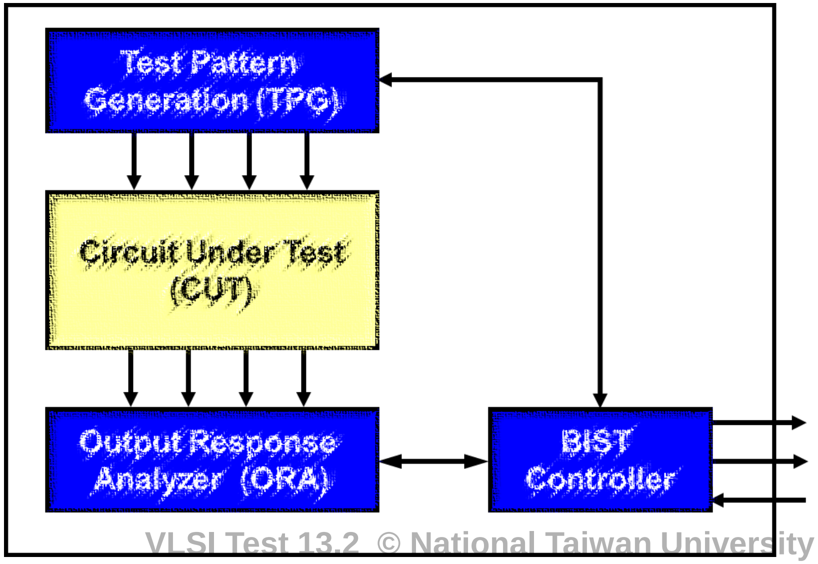
BIST Part1 - TPG

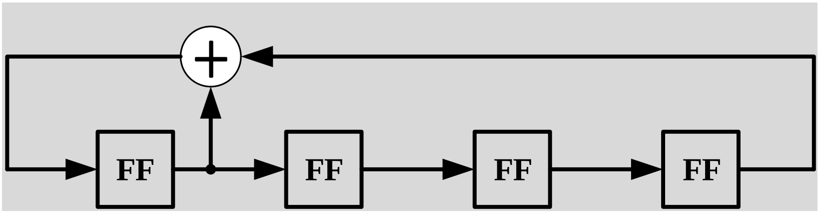
- Introduction
- Test Pattern Generation (TPG)
  - Deterministic: ROM Algorithm Counter
  - **Pseudo Random**
    - ***Linear Feedback Shift Register, LFSR* (1977)**
      - **Two types of LFSR**
      - Design of LFSR
    - Cellular Automata, CA (1984)



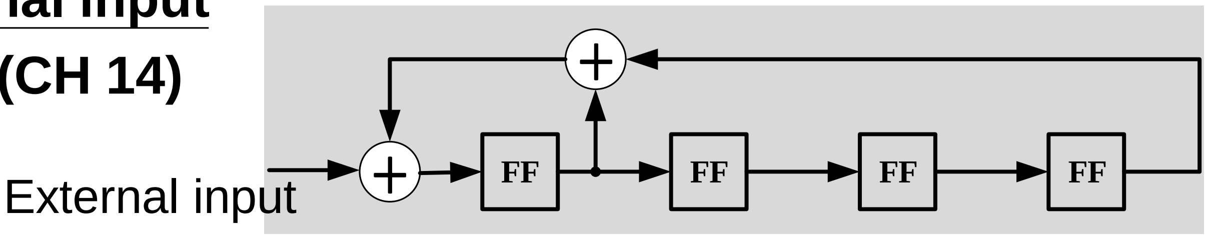
Linear Feedback Shift Register [Frohwerk 77]

- **LFSR consist of unit delays (flip-flops, FF) and feedback (XOR)**
- **Two applications of LFSR:**

  1. LFSR without external input
  - **Used for TPG**
  - **aka *Autonomous LFSR***



  2. LFSR with external input
  - **Used for ORA (CH 14)**



⊕ =XOR

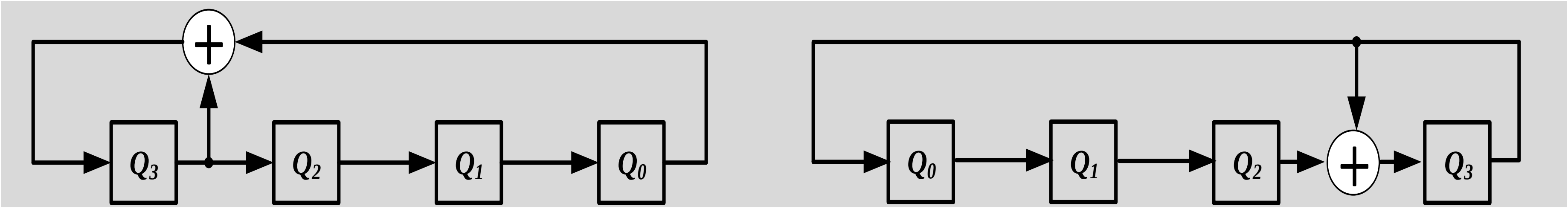
- ***"Linear"* because XOR is *mod-2 addition***

