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Optimal design and planning for compact automated parking systems

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ABSTRACT

Compact automated parking (CAP) systems are fully automated parking systems, which store cars densely. Such systems are mainly used in congested cities all over the world, providing rapid parking access and safe vehicle storage. We study a prominent new technology, with relatively low cost and rapid response. The system has a rotating ring equipped with shuttles in each tier for horizontal transport, and uses two lifts in the middle of the CAP system for vertical transport. We present a dedicated lift operating policy under which it uses one lift for storage and another for retrieval, and a general operating policy under which it uses both lifts for storage and retrieval. We propose queuing networks for single-tier and multi-tier systems based on two different policies for operating the lifts (a dedicated and general operating policy). We validate the analytical models using simulation based on a real application. We also conduct a sensitivity analysis in which we vary speeds of lifts and car rotation. Then we use the analytical models to optimize the system layout by minimizing the retrieval time. Furthermore, combining time efficiency and system cost, we find an appropriate system layout for designers. Third, we compare two lifts under dedicated and general operating policies. Forth, we find the optimal number of the lifts through a general compact automated parking system. Finally, we calculate the investment cost of a CAP system under different system configurations and compare it with an alternative design: a cubic parking system.

1. Introduction

Traditional parking systems require much space, and lead to pollution and congestion, since drivers often keep circulating to find an empty parking spot. As a response, compact automated parking (CAP) systems have been introduced. A CAP system is an automated car parking system with a compact storage area and automated car handling technology. Such systems combine rapid access and safe vehicle storage, with a minimum of floor space needed. Such a system (with circular design) has been conceptualized for storing sea containers in the stack at a container terminal in the warehouse (see U024, 2012). In the past decades, several storage and handling technologies for such systems have been developed. They are used in major cities all around the world, particularly in Europe, Asia, and North America in areas where parking space is limited and expensive.

This paper studies a new car handling technology that combines high efficiency with speedy response. The system consists of an inner rotating ring equipped with shuttles in each tier and has two lifts equipped with platforms in the middle of the system (see Fig. 1(a) and (b)). We take a storage transaction as an example. The lift transports the car from the ground floor tier to the storage tier. There, in the high tier, the car is transferred to a shuttle and the rotating ring moves the shuttle to a storage position in the outer ring (see Fig. 1(d)). The first tier on the ground floor, is different from other tiers. There, the outside ring consists of several car rotating tables where cars enter and leave the system. The rotating table is used to orient the car for easy access by the driver and the shuttle. The inside rotating ring consists of multiple shuttles that transport cars from rotating tables to lifts (see Fig. 1(c)). Some variants of these systems can be found e.g. on YouTube (see Emirates, 2016; Liechtenstein, 2014; Nctv7, 2012).

Compared with a conventional parking system, a CAP system not only saves floor space, but also has high throughput capacity

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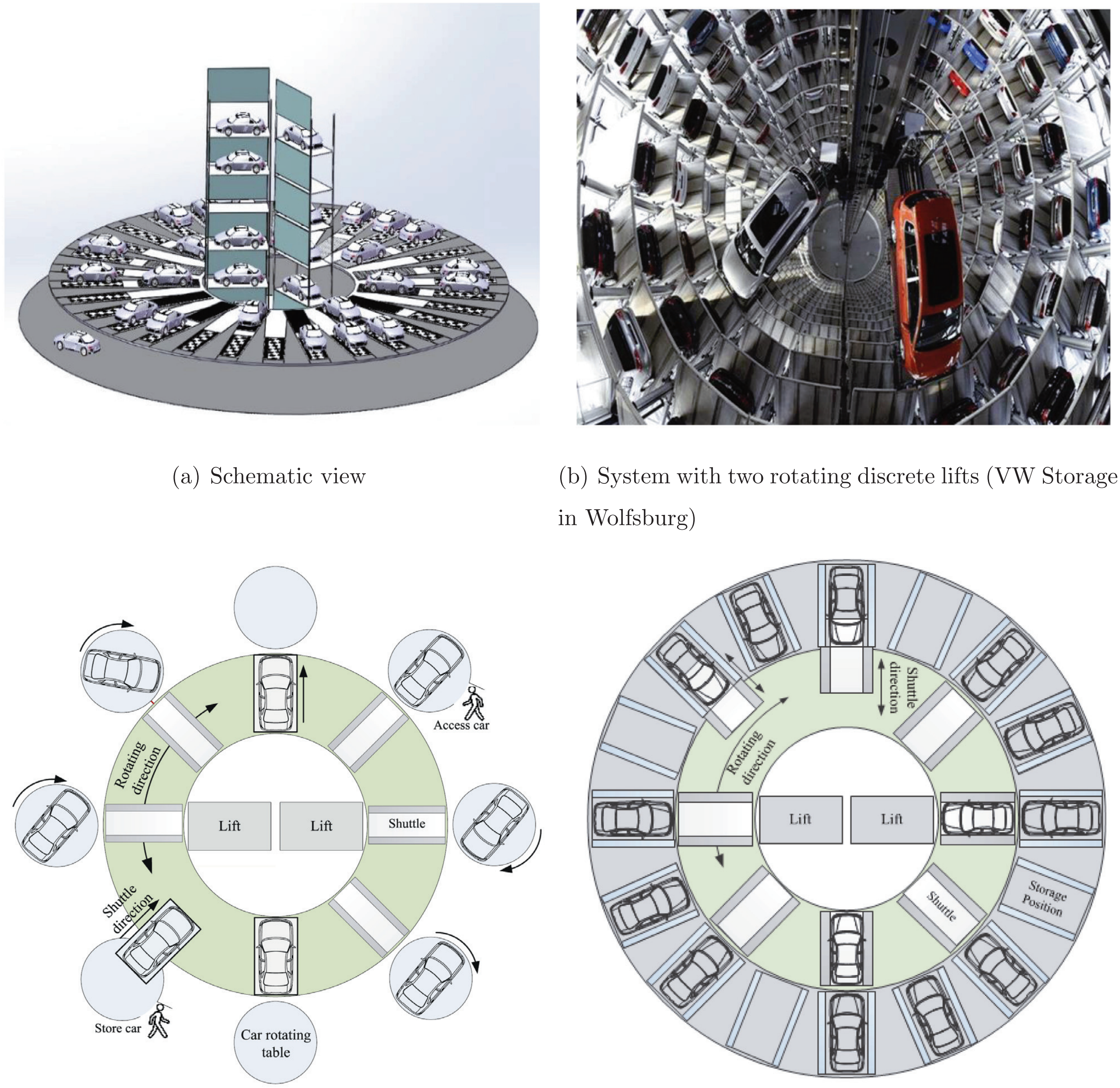


(a) Schematic view

(b) System with two rotating discrete lifts (VW Storage in Wolfsburg)

(c) Top view of ground floor tier

(d) Top view of a higher tier

Fig. 1. Compact automated parking system.

and short response times, due to the multiple lifts and multiple shuttles used in the circular rings. The lift can be discrete (containing one platform, see Fig. 1(b)) or continuous (containing multiple platforms on a rotating chain). A continuous lift operates similarly to a vertical conveyor, and cars only have to wait for an open space to be transported. However, such an automated system requires a higher investment and it may have other drawbacks, such as dependency on automated equipment, which may fail. In order to find out whether the investment is justified, it is therefore necessary to first model and analyze the systems for performance impact.

Different technologies are used in compact automated parking systems. Many of them store cars on autonomous shuttles (Gue & Kim, 2007; Zaerpour, Yu, & De Koster, 2015b). This implies that each car is stored on its own shuttle that can move to a lift for vertical transport. As these shuttles are quite expensive (30,000–60,000/shuttle, depending on movement abilities), a system like the one sketched in Fig. 1 (with fewer and simpler shuttles) is cheaper. The analysis of such vehicle-based storage and retrieval systems with autonomous shuttles differs from our analysis, due to the different layouts and technologies used. The most important operational performance measures of a CAP system are its response time (how long does a customer have to wait for her car) and throughput capacity (how many cars can be handled per hour). Estimating the performance to find a system design with required throughput capacity and acceptable costs for a given storage capacity is a key issue in deciding for a CAP system. We therefore aim to answer the following research questions in this paper:

Table 1
Abbreviations used in this paper.

