^2 + 3\lambda + 1$$

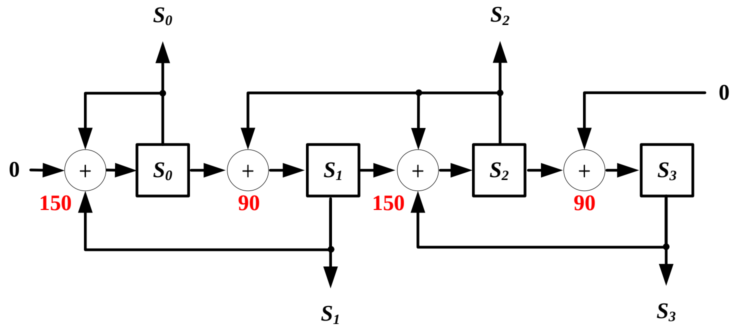
$$= \lambda^4 + \lambda + 1 \quad \text{(mod-2 arithmetic)}$$

This CA has a primitive polynomial

LFSR ↔ CA *not in exam

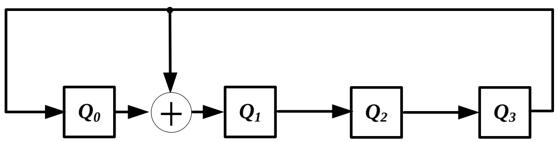
- If irreducible characteristic polynomial are equal: $f_{CA}(x) = f_{LFSR}(x)$
  - Then their matrices are *similar* : $T_{CA} \leftrightarrow T_{LFSR}$
  - *i.e.* $T_{CA} = V^{-1} T_{LFSR} V$ (aka. *isomorphism transformation*)
  - [Serra 88][Bardell 90]

- Implications:
- 1. If characteristic polynomial $f_{CA}(x)$ is primitive polynomial
  - It generates *m-sequence* of length 2^N-1
- 2. CA and LFSR of same polynomial generate same sequence
  - Just phase difference



$f_{CA}(x) = x^4+x+1$
is primitive polynomial
so it generates m-sequence

LFSR ↔ CA, same $f(x)=x^4+x+1$



state	Q_0	Q_1	Q_2	Q_3
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	1	1	0	0
5	0	1	1	0
6	0	0	1	1
7	1	1	0	1
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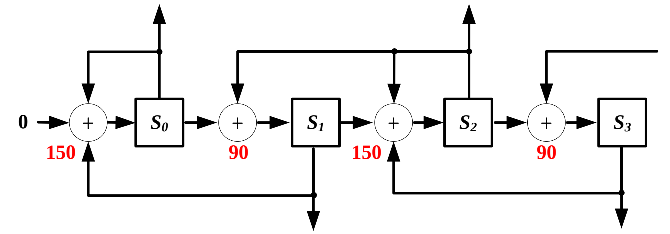
13	1	0	1	1
14	1	0	0	1

same m-sequence

Phase diff. 12 1 1



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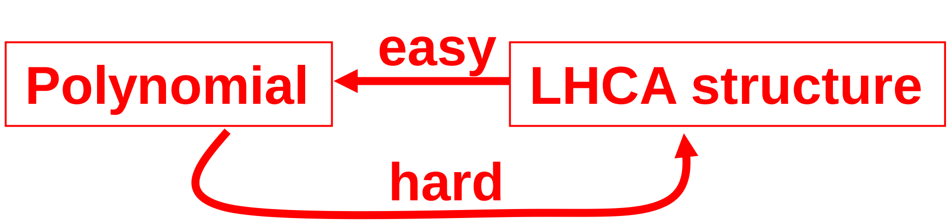
13	0	0	1	1
14	0	1	0	1

same m-sequence

Phase diff. 11 9 1

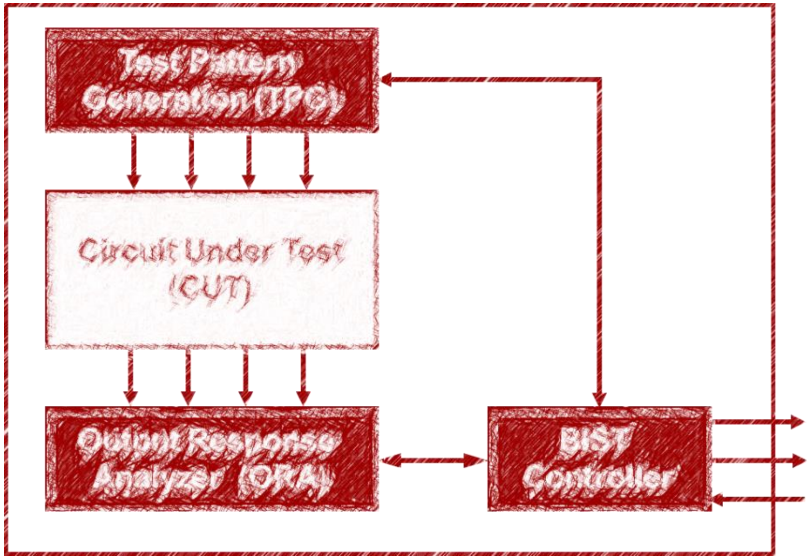
Pros and Cons of CA

- Advantages of CA (over LFSR)
  - + More randomness than LFSR
    - ⇒ Large phase shift
  - + Potentially faster than LFSR
    - ⇒ short feedback loop
  - + Small XOR
    - ⇒ no more than 3-input XOR
- Disadvantages of CA (over LFSR)
  - – No easy design methodology
    - ⇒ No good method to construct LHCA from polynomial



Summary - CA

- CA are named by its function
- 90/150 LHCA very popular
- Polynomial of CA is $f(\lambda) = \det(C-\lambda I)$
- CA are good but hard to design



Reference

- **(BMS 87) P.H. Bardell, W.H. McAnney, J. Savior, *Built-in Test for VLSI: Pseudorandom Techniques*, Wiely Interscience, 1987.**

- **[Könemann 91] B Könemann, "LFSR-coded test patterns for scan designs," European Test Conference, 1991**