Two Types of Autonomous LFSR

- Autonomous LFSR is *Modular Counter*
  - Very small area, generate *pseudo random* outputs
- Two structures:
  - *Type 1: Standard Form* (aka *external XOR*) LFSR
  - *Type 2: Modular Form* (aka *internal XOR*) LFSR

Type-1 Standard LFSR

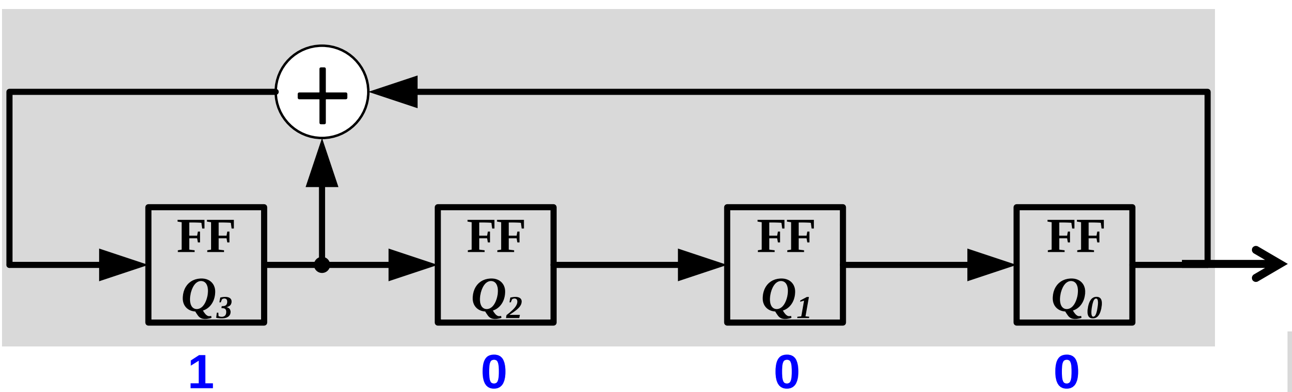
Type-2 Modular LFSR



LFSR is Good for TPG

Type-1, Standard Form LFSR

- **Three ways to describe LFSR**
  1. ***Next state equation*:** $Q_3^+ = Q_3 \oplus Q_0$
     - **Q_3^+ means next state of FF Q_3**
  2. ***Recurrence equation*:** $q_m = q_{m-1} \oplus q_{m-4}$
  3. ***Characteristic polynomial*:** $f(x) = x^4 + x^3 + 1$
     - **Most popular. will see why 13.4**



output bit stream (q_0 is first)

$q_0\ q_1\ q_2\ q_3\ q_4\ q_5\ q_6\ q_7$

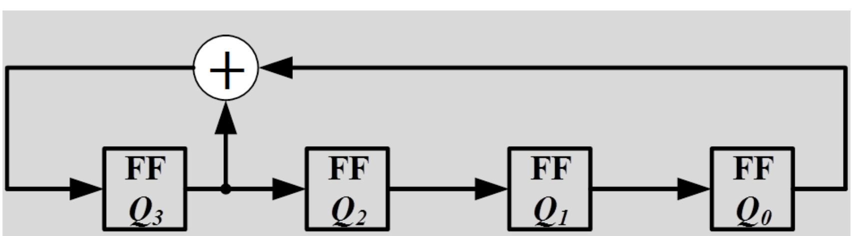
0 0 0 1 1 1 1 0

$q_4 = q_3 \oplus q_0$
$q_5 = q_4 \oplus q_1$
...
$q_m = q_{m-1} \oplus q_{m-4}$

- **NOTE: Modular-2 arithmetic: ⊕ + - are same**
  - **1⊕1=0; 0⊕1=1; 1⊕0=1; 0⊕0=0**

State Sequence of x^4+x^3+1 LFSR

- *Seed* = Initial state of LFSR
  - must be non-zero
- Total $2^4-1 = 15$ distinct states
  - All-zero state not included
- Periodical.
  - Cycle length $L_c = 15$