Notations	Implication	Notations	Implication
CAP	Compact automated parking	I/O	Input and output
AVS/R	Autonomous vehicle storage and retrieval	POSC	Point-of-service-completion
VHDS	Very high-density storage	FCFS	First-come-first-service
SBCS	Shuttle-based compact storage	VW	Volkswagen
S/R	Storage and retrieval	DD	Discrete lift dedicated operating
AS/R	Automated storage and retrieval	DG	Discrete lift general operating
		CD	Continuous lift dedicated operating

1. How can we build accurate and effective analytical models to estimate the response time and throughput capacity of a CAP system?
2. Which lift operating policy maximizes the performance of a CAP system?
3. Considering the minimum retrieval time, what layout (system height, number of storage positions per tier) is the most beneficial for a system with a given storage capacity? Furthermore, considering the time efficiency and investment cost, which layout is appropriate for warehouse designer?
4. For a more general CAP system, how does the number of the lifts affect system performance?
5. Compared with a cubic parking system, is the investment cost of CAP system higher or lower?

To answer these questions, we first derive closed-form formulas and model a single-tier and multi-tier CAP system using queuing networks. We apply an approximation method to derive the response time, the utilizations of lifts and shuttles, and the waiting time of lifts and shuttles. Both discrete lifts (with one platform) and continuous lifts (with multiple platforms) are examined. We consider two operating policies for discrete lifts: A dedicated policy under which it uses one lift for storage and the other for retrieval, and a general policy under which it uses both lifts for storage and retrieval. For continuous lifts, only the dedicated policy is considered since such a lift cannot process storage and retrieval transactions simultaneously. We build simulation models and used the data obtained from the analysis models evaluation of the lift and shuttle movements as the service time of the queueing network. Second, these models are used to compare the performance of discrete and continuous lifts, and dedicated and general lift policies. Third, we optimize the system layout using the analytical models by minimizing the retrieval transaction time. Furthermore, combining the time efficiency and investment cost, we consider an appropriate system layout for warehouse designer. Forth, we also study one rotating lift and a general parking system with multiple lifts. We find the optimal number of the lifts for a general parking system with multiple lifts. Finally, We compare the cost of footprint area with a live-cube parking system based a case described in literature.

The contribution of this paper is that:

1. We formulate accurate performance estimation models for a CAP system using two kinds of lifts and lift operating policies. Moreover, we provide design insights for a CAP system through minimizing retrieval time. We show that the system performance is sensitive to the system structure and transaction arrival rate. We show a critical retrieval transaction arrival rate exists: When the rate exceeds this value, a continuous lift performs better in terms of retrieval transaction throughput time. Otherwise, discrete lifts are preferred.
2. We consider an appropriate system layout, combining time efficiency and investment cost under different system configurations. The results show that a designer should not go for a configuration with minimum retrieval time, systems with slightly higher retrieval times may have substantially lower cost.
3. We investigate this kind of CAP system through varying the number of the lifts from 1 to n, and find the optimal number of lifts are always two when the tiers exceed 5. The results show that the service capacity of the two lifts is adequate for this CAP system under the optimum system configuration.
4. We conduct sensitivity analysis and vary speeds of lifts and the car rotation based on a real CAP (Nctv7, 2012). Our research shows that the results are insensitive to speeds of lifts and shuttles.
5. We compare the CAP system with a live-cube parking system (Zaerpour et al., 2015b). The results indicate that when the number of tiers exceed 4, the investment cost of CAP system is lower than live-cube parking system.

Below, We give all abbreviations used in this paper in Table 1.

2. Literature review

Many papers have studied automated storage systems with lifts and shuttles, including autonomous vehicle-based storage and retrieval (AVS/R) systems (Malmborg, 2002), very high-density storage (VHDS) systems (Gue & Kim, 2007) and shuttle-based compact storage (SBCS) systems (Lerher, Ekren, Dukic, & Rosi, 2015a and Lerher, Ekren, Sari, & Rosi, 2015b). All these systems use shuttles for horizontal load movements and lifts for vertical transport, but they differ in the type of lifts, the type of shuttles, and the load movement pattern. Lifts can be discrete or continuous (i.e. containing multiple platforms). In an AVS/R system, shuttles are tier-captive (a shuttle is dedicated to its own tier) (Heragu, Cai, Krishnamurthy, & Malmborg, 2011) or tier-to-tier (a shuttle can visit other tiers) (Kuo, Krishnamurthy, & Malmborg, 2007). In a VHDS system, shuttles are load-dedicated, i.e. each load is stored on its own shuttle (Kota, Taylor, & Gue, 2015). In an SBCS system, different types of shuttles are used to transport the loads along the storage lane (x-direction) and the cross aisle (y-direction), respectively (Lerher, 2016).

We first review literature on parking system related to our study. Then, we review literature on performance estimation of other systems, focusing on response time, throughput capacity, and differences and similarities with the system examined in this study.

While literature on parking systems and parking design exists, the literature on automated parking systems from a storage perspective is limited. In addition, cylindrical CAP systems have not been analyzed yet. We consider this parking design based on a real VW Storage in Wolfsburg (see Fig. 1). Evidence of the increased use of such automated parking systems can be found on internet (see, e.g. Emirates, 2016; Nctv7, 2012; Liechtenstein, 2014). We only review the literature on automated parking systems that closely relates to our study. Zaerpour et al. (2015b) study a live-cube parking system. They first use closed-form formulas for retrieval transaction time and optimize system dimensions by minimizing the retrieval time. Then, they compare the operational, investment costs, and energy consumption of live cubic systems with traditional systems based on a real application. Heimberger, Horgan, Hughes, McDonald, and Yogamani (2017) describe the use ofcomputer vision in automated parking systems: design, implementation and challenges. They discuss the design and implementation of an automated parking system from the perspective of computer vision algorithms. Designing a low-cost system with functional safety is challenging and leads to a gap between the prototype and the end product, in order to handle all the corner cases. Some literature studies one-layer storage parking systems. Lam, Li, Huang, and Wong (2006) propose a time-dependent network equilibrium model. They consider a travelers choice of departure time, route, parking location and parking duration in road networks with multiple user classes and multiple parking facilities. Liu, Yang, and Yin (2014) consider a parking reservation scheme with expiration times. Customers with a parking reservation must arrive at the reserved parking locations before a predetermined expiration time. Iranpour and Tung (1989) derive a methodology for optimal design of a parking system with a single layer.

In the studies of AVS/R system, both probabilistic and queuing models are used for performance estimation. Probabilistic models provide closed-form expressions for both single (i.e. storage or retrieval transaction) and dual-command (i.e. a combination of storage followed by retrieval) cycle times. Malmborg (2002) was the first to investigate AVS/R system. He calculated the expected S/R throughput time (weighted sum of both single and dual command cycle times) by building continuous markov chain models for horizontal and vertical material flows. Lerher, Ekren, Dukic, and Rosi (2015a) derived closed-form expressions for both single and dual-command cycle time, formulating cumulative density functions for traveling time of both lifts and shuttles. Manzini, Accorsi, Baruffaldi, Cennerazzo, and Gamberi (2016) investigate the number and depth of the lanes and the optimal shape ratio to aid the design and control of such a storage system. They build travel time model for AVS/R system. Marchet, Melacini, Perotti, and Tappia (2013) investigate the key design differences between the two types (i.e. tier-captive versus tier-to-tier) of AVS/RS configurationusing simulation, and propose a comprehensive design framework.

Due to the effects of queuing between different resources in the AVS/R system, probabilistic models cannot accurately calculate the throughput time of AVS/R systems. Some papers have emerged using queuing models for performance estimation. Fukunari and Malmborg (2008) calculated the lift service process (vehicles are customer and lifts are servers) as an M/G/L queue, nested within an M/G/V queue modeling the vehicle service process. They developed efficient approximations to derive the transaction cycle time. To address the drawbacks of using a nested queuing model, Fukunari and Malmborg (2009) built closed queuing networks for AVS/R systems, additionally considering the vehicle movement outside the system and vehicle unavailability due to maintenance and repair.

Besides single queue systems and closed queuing networks, open queuing networks are also used for performance estimation since open queuing networks can capture the effect of waiting transactions on the throughput time. Heragu et al. (2011) formulated open queuing networks for AVS/R system using tier-captive shuttles, and analyzed the advantages of AVS/R system over traditional AS/R system (using an aisle-captive crane). Marchet (2012) also investigated open queuing networks to estimate the throughput time of tier-captive systems, which considers acceleration and deceleration of lifts and vehicles. Epp, Wiedemann, and Furmans (2017) present a discrete-time open queueing network approach for performance evaluation of autonomous vehicle storage and retrieval systems (AVS/RSs) with tier-captive single-aisle vehicles.

Although open queuing networks can accurately derive the system operating time and throughput capacity, formulating vehicles as a circulating resource may be a better approach to analyze the effect of number of vehicles on the system performance. Based on this idea, recent studies start to use semi-open queuing networks to estimate AVS/R systems performance. Roy, Krishnamurthy, Heragu, and Malmborg (2012) formulated semi-open queuing networks for a single-tier AVS/R system and derived an approximation method to calculate the transaction cycle times and utilizations of both vehicles and lifts. Roy, Krishnamurthy, Heragu, and Malmborg (2015) analyzed the effect of vehicle dwell points and cross-aisle locations on the system performance, also using a semi-open queuing network approach. They derived that the end-of-aisle of the cross-aisle location is a better choice, also the load/unload dwell point policy. Cai, Heragu, and Liu (2014) used semi-open queuing networks for multi-tier AVS/R systems and matrix-geometric methods to analyze the system performance. Zou, Xu, Gong, and De Koster (2016) propose a parallel processing policy for AVS/R system and formulate a fork-join queueing network to estimate the system performance.

Different from an AVS/R system, a VHDS system stores loads in a compact area. Each load is stored on a shuttle. To store or retrieve a load, shuttles have to cooperate to create a virtual aisle. Gue and Kim (2007) were the first to analyze a VHDS system. They focused on a single-tier VHDS system and derived a closed-form expression of the expected retrieval throughput time (expressed in number of movements) for the systems with one empty location that located near the I/O point, and proposed heuristics for systems with multiple empty locations located near the I/O point. Kota et al. (2015) studied a single-tier VHDS system which has randomly located empty locations. They investigated a closed-form expression of the expected retrieval time for systems which have one or two empty locations, and derived heuristics with worst-case bounds for systems when the systems have more than two empty locations. Zaerpour et al. (2015b) investigated a multi-tier VHDS system and derived closed-form formulas for the expected retrieval throughput time for any system configuration. Moreover, they derived the optimal system dimensions by minimizing the system response time.

An SBCS system is a hybrid between an AVS/R system and a VHDS system. It consists of a cross aisle in the middle of each tier and has storage lanes besides the cross-aisle. The loads within a storage lane (usually of the same product) are stored densely. Such systems are commonly used for storage of refrigerated products. Tappia, Roy, De Koster, and Melacini (2016) studied the performance and obtained design insights for SBCS systems, considering both discrete and continuous lifts. They examined both generic (a shuttle can visit multiple storage lanes) and specialized (a shuttle can only move to another storage lane using a second vehicle) shuttles. They find that a multi-tier SBCS system prefers specialized shuttles, since the higher cost of generic shuttles cannot be balanced by savings in reduced throughput time.

Table 2 summarizes the above mentioned researches, and shows that while many papers focus on performance estimation of automated storage systems using lifts and shuttles, the systems with shuttles moving on a rotating ring have not yet been studied.

3. CAPS description and modeling preparations

In this section, we first describe the operations of the CAP systems examined in this study. Then, we specify the components of storage and retrieval transaction cycle times followed by assumptions and notations.

3.1. System description

The CAPS examined in this study is a multi-tier automated car parking system (Fig 1(a) and (b)). The first tier, sketched in Fig 1(c), has multiple input/output stations where drivers can leave their

Table 2
Literature in automated storage systems with lifts and shuttles.

Reference	Type of lifts	Type of shuttles	Load movement
AVS/R system:Cai et al. (2014); Fukunari and Malmborg (2008, 2009); Kuo et al. (2007); Malmborg (2002); Roy et al. (2012, 2015)	Discrete	Tier-to-tier	Horizontal: by roaming shuttles Vertical: by lifts (loads carried by shuttles)
AVS/R system: Heragu et al. (2011); Lerher et al. (2015a, 2015b); Lerher (2016); Marchet et al. (2012)	Discrete	Tier-captive	Horizontal: by roaming shuttles Vertical: by lifts
VHDS system: Gue and Kim (2007); Kota et al. (2015); Zaerpour et al. (2015b)	Discrete	Load-dedicated	Horizontal: by load-dedicated shuttles Vertical: by lifts(loads carried by shuttles)
SBCS system: Tappia et al. (2016)	Discrete and continuous	Tier-captive	Horizontal: by roaming shuttles Vertical: by lifts
This paper	Discrete and continuous	Tier-captive	Horizontal: by roaming shuttles Vertical: by lifts

Table 3
Operational steps included in a storage and retrieval transaction.

Step	Storage step description	Step	Retrieval Step Description
a	The car to be stored requests a shuttle in the first tier, then waits for the shuttle.	*j*	The car to be retrieved requests a shuttle in the designated tier, then waits for the shuttle.
b	The rotating ring rotates the assigned shuttle to the car. Then the shuttle loads the car.	*k*	The rotating ring rotates the assigned shuttle to the car. Then the shuttle loads the car.
c	The rotating ring rotates the shuttle with the car to the lift.	*l*	The rotating ring rotates the shuttle with the car to the lift.
d	The car requests a lift platform, then waits for the platform.	*m*	The car asks for a lift platform. The car waits for the platform.
e	The lift transports the assigned platform to the first tier. Then the platform loads the car.	*n*	The lift transports the assigned platform to the retrieval tier. Then the platform loads the car.
f	The lift transports the car to the storage tier.	*o*	The lift transports the car to the first tier.
g	The car requests a shuttle in the storage tier. The lift and car wait for the shuttle.	*p*	The car requests a shuttle in the first tier. The car waits for the shuttle.
h	The rotating ring rotates the assigned shuttle to the entrance of the lift. Then the shuttle loads the car from the lift platform.	*q*	The rotating ring at the first tier rotates the assigned shuttle to the entrance of the lift. Then the shuttle loads the car.
i	The rotating ring rotates the shuttle with the car to the designated storage location. The shuttle unloads the car into the storage position.	*r*	The rotating ring rotates the car to the car rotating table. The shuttle unloads the car into the car rotating table.

car for storage or pick it up. Each station is equipped with a rotating table that can rotate the car for easy access by the shuttle and the driver. A rotating ring with multiple shuttles transports the cars between lifts and car rotating tables. The higher tiers store the cars (Fig 1(d)). At the higher tiers, the outer ring serves as storage area and the inside ring is a rotating ring equipped with multiple shuttles. The shuttles pick up the cars from the lifts and move them into storage positions, or vice versa. The rotating ring transports the car on the shuttle between the lifts and the storage positions.

Two lifts in the middle of the system transport loads between the first tier and higher tiers. A lift can be discrete (i.e. with one platform) or continuous (i.e. with multiple platforms). A continuous lift operates like a vertical carousel (similar to a circular conveyor), with the difference that there are inputs and outputs at every tier. When a retrieval car on a tier requests a platform, the lift will transfer the closest empty platform to the target tier. In a CAPS with continuous lifts, the number of platforms in the lift is so large that the car to be stored or retrieved can get a platform without waiting. It takes a constant time for the shuttles to load/unload a car. The continuous lift will stop by a constant time once it moves a unit distance (the height of one tier), since one platform should be replenished at the first tier and one car should be moved out a platform at the top of the lift. We consider two lifts operating policies: dedicated and general policies. Under the dedicated policy, one lift stores cars, while the other retrieves them. Under the general policy, both lifts can store and retrieve cars. A continuous lift cannot process storage and retrieval transactions simultaneously (they move in opposite directions). We therefore only consider the dedicated policy for continuous lifts.