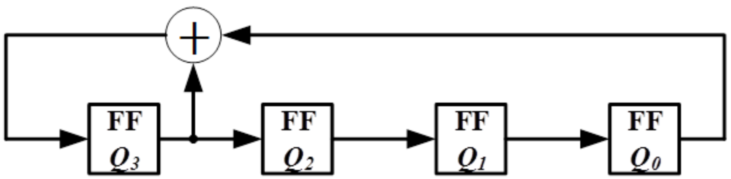


state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15 (=0)	1	0	0	0

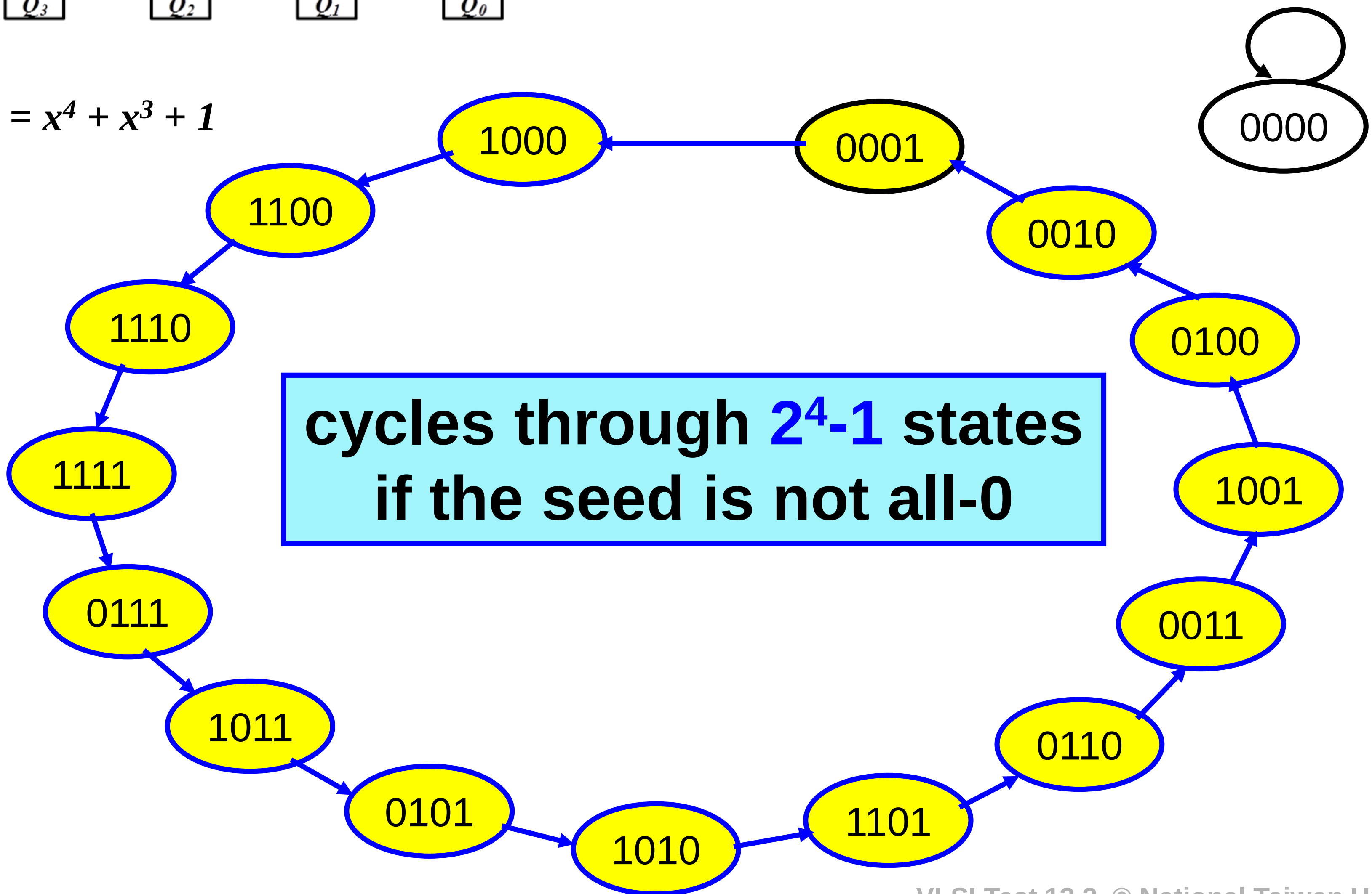
seed

Back to seed after 15 cycles

State Diagram



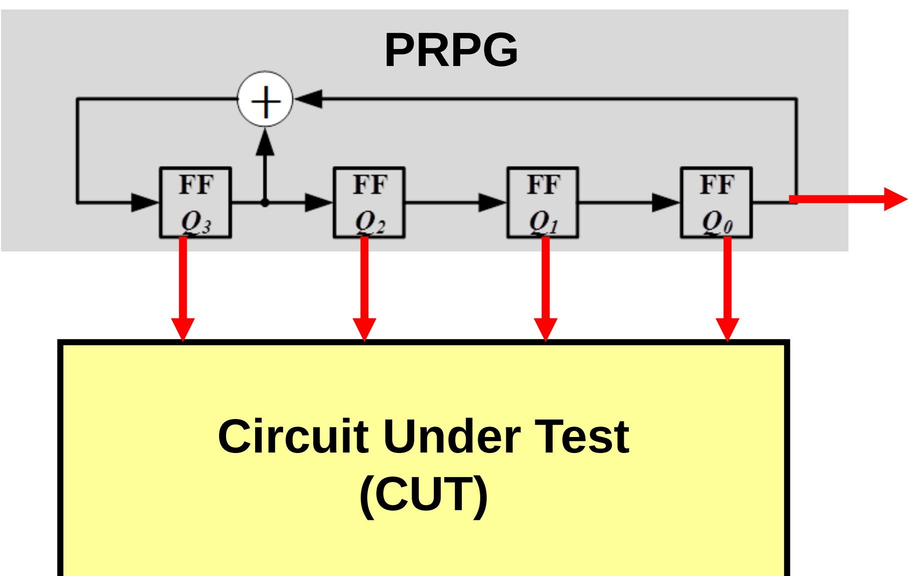
$f(x) = x^4 + x^3 + 1$



cycles through 2^4-1 states if the seed is not all-0

Pseudo Random Pattern Generator (PRPG)