For a storage and retrieval transaction, the steps included are indicated in Table 3.

3.2. Notations and assumptions

The notations used in this study are defined in Table 4. We denote a discrete lift with the dedicated operation policy as DD, a discrete lift with the general operation policy as DG, and a continuous lift with the dedicated operation policy as CD.

We make the following assumptions for CAP systems:

1. The cars are randomly stored in the system, which means the car to be retrieved will be equally likely stored in any storage position.
2. The assignment of shuttles in the rotating ring to cars follows a random rule, i.e., the available shuttle will be positioned on the rotating ring, in any position with the same probability.
3. The arrival of storage and retrieval transactions both follow a Poisson process, i.e., the inter-arrival times of any two adjacent retrieval transactions follow an exponential distribution.
4. The service rule of the lifts and rotating rings are both First-Come-First-Service (FCFS).
5. The dwell point of both the platforms in the lift and the shuttles in the rotating ring is the Point-of-Service-Completion (POSC), they will dwell at the position where the last operation finishes.
6. We do consider the acceleration/deceleration of the lifts and the rotating ring.
7. The times for the shuttle to load/unload to/from a storage position and a lift platform are the same.
8. The number of shuttles at each tier is the same and the number of storage position at each tier is the same.

3.3. Components of storage and retrieval transaction cycle time

Based on the operational processes of storage and retrieval transactions, we write the storage transaction cycle time under the DD operating policy, the DG operating policy and the CD operating

Table 4
Notations used in this paper.

Notation	Description	Notation	Description
ε	The time for a shuttle to load/unload a car (s).	d	Diameter of the rotating ring (m).
v_s	Maximum speed of rotating ring (m/s).	D	Diameter of storage area at each tier (m).
v_l	Maximum speed of lift (m/s).	N_p	The number of platforms in each continuous lift.
h	The height of each tier (m).	T	The number of tiers.
N_c	The number of input/output carousels.	λ_s/λ_r	Arrival rate of storage/retrieval transactions (per hour).
N_s	The number of shuttles in a rotating ring at each tier.	n_{sp}	The number of storage positions at each tier.
w_{sp}	The width of a storage position.	N_{sp}	The number of total storage positions, $N_{sp} = (T-1) * n_{sp}$.
w_c	The width of a car.	ε_s	The unit stop time of the continuous lift (s).
C	The system storage capacity.		
T_S, T_R	The storage/retrieval transaction throughput time.	W_{S1}, W_{R1}	The waiting time of cars for shuttles in the first tier (W_{S1} for step a, W_{R1} for step p).
t_{S1}, t_{R1}	The storage/retrieval operating time in the first tier (t_{S1} for steps b, c, t_{R1} for steps q, r).	W_{Sl}, W_{Rl}	The waiting time of cars for discrete lift (W_{Sl} for step d, W_{Rl} for step m).
t_{Sl}, t_{Rl}	The storage/retrieval operating time of the lift (t_{Sl} for steps e, f, t_{Rl} for steps n, o).	W_{ST}, W_{RT}	The waiting time of cars for shuttles in the target tier (W_{ST} for step g, W_{RT} for step j).
t_{ST}, t_{RT}	The storage/retrieval operating time in the target tier (t_{ST} for steps h, i, t_{RT} for steps k, l).	D_{l,t_1}	The travel distance that the lift moves to the first tier (step e).
$D_{s_1,c}$	The travel distance that the shuttle in the first tier moves from the dwell point to input/output carousels (step b).	$D_{s_t,sp}$	The travel distance that the shuttle at the target tier moves from its dwell point to the storage position (steps k).
$D_{c,l}$	The travel distance that the shuttle in the first tier moves from the carousel to the lift (step c). $D_{l,c}$ (step r) is identical to $D_{c,l}$.	$D_{l,sp}$	The travel distance that the shuttle at the target tier moves from the lift to the storage position (step i). $D_{sp,l}$ (step l) is identical to $D_{l,sp}$.
$D_{s_1,l}$	The travel distance that the shuttle in the first tier moves from the dwell point to the lift (step q).	D_{t_1,t_t}	The travel distance that the lift transport the cars from the first tier to the target tier (step f), D_{t_t,t_1} (step o) is identical to D_{t_1,t_t}.
$D_{s_t,l}$	The travel distance that the shuttle at the target tier moves from the dwell point to the lift (step h).	D_{l,t_t}	The travel distance that the lift transport the cars to the target tier, (step n).

policy as Eq. (1)

$$T_S = t_{S1} + t_{Sl} + t_{ST} + W_{S1} + W_{Sl} + W_{ST} \tag{1}$$

We can write the retrieval transaction cycle time as Eq. (2)

$$T_R = t_{R1} + t_{Rl} + t_{RT} + W_{R1} + W_{Rl} + W_{RT} \tag{2}$$

Note that $W_{Sl} = 0$, and $W_{Rl} = 0$ in the case of continuous lift. In Section 4, we calculate the operating times (t_{S1}, t_{Sl}, t_{ST}, t_{R1}, t_{Rl}, t_{RT}) and build queuing networks to estimate the expected waiting times (W_{S1}, W_{Sl}, W_{ST}, W_{R1}, W_{Rl}, W_{RT}).

4. Analytical models for performance estimation

We build open queuing networks for performance estimation in Section 4.1, calculate the service time of the nodes in the queuing network in Section 4.2, and give the solution method in Section 4.3.

4.1. Queuing network models for the CAP system

The operations of the CAP system can be modeled by a queueing network. We do this for the continuous lift, and the discrete lift with dedicated and general policies, respectively.

Continuous lift

In Fig. 2 lifts and shuttles are modeled as servers, as they move cars vertically and horizontally, respectively. Storage and retrieval transactions are modeled as customers. Both storage and retrieval transactions can be divided into $T-1$ classes, based on the designated storage or retrieval tier. In the remainder of the paper, the storage tier t ($t = 2, \ldots, T$) in which a storage or retrieval transaction has to be performed also defines the customer type.

Fig. 2 shows the queuing network for the CAP system with a continuous lift. The storage transactions arrive at the first tier with arrival rate λ_s, leading to arrival rates at different tiers $\lambda_{s2}, \lambda_{s3}, \ldots, \lambda_{sT}$, with $\sum_{i=2}^{T} \lambda_{si} = \lambda_s$. The storage transaction requests a shuttle at the first tier, and then the claimed shuttle moves from its dwell point to the storage car. We model this operation as the service node $\mu_{s_1 1}$, which has N_s servers, since each tier has N_s shuttles. After the shuttle reaches the storage car, it

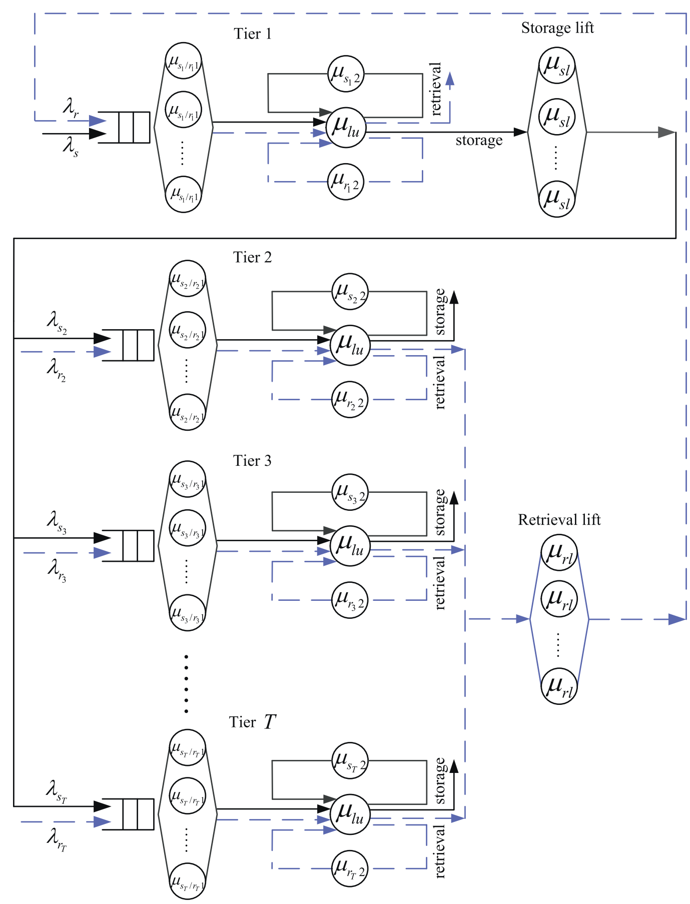


Fig. 2. Continuous lift.

loads the car. This operation is modeled as the service node μ_{lu}. We model μ_{lu} as an infinite server node without a queue. Then the storage transaction goes to the station $\mu_{s_1 2}$ where the rotating ring rotates the shuttle to the lift. The shuttle unloads the car

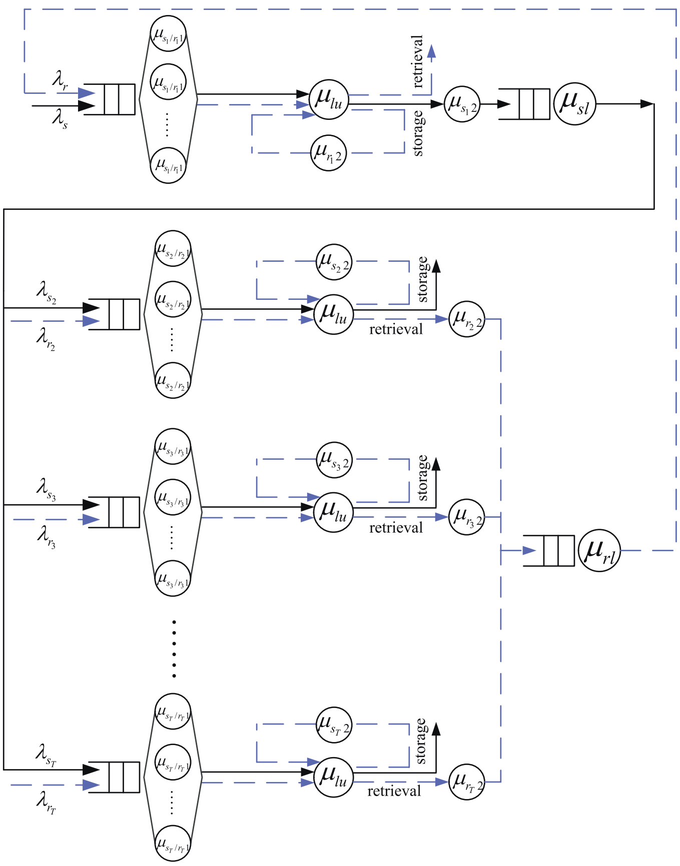


Fig. 3. Discrete lifts and dedicated lift policy.

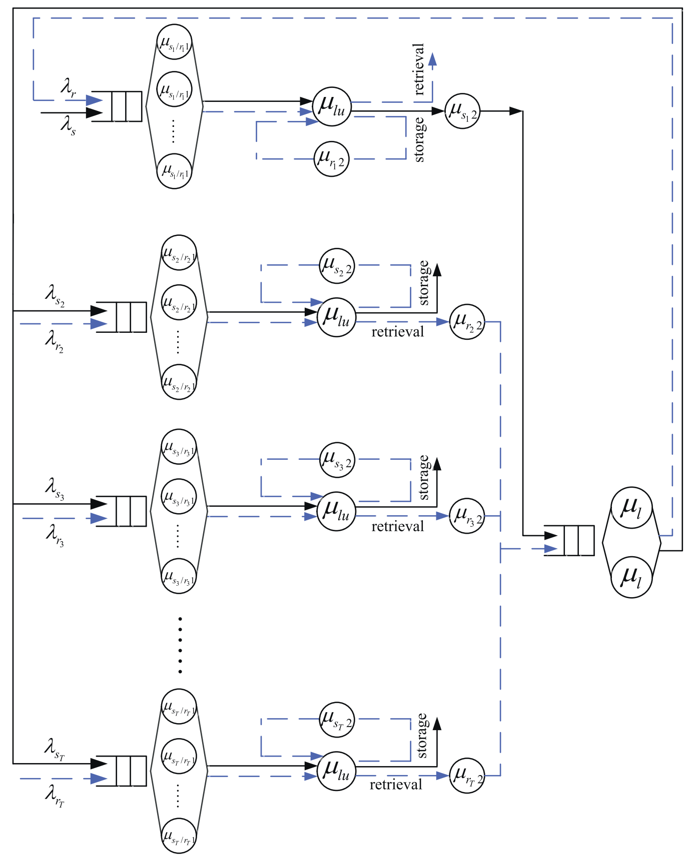


Fig. 4. CAP system with discrete lift and general lift policy.

when it reaches the lift, which is also modeled by the service node μ_{lu}. Next, the transaction goes to the station μ_{sl} representing the vertical transport of the storage car from the first tier to its designated tier. Since the lift is continuous, the transaction is servered at node μ_{sl} without waiting. Therefore, we model service node μ_{sl} as an infinite server. When the storage car reaches the designated tier, it claims a shuttle and then the shuttle moves from its dwell point to the lift, which is modeled as the service node $\mu_{s_t 1}$, where $t = 2, 3 \ldots T$. Then, the shuttle loads the car (μ_{lu}) and transports it to the storage position ($\mu_{s_t 2}$, $t = 2, 3 \ldots T$). Next, the shuttle releases the car (μ_{lu}). Finally, the transaction is stored at its designated tier, and the transaction leaves the queuing system.

The retrieval transaction process is the reverse of the storage process. We use the notation λ_{r_t}, $\mu_{r_t,1}$, $\mu_{r_t,2}$ $(t = 2, 3 \ldots T)$ to represent the arrival rate and the service rates at each service node insteading of the storage process node notation λ_{s_t}, $\mu_{s_t,1}$, $\mu_{s_t,2}$, $(t = 2, 3 \ldots T)$.

Discrete lift with dedicated lift policy

The difference between continuous and discrete lifts is the number of platforms available. A car may need to wait for the platform of the discrete lift, while no waiting is required for a continuous lift. This difference reflects in the service nodes μ_{sl} and μ_{rl} (Fig. 3). For a continuous lift, both μ_{sl} and μ_{rl} are infinite servers, while they are both single server for the case of a discrete lift. For DD operating policy, the transaction goes to the station μ_{sl} where the lift loads the car, and then transports the car from the first tier to the designated tier.

Discrete lift with general lift policy

Under the general lift policy, the two lifts process both storage and retrieval transactions (see Fig. 4). With the dedicated lift policy, the storage lift and the retrieval lift are both modeled as a single server. Under the general lift policy, the lifts are modeled as a single queueing system with two servers.

4.2. Operating time in a transaction cycle

We calculate the horizontal and vertical operating times, respectively.

Horizontal operating time

The horizontal operating time differs between the first tier and the target tier. Both DD and CD systems have the same horizontal operating time, since the car always moves to a designated lift, but they differ from DG in which the car can go to any lift.

Based on the number of shuttles used at each tier, we divide the rotating ring into N_s. We denote the shuttle position as s_1. As all movements in horizontal direction use the rotating ring, the storage position (*sp*), the lift position (*l*) and the carousel input/output position (*c*) can all be converted to points on the circumference of the rotating ring.