- ***Pseudo random.*** **NOT truly random**
- **Serial PRPG:** $q_0, q_1, q_2, q_3, \ldots$
  - **Periodical.** $L_c = 15$
- **Parallel PRPG:** $(Q_3\ Q_2\ Q_1\ Q_0)$
  - **Each output shifted by one cycle**
  - ***Phase difference*** **=1**



state	Q_3	Q_2	Q_1	Q_0	
0	1	0	0	0	q_0
1	1	1	0	0	q_1
2	1	1	1	0	q_2
3	1	1	1	1	q_3
4	0	1	1	1	q_4
5	1	0	1	1	q_5
6	0	1	0	1	q_6
7	1	0	1	0	q_7
8	1	1	0	1	q_8
9	0	1	1	0	q_9
10	0	0	1	1	q_{10}
11	1	0	0	1	q_{11}
12	0	1	0	0	q_{12}
13	0	0	1	0	q_{13}
14	0	0	0	1	q_{14}
15 (=0)	1	0	0	0	q_{15}

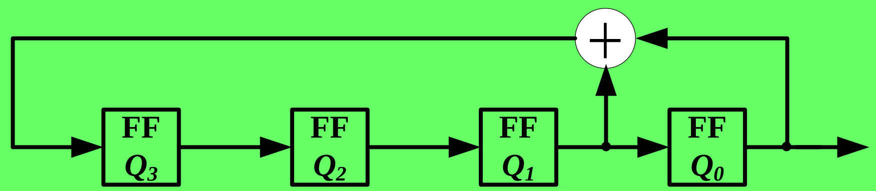
↔ ↔ ↔

Phase diff. =1

Quiz

Q: Given this new LFSR, show the state sequences with seed [1 0 0 0]

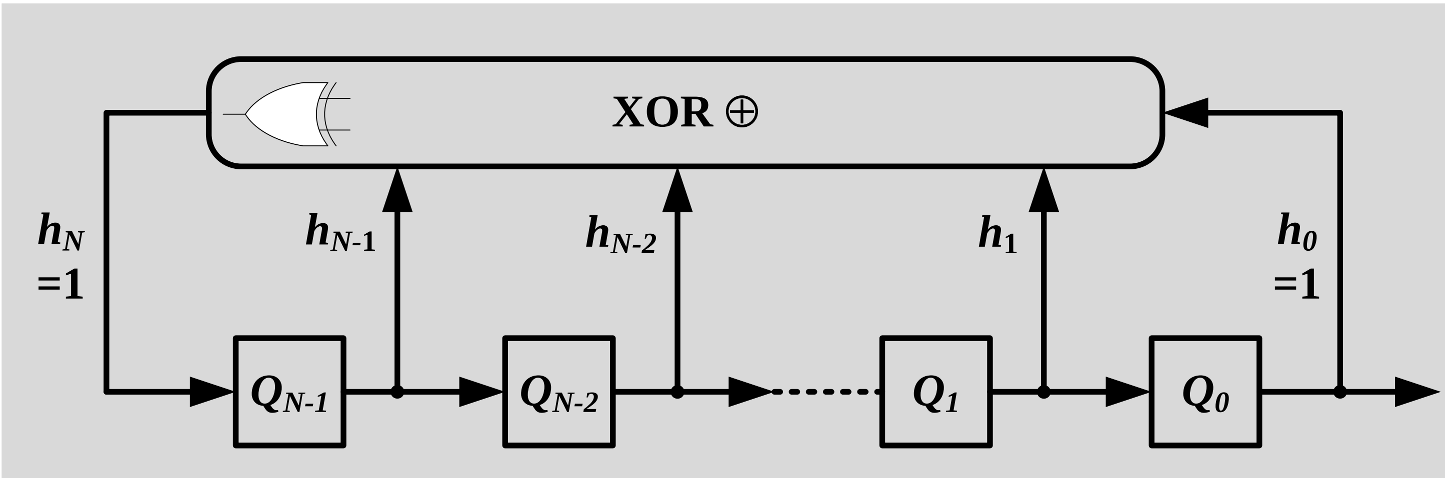
Q2: What is cycle length L_c=?