(1) The first tier

At the first tier, the horizontal storage operating time corresponds to two movements: the move of a shuttle from its dwell point to the target car, followed by the move of the car to the lift. For ease of simplification, we here assume the shuttle is selected randomly. It is also possible to assign the closest shuttle, revising Eq. (3) by the method of Appendix A). The distance for the first movement is $D_{s_1,c}$, which can be calculated by Eq. (3)

$$D_{s_1,c} = \frac{\pi d}{N_s} \min\{|s_1 - c|, N_s - |s_1 - c|\} \tag{3}$$

where c is the position of storage car (carousel input). Since we assume a random storage policy and POSC dwell point policy for the shuttle, s_1 follows a discrete uniform distribution $[1, N_s]$ with a step size of one and $c$follows a discrete uniform distribution on the set $\{N_s/N_c, 2N_s/N_c, \ldots, N_s\}$, with a step size of $\frac{N_s}{N_c}$. The shuttle will move to the car by the shortest path, which is chosen from either clockwise path ($\mid s_1 - c\mid$) or anti-clockwise path ($N_s - \mid s_1 - c\mid$). The distance for the second movement can be calculated by following equation

$$D_{c,l} = \frac{\pi d}{N_s}\min\{|c-l|, |N_s - |c-l|\} \tag{4}$$

where l is the lift position. Under the dedicated lift policy (for both continuous and discrete lift), one lift serves for storage transactions with position $l = 0$, the other lift serves for retrieval transactions with position $l = \frac{N_s}{2}$. Under the general lift policy, l can be either 0 or $\frac{N_s}{2}$, with a probability 0.5.

Since we consider a constant speeds of lifts and the rotating ring, the travel time can be calculated by $t_{s_1,c} = \frac{D_{s_1,c}}{v_s}$, $t_{c,l} = \frac{D_{c,l}}{v_s}$. It is possible to include lift and shuttle acceleration and deceleration in a straightforward manner (Appendix B). The expected storage time at the first tier can be calculated by following equation

$$ESC(t_{S1}) = \sum_{s_1}\sum_{c} p(s_1) * p(c)(t_{s_1,c} + \varepsilon) + \sum_{c}\sum_{l} p(c) * p(l) * t_{c,l} \tag{5}$$

where $p(s_1) = \frac{1}{N_s}$, $p(c) = \frac{1}{N_c}$. For the general lift policy, $p(l) = \frac{1}{2}$. For the dedicated lift policy, $p(l) = 1$.

A retrieval transaction at the first tier is processed in reverse order compared to a storage transaction. The travel distance can be derived by the same method. At the first tier, two movements are needed to retrieve a car: a shuttle moves from its dwell point to the designated lift and then transports the car to the carousel door. The distance for the first movement can be calculated by Eq. (6)

$$D_{s_1,l} = \frac{\pi d}{N_s}\min\{|s_1 - l, N_s - |s_1 - l|\} \tag{6}$$

The second movement is in reverse direction of the movement corresponding to the distance $D_{c,\ l}$, they are equal in distribution. So, we have $ESC(D_{l,c}) = ESC(D_{c,l})$. The expected retrieval time at the first tier can now be calculated by Eq. (7)

$$ESC(t_{R1}) = \sum_{s_1}\sum_{l} p(s_1) * p(l)(t_{s_1,l} + \varepsilon) + \sum_{c}\sum_{l} p(c) * p(l)(\varepsilon + t_{l,c}) \tag{7}$$

where $t_{s_1,l} = \frac{D_{s_1,l}}{v_s}$, and $t_{l,c} = \frac{D_{l,c}}{v_s}$.

(2) The target tier

At the target tier, the horizontal storage operating time corresponds to two movements: a shuttle first moves from its dwell point to the lift and then transports the car to the storage position. The first movement is reverse to the movement corresponding to $D_{s_1,l}$. So, we have $ESC(D_{s_t,l}) = ESC(D_{s_1,l})$. The distance for the second movement can be calculated by Eq. (8)

$$D_{l,sp} = \frac{\pi d}{N_s}\min\{|l - sp|, N_s - |l - sp|\} \tag{8}$$

where sp is the storage position, which follows a uniform distribution on $\{N_s/N_{sp}, 2N_s/N_{sp}, \ldots, N_s\}$, with a step size of N_s/N_{sp}. The expected storage time at the target tier can be calculated by Eq. (9)

$$ESC(t_{ST}) = \sum_{s_1}\sum_{l} p(s_1) * p(l)(t_{s_t,l} + \varepsilon) + \sum_{sp}\sum_{l} p(sp) * p(l)(\varepsilon + t_{l,sp}) \tag{9}$$

where $p(sp) = \frac{1}{N_{sp}}$. $t_{s_t,l} = \frac{D_{s_t,l}}{v_s}$, $t_{l,sp} = \frac{D_{l,sp}}{v_s}$.

At the target tier, the horizontal retrieval operating time corresponds to two movements: the shuttle first moves from its dwell point to the storage position and then transports the car to the designated lift. The distance for the first movement can be calculated by Eq. (10)

$$D_{s_t,s_p} = \frac{\pi d}{N_s}\min\{|s_1 - s_p|, N_s - |s_1 - s_p|\} \tag{10}$$

The second movement is the reverse of the movement corresponding to $D_{l,\ sp}$. So, we have $ESC(D_{sp,l}) = ESC(D_{l,sp})$. The expected retrieval time at the target tier can be calculated by Eq. (11)

$$ESC(t_{RT}) = \sum_{s_1}\sum_{sp} p(s_1) * p(sp)(t_{s_t,sp} + \varepsilon) + \sum_{sp}\sum_{l} p(sp) * p(l) * t_{sp,l} \tag{11}$$

where $t_{s_t,sp} = D_{s_t,sp}/v_s$, $t_{sp,l} = D_{sp,l}/v_s$.

Vertical operating time

(1) The CD system

The vertical storage operating time corresponds to two movements: the claimed platform moves to the first tier and then transports the car to the target tier. The distance for the first movement is D^{CD}_{l,t_1}, which can be calculated by Eq. (12)

$$D^{CD}_{l,t_1} = (T * h)/N_p * n_p \tag{12}$$

where n_p is the shuttles position on the continuous lift, it follows a discrete uniform distribution $[1, N_p]$.

Among the operational process of the lift to transport the car to the target tier, we can calculate the single tier travel distance by Eq. (13)

$$D_{t_1,t_2} = h \tag{13}$$

Since the lift works at a constant speed, the travel time equals $t_{l,t_1} = D^{CD}_{l,t_1}/v_l$, $t_{t_1,t_2} = D_{t_1,t_2}/v_l$. Appendix B treats the case with lift acceleration and deceleration. The expected operating time of a storage transaction for the continuous lift can now be calculated by Eq. (14)

$$ESC(t^{CD}_{SL}) = \sum_{n_p}\sum_{t} p(n_p) * p(t)\left(\varepsilon + t_{l,t_1} + t_{t_1,t_2} * (t-1) + (t-2) * \varepsilon_s\right) \tag{14}$$

where t is the target tier, and it follows a discrete uniform distribution [1,T], $p(t) = \frac{1}{T}$, $p(n_p) = \frac{1}{N_p}$.

The retrieval transaction movement for the lift is similar to a storage transaction, except that the lift first moves to the target tier, traveling distance D^{CD}_{l,t_t}, which can be calculated by Eq. (15)

$$D^{CD}_{l,t_t} = |((T * h)/N_p) * n_p - (t-1) * h| \tag{15}$$

The expected operating time of a retrieval transaction for the continuous lift can now be calculated by substituting t_{l,t_t} instead of t_{l,t_1} into Eq. (14).

(2) The DD system

The vertical storage operating time of the discrete lift includes two movements. The distance for the lift to move from its dwell point to the first tier is D^{DD}_{l,t_1}, which can be calculated by Eq. (16)

$$D^{DD}_{l,t_1} = (t_l - 1) * h \tag{16}$$

where t_l is the dwell point of lift, and it follows a discrete uniform distribution [1,T], $p(t_l) = \frac{1}{T}$. The distance for the lift to transport the car to the target tier can be calculated by Eq. (17)

$$D_{t_1,t_t} = (t-1) * h \tag{17}$$

The expected operating time of a storage transaction for the DD system can be calculated by Eq. (18)

$$ESC(t^{DD}_{SL}) = \sum_{t_l} p(t_l) * \left(\varepsilon + t_{l,t_1}\right) + \sum_{t} p(t) * t_{t_1,t_t} \tag{18}$$

where $t_{l,t_1} = D^{DD}_{l,t_1}/v_l$, $t_{t_1,t_t} = D_{t_1,t_t}/v_l$.

The expected retrieval transaction time equals the expected storage transaction time. We have $ESC(t_{RL}^{DD}) = ESC(t_{SL}^{DD})$.