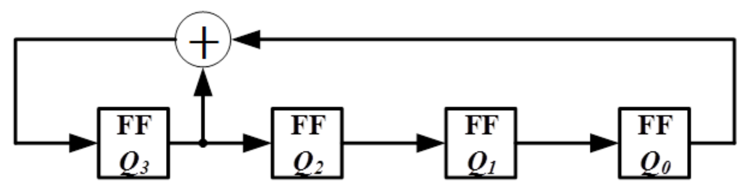
state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

N-degree Standard Form LFSR

- ***N* = *LFSR degree*** **= number of FF = characteristic polynomial degree**



h_i = 1 if feedback exists
h_i = 0 if no feedback
h_N =1, h_0 =1



$f(x) = x^4 + x^3 + 1$

- ***Characteristic polynomial*****:** **(details see appendix)**

$$f(x) = \sum_{i=0}^{N} h_i x^i$$

***f(x)* Can Be Written from LFSR Structure**

Matrix Representation (Type-1 LFSR)

Identity matrix

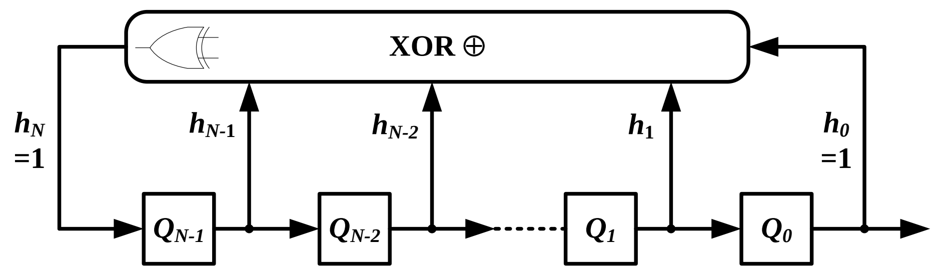
$$
\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \vdots \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & 0 & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & 0 \\
0 & 0 & 0 & \cdots & 0 & 1 \\
h_0 & h_1 & h_2 & \cdots & h_{N-2} & h_{N-1}
\end{bmatrix}
\begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}
$$

$Q^+ = T\,Q$ (mod-2)

Q = current state

Q^+ = next state

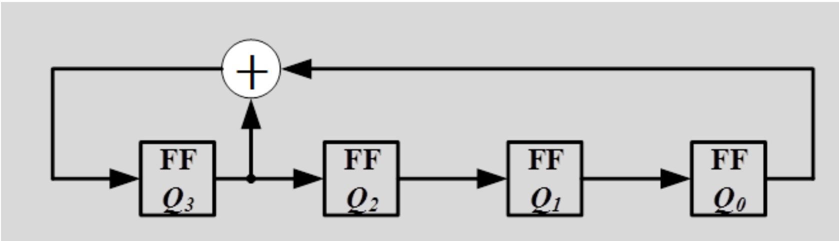
T = *companion matrix*



$Q_{N-1}^+ = \Sigma\, h_i Qi$

$Q_0^+ = Q_1$

Example



state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

mod-2 arithmetic:
1x1=1 1x0=0 0x0=0
1+1=0 1+0=1 0+0=0

$$Q_{seed} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$TQ_{seed} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

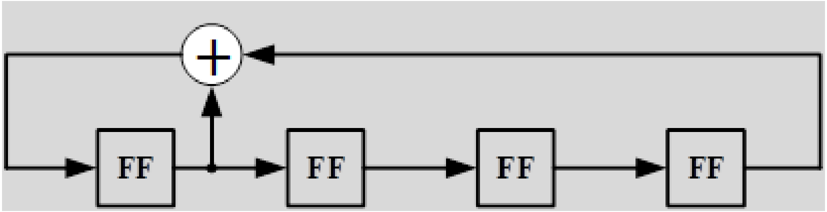
$$T^2Q_{seed} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T^3Q_{seed} == T^2TQ_{seed} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Matrix Multiplication Same as Simulation

Linear Relationship between States

- 0_{th} *cycle: LFSR state* = Q_{seed}
- 1_{st} cycle: TQ_{seed}
- 2_{nd} cycle: $TTQ_{seed} = T^2Q_{seed}$
- 4_{th} cycle: $T^4Q_{seed} = (T^2)^2Q_{seed}$
- 16_{th} cycle: $T^{16}Q_{seed} = (T^4)^2Q_{seed} = ((T^2)^2)^2Q_{seed}$
  - only 3 matrix squares needed



- After L_c cycles, LFSR returns to initial state
  - $T^{Lc}Q_{seed} = Q_{seed}$
  - In this case L_c=15. $T^{15} = I$, so $T^{16} = T$

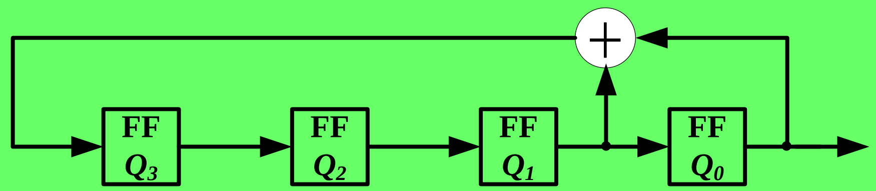
Matrix Faster Than Simulation

Quiz

Q1: Given this LFSR, What is its characteristic polynomial?

Q2: What is its companion matrix *T*?