(3) The DG system

The storage transaction in the DG system is the same with the DD system. So, we have $ESC(t_{SL}^{DG}) = ESC(t_{SL}^{DD})$. For a retrieval transaction, the lift first moves to the target tier, traveling distance D_{l,t_t}^{DG}, which can be calculated by Eq. (19)

$$D_{l,t_t}^{DG} = |t_l - t| * h \tag{19}$$

The distance for the lift to transport the car to the first tier equals D_{t_1,t_t}. So, we have $ESC(D_{t_t,t_1}) = ESC(D_{t_1,t_t})$. The expected operating time of a retrieval transaction for the DD system can now be calculated by Eq. (20)

$$ESC(t_{RL}^{DG}) = \sum_{t_l}\sum_{t} p(t_l) * p(t) * (\varepsilon + t_{l,t_t}) + \sum_{t} p(t) * t_{t_t,t_1} \tag{20}$$

where $t_{l,t_t} = D_{l,t_t}^{DG}/v_l$, $t_{t_t,t_1} = D_{t_t,t_1}/v_l$.

4.3. Approximation method for waiting time estimation

We first get the waiting time of the discrete lifts with general lift policy. Secondly, we extend the approximation method to discrete lifts with a dedicated lift policy.

Discrete lift with general lift policy

We first consider retrieval transactions; storage transactions are carried out in reverse sequence. We first calculate the load waiting time for the lift, based on the steps in Bolch (2006) and in Melacini et al. (2011). There are $2 + N_s * T$ servers for both retrieval and storage in this stage. From Fig. 4 is appears there are four steps to calculate the waiting time for lifts. Step 1, we decompose the queuing network into -/G/1 queues that are mutual independence, and derive the arrival rates, the utilizations and the mean service time of each server. Step 2, we calculate the square of the variation coefficient of the service times for each serves. Step 3, we calculate the square of the variation coefficient of the inter-arrival times for each server. Step 4, we calculate the mean queue length and the mean waiting time of each server. The steps are described in Appendix C in detail. The mean load waiting time for the lift can be derived by:

$$W_{Sl}^{DG} = \frac{\sum_{t=2}^{T} \overline{W}_{s_t,t} + \sum_{t=2}^{T}\sum_{j=1}^{2} \overline{W}_{l_j,t}}{T-1}$$

The arrival of storage and retrieval transactions both follow a stationary Poisson process in this paper. In reality, car arrivals and retrievals may come in peaks. Recently, a technique has been introduced to handle non-stationary job arrivals in queuing networks (Dhingra, Kumawat, Roy, & de Koster, 2017). Also scheduled car pick ups may be modeled (see Zaerpour, Yu, & De Koster, 2015a). In fact, we can extend our analytical models to other arrival rate distributions, such as a Beta distribution. The arrival of customers and retrieval transactions follow a non-Poisson process. The difference between a Poisson distribution and a non-Poisson distribution is the second moment of the inter-arrival time between any two adjacent customers, i.e., cv^2. Our queueing network models and the solution approach can handle both cases.

We test the sensitivity of the inter-arrival process for an arrival distribution using $20.5 + 16 * Beta(1.15, 1.13)$. We get this distribution based on a stochastic simulation of the inter-arrival times of any two adjacent customers and we get the distribution: $20.5 + 16 * Beta(1.15, 1.13)$. This distribution has $cv^2 = 0.024$ and guarantees a lift utilization larger than 80% . We filled out the $c_{0s_i,i}^2$ in formula ($c_{As_i,i}^2 = \frac{1}{\lambda_{s_i,i}}(\sum_j c_{js_i,i}^2 * \lambda_{j,i} * p_{js_i,i} + c_{0s_i,i}^2 * \lambda_{0,i} * p_{0s_i,i}), i = 2, 3 \ldots T$) in Appendix D and compared the results of Poisson distribution with Beta distribution. The results can be found in Appendix G. We find that the variation trend of the three lift operating policies and optimum tiers are insensitive for the arrival of customers.

Discrete lift with dedicated lift policy

In case of a discrete lift with dedicated lift policy, both storage and retrieval transaction face $1 + N_s * T$ servers. Although the lifts share the shuttles, each lift is dedicated to either storage or retrieval. Using Fig. 3, we can calculate the waiting time for lifts. This can be done by slightly modifying the calculation steps explained in DG operating policy. By reducing the number of lifts from two to one and repeating the steps in CD operating policy, we derive the waiting time of retrieval commands for the discrete lift dedicated policy. Calculating the waiting time for storage commands is similar as for retrievals.

5. Model validation

This section validates the analytical models by simulation, using Arena (version 14.0) software. The system parameters for simulation validation are presented in Table 5. They are based on VWs Compact Automated Parking system (see Nctv7 (2012)). A car measures $5.2\text{m} \times 2.0\text{m} \times 1.6\text{m}$ (length $\times$ width(W_c) $\times$ height), and the size of a storage position is $6.5\text{m} \times 2.7\text{m} \times 2.1\text{m}$ (length $\times$ width(W_{sp}) $\times$ height). v and a represent the velocity and acceleration/deceleration of the shuttles and the lifts,respectively. h is the height of the tier. $\varepsilon/\varepsilon_s$ is the time for loading/unloading a car.

We consider 6 system scenarios by varying the total storage capacity N_{sp} and the system structure (T and D). The details of these system scenarios are presented in Table 6. Generally, the lift is the bottleneck of a CAP system. This holds in particular for systems with a discrete lift and a dedicated lift policy. For each scenario, we assume storage and retrieval transaction arrival rates which guarantee a lift utilization larger than 80%.

We assume that one shuttle serves three storage positions. The width of a storage position equals 2.67 m. This results in a storage system diameter $D = \lceil (2.67 * n_{sp}/\pi) \rceil$, where $\lceil\rceil$ means rounding up. Since we consider a system with two lifts, the rotating ring diameter must be at least 4 cars wide (20.8 meters). According to Pypno, John, and Sierpiski (2014), the storage capacity of a CAP system can be 1,000 cars. We therefore study systems with 400 and 1,000 car storage positions(N_{sp}).

In simulation validation and numerical experiments, we consider acceleration/deceleration of the rotating ring and the lifts, the acceleration/deceleration rate is a and speed rate is v. The movement time corresponding with a distance D' is t, with

$$t = \begin{cases} D'/v + v/a, & \text{if } t > 2t_1 \\ 2\sqrt{D'/a}, & \text{otherwise } t \le 2t_1 \end{cases}$$

where the calculation method of the time t can be explained in detail in Appendix B.

Fig. E1 in Appendix E illustrates the simulation flow chart of retrieval and storage transactions. The assumptions of the simulation model are the same as those of the analytical model described in Section 3.1. We consider sequential movement of the lift platforms and the shuttles. For each scenario, 100 replications are run with a warm-up period of 100 hours and a running time of 1,000 hours per replication, leading to a 95% confidence interval where the half-width is within 2% of the average. We collected several performance measures, including the storage transaction throughput time T_S^{CD}, T_S^{DD}, T_S^{DG}, the retrieval transaction throughput time T_R^{CD}, T_R^{DD}, T_R^{DG}, the utilization of shuttles at the first tier ρ_1 and of lifts ρ_{rl}, ρ_{sl}, and the expected waiting time of storage and retrieval transactions for lifts (W_{Rl}^{DD}, W_{Sl}^{DD}, W_{Rl}^{DG} and W_{Sl}^{DG}). The relative error of analytical results (A) to simulation results (S) is calculated

Table 5
Literature in automated storage systems with lifts and shuttles.

$\varepsilon/\varepsilon_s$	w_c	w_{sp}	v_s	a_s	v_l	a_l	h
$12(s)$	$2(m)$	$2.67(m)$	$1.8(m/s)$	$0.6(m/s^2)$	$1.6(m/s)$	$0.5(m/s^2)$	$2.13(m)$

Table 6
System scenarios examined in simulation validation.