Q3: Use *T* to derive state after three cycles, starting from seed [1 0 0 0]



state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	1	0	0	1

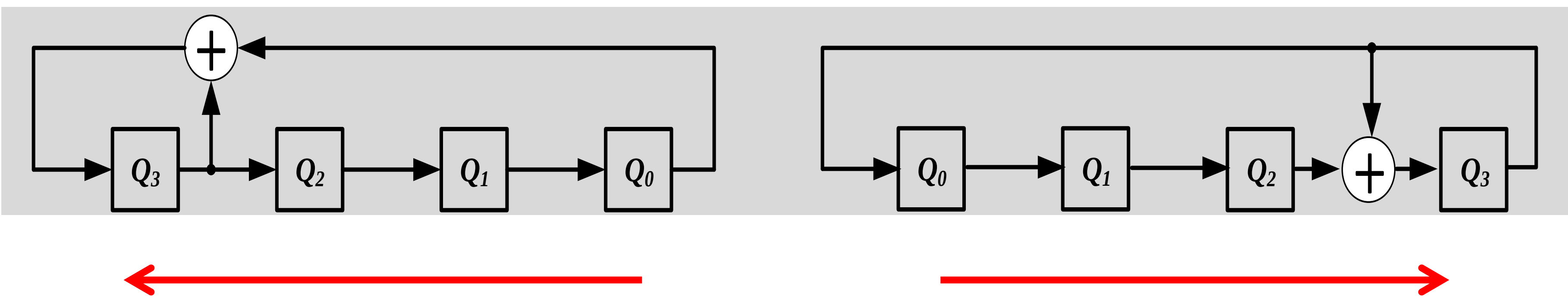
ANS

LFSR Types

- *Type 1: Standard Form* (aka *external XOR*) LFSR
- *Type 2: Modular Form (aka internal XOR) LFSR*
    - XOR gates are internal to LFSR
    - as opposed to standard form LFSR, in which XOR are external

Type-1 Standard LFSR

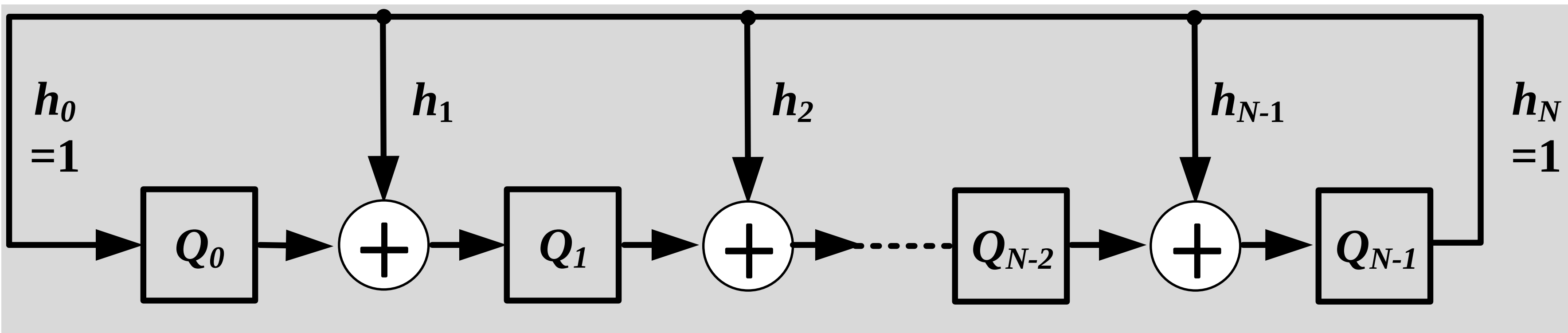
Type-2 Modular LFSR



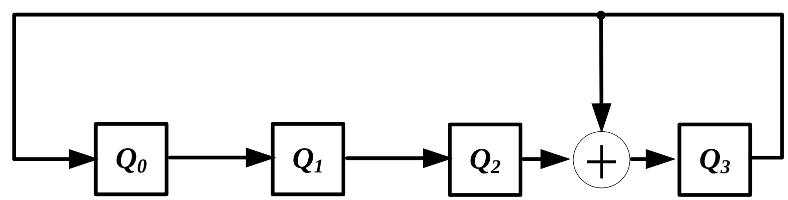
NOTE: Order of FF Different

N-degree Modular Form LFSR

- ***N = LFSR degree*** = number of FF = characteristic polynomial degree



h_i = 1 if feedback exists
h_i = 0 if no feedback
h_N =1, h_0 =1



$f(x) = x^4 + x^3 + 1$

- ***Characteristic polynomial***: $f(x) = \sum_{i=0}^{N} h_i x^i$

$f(x)$ Same as Type-1 LFSR

(remember correct order of FF !)

Matrix Representation (Type-2 LFSR)

$$
\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \cdot \\ \cdot \\ \cdot \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & \dots & 0 & h_0 \\
1 & 0 & 0 & \dots & 0 & h_1 \\
\vdots & 1 & \vdots & & \vdots & h_2 \\
\vdots & \cdot & \vdots & & \vdots & \vdots \\
0 & 0 & 1 & \dots & 0 & \vdots \\
0 & 0 & 0 & \dots & 0 & h_{N-2} \\
0 & 0 & 0 & \dots & 1 & h_{N-1}
\end{bmatrix}
\begin{bmatrix} Q_0 \\ Q_1 \\ \cdot \\ \cdot \\ \cdot \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}
$$

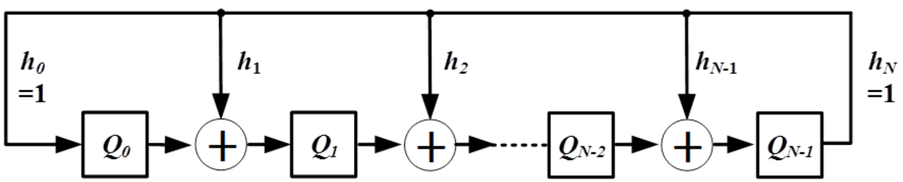
Identity matrix

$Q^+ = T\,Q$

Q = current state

Q^+ = next state

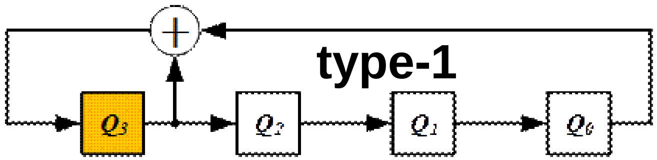
T = Companion matrix

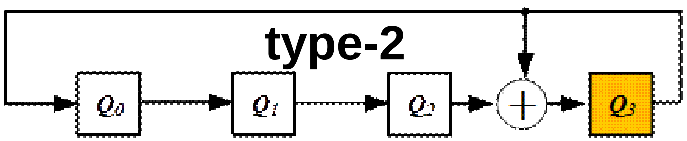


$$Q_1^+ = Q_0 + h_1 Q_{N-1}$$

Quiz

Q: Given two types LFSR of x^4+x^3+1, same seed 1000, fill in table.



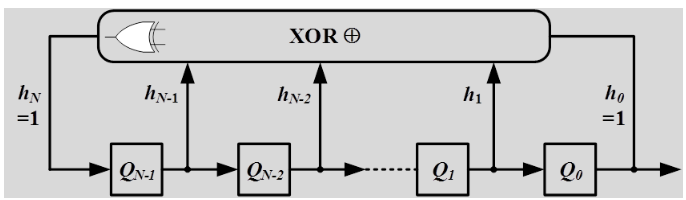
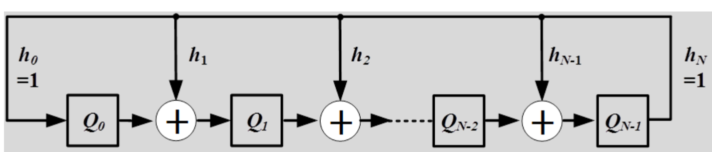


state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15 (=0)	1	0	0	0

state	Q_0	Q_1	Q_2	Q_3
0	1	0	0	0
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15 (=0)				

Summary – LFSR

- **Two types** of LFSR. **Simple** structure, good for **TPG**
- LFSR generate **pseudo random test patterns.** Repeat every L_c cycles
- Linear relationship between states: $Q^+ = TQ$
- **Characteristic polynomial** is often used to describe LFSR

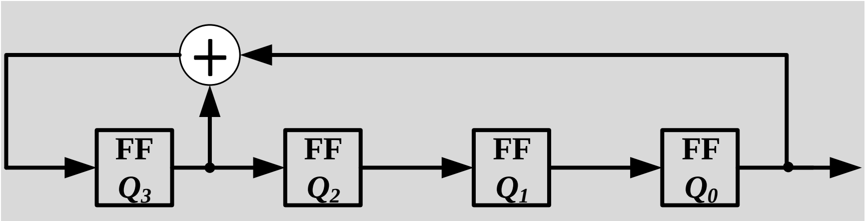
	Structure	Matrix
type1 standard-form	 index right to left	$\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \vdots \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & 0 & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ h_0 & h_1 & h_2 & \cdots & h_{N-2} & h_{N-1} \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}$
type2 modular-form	 index left to right	$\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \vdots \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & h_0 \\ 1 & 0 & 0 & \cdots & 0 & h_1 \\ \vdots & 1 & \vdots & & \vdots & h_2 \\ 0 & 0 & 1 & \cdots & 0 & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h_{N-2} \\ 0 & 0 & 0 & \cdots & 1 & h_{N-1} \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}$
Char. Poly	$f(x) = \sum_{i=0}^{N} h_i x^i$	$f(\lambda) = \det(T - \lambda I)$

FFT 1

- **Characteristic polynomial can also be derived from matrix *T***

$$\boxed{f(\lambda) = \det(T - \lambda I)}$$

- **Q1: Given this type-1 LFSR, Please verify that $\det(T-\lambda I)=\lambda^4 - \lambda^3 - 1$**
- **Q2: From structure, we know = $f(x) = x^4+x^3+1$**
  - **Not the same, why?**

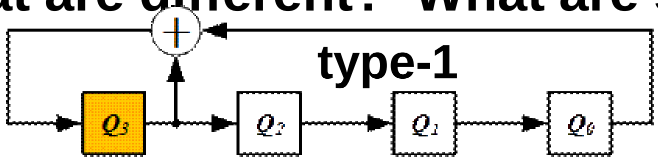


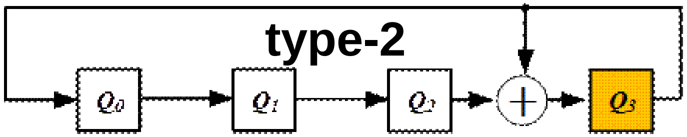
$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\det(T - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 1 & 0 & 0 & 1-\lambda \end{vmatrix} = ?$$

FFT 2

- Q: Given two types LFSR of same x^4+x^3+1, same seed 1000.
- What are different? What are same?

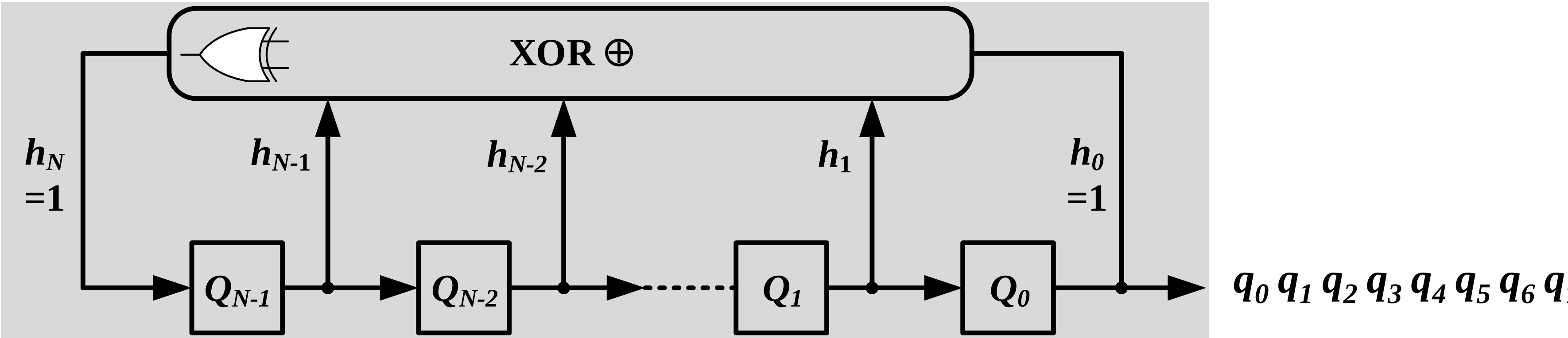




state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15 (=0)	1	0	0	0

state	Q_0	Q_1	Q_2	Q_3
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	1	0	0	1
5	1	1	0	1
6	1	1	1	1
7	1	1	1	0
8	0	1	1	1
9	1	0	1	0
10	0	1	0	1
11	1	0	1	1
12	1	1	0	0
13	0	1	1	0
14	0	0	1	1
15 (=0)	1	0	0	0

APPENDIX: Characteristic Polynomial (1)



output bit stream: $q_0,\ q_1,\ q_2,\ \ldots$ (q_0 first)

recurrence equation: $q_m = \sum_{i=1}^{N} h_i q_{m-i}$

generating function: $G(x) = q_0 x^0 + q_1 x^1 + q_2 x^2 + \ldots = \sum_{m=0}^{\infty} q_m x^m$

$$= \sum_{m=0}^{\infty}\left[\left(\sum_{i=1}^{N} h_i q_{m-i}\right)\cdot x^m\right] = \sum_{i=1}^{N} h_i x^i \cdot \left[\sum_{m=0}^{\infty} q_{m-i} x^{m-i}\right]$$

$$= \sum_{i=1}^{N} h_i x^i \cdot \left[q_{-i} x^{-i} + \ldots + q_{-1} x^{-1} + \sum_{m=0}^{\infty} q_m x^m\right]$$

APPENDIX: Characteristic Polynomial (2)

$$G(x) = \sum_{i=1}^{N} h_i x^i \cdot \left[q_{-i} x^{-i} + \ldots + q_{-1} x^{-1} + G(x) \right]$$

$$G(x) = \frac{\sum_{i=1}^{N} h_i x^i \cdot [q_{-i} x^{-i} + \ldots + q_{-1} x^{-1}]}{1 - \sum_{i=1}^{N} h_i x^i}$$

LFSR seed. $q_{-N}, \ldots, q_{-1}$

LFSR structure

when seed $q_{-N} \ldots q_{-1}$=[1000...0]

$$G(x) = \frac{1}{1 - \sum_{i=1}^{N} h_i x^i} = \frac{1}{f(x)}$$

characteristic polynomial: $f(x) = 1 - \sum_{i=1}^{N} h_i x^i = \sum_{i=0}^{N} h_i x^i$