Scenario	N_{sp}	T	n_{sp}	$D(m)$	$d(m)$	N_s	N_c	$\lambda_r/\lambda_s(/h)(CD)$	$\lambda_r/\lambda_s(/h)(DD)$	$\lambda_r/\lambda_s(/h)(DG)$
A	400	8	58	50	25	20	6	200	150	170
B	400	9	50	43	22	17	6	250	130	150
C	400	10	45	39	21	15	6	300	120	140
D	1000	8	143	122	61	48	6	200	155	170
E	1000	9	125	107	54	42	6	250	130	150
F	1000	10	112	96	48	38	6	300	120	140

Table 7
Simulation validation results of the CAP system with continuous lifts.

Scenario	T_R^{CD}			T_S^{CD}			ρ_1		
	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$
A	163.7	168.9	3.1	165.9	171.0	3.0	15.1	15.1	0.1
B	167.3	166.6	0.4	169.7	168.6	0.6	20.1	20.1	0.0
C	172.6	181.8	5.0	175.2	184.4	5.0	25.3	26.0	2.7
D	228.7	230.2	0.7	230.8	234.0	1.4	13.8	13.6	1.4
E	223.6	229.5	2.6	225.9	229.7	1.7	17.4	17.4	0.1
F	222.0	229.2	3.1	224.6	232.0	3.2	20.8	20.8	0.0

Table 8
Simulation validation results of the CAP system with discrete lift with dedicated lift policy.

Scenario	T_R^{DD}			T_S^{DD}			W_{Rl}^{DD}			W_{Sl}^{DD}		
	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	145.2	145.3	0.1	201.3	198.9	1.2	21.4	20.9	2.7	73.5	71.5	2.8
B	138.1	137.8	0.2	180.0	180.8	0.4	19.4	18.8	3.5	56.7	57.8	2.0
C	136.5	136.1	0.3	184.1	186.3	1.2	20.6	18.6	10.7	62.9	63.9	1.6
D	212.7	209.6	1.5	292.3	287.2	1.8	24.0	23.6	1.6	99.6	96.1	3.6
E	194.2	194.1	0.0	236.5	235.6	0.4	19.4	18.9	3.1	57.1	58.0	1.5
F	185.7	183.6	1.2	233.8	233.7	0.0	20.7	18.7	10.7	63.4	63.4	0.0
Scenario	ρ_{rl}			ρ_{sl}			ρ_1					
	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$			
A	70.5	70.8	0.4	87.4	87.1	0.4	11.3	11.3	0.0			
B	66.0	66.2	0.3	83.0	83.1	0.1	10.4	10.4	0.0			
C	65.4	64.2	1.9	83.3	83.3	0.0	10.1	10.5	3.8			
D	72.9	73.2	0.4	90.3	90.1	0.3	10.7	10.5	1.4			
E	66.0	66.2	0.3	83.0	83.2	0.2	9.0	9.0	0.0			
F	65.4	64.2	1.8	83.3	83.3	0.0	8.3	8.3	0.1			

by Eq. (21)

$$\delta = \frac{|A - S|}{S} \times 100\% \tag{21}$$

The results are presented in Tables 7–9. The relative errors of storage and retrieval throughput time are all less than 5%. The relative error of expected waiting time of cars for lifts is less than 10%. The relative error of the utilizations of both lifts and shuttles are less than 3.7%. The relative errors mainly come from two sources: the service rate of each service node is captured by only the first two moments and from the approximate method to estimate the waiting time. While the relative errors of expected waiting time are large, the absolute errors are acceptable (smaller than 3 seconds). Therefore, we can say that our analytical models can estimate the system performance of the CAP system with accuracy.

The results in Tables 7–9 show that the storage and retrieval throughput time is influenced by the design parameters of the CAP system, i.e.,N_{sp}, T and D. Once N_{sp} increases, the throughput time increases. In the small-size CAP system with 400 storage positions (eg. In Table 7), the throughput time increases with an increase of T ($T = 8, 9, 10$), since the increase of vertical travel time dominates the decrease of horizontal travel time. Therefore, the total storage and retrieval throughput time increases. In the large-size CAP system with 1,000 storage positions, the throughput time decreases with an increase of T ($T = 8, 9, 10$), since the decrease of horizontal travel time dominates the increase of vertical travel time. Therefore, the total storage and retrieval throughput time decreases. Also the system throughput time, which consists of travel and waiting time, is significantly affected by the lift policy used in the system. These observations motivate us to investigate the optimal system structure further and compare the performance of different lift policies in the next section.

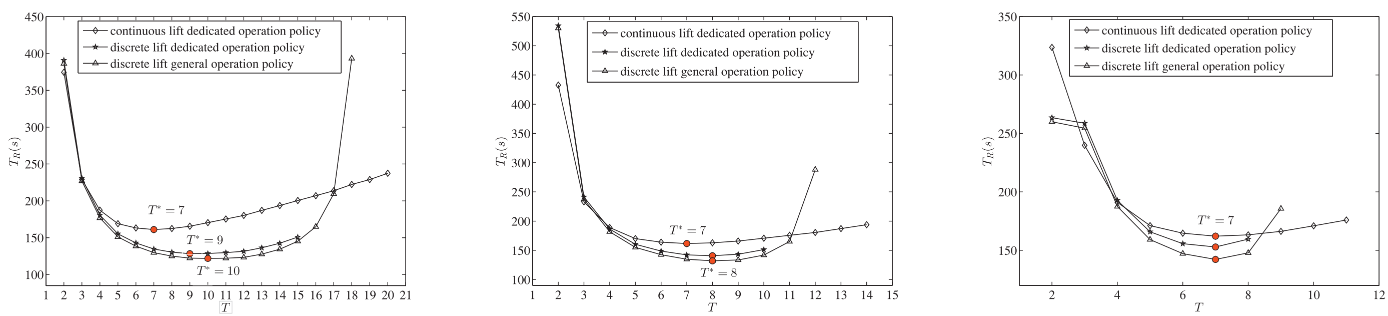
6. Results

6.1. Optimal CAP systems

This section investigates the optimal CAP system structure. Basic system parameters are identical to those in Tables 5 and 6. The storage capacity of the CAP system takes two values, i.e., $C = 400$, or $C = 800$. For each value of C, the retrieval transaction arrival rate

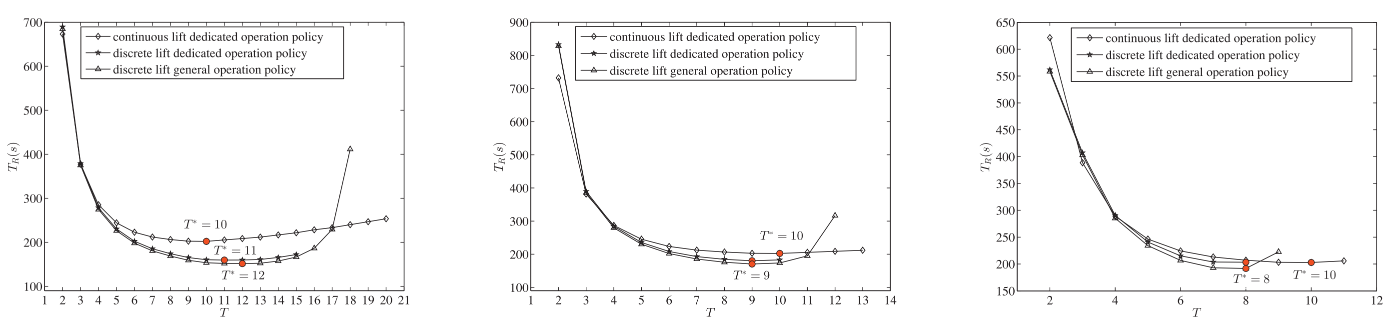
Table 9
Simulation validation results of the CAP system with discrete lift with general lift policy.

Scenario	T_R^{DG}			T_S^{DG}			W_{rl}^{DG}		
	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	147.9	146.2	1.1	153.7	151.9	1.1	25.1	22.9	9.8
B	140.4	138.9	1.0	146.8	145.2	1.1	22.9	21.1	8.6
C	141.9	140.0	1.3	148.9	146.9	1.4	27.2	25.2	8.2
D	212.6	208.6	1.9	218.4	214.6	1.8	25.2	22.9	9.8
E	196.5	195.1	0.7	202.9	199.8	1.5	22.9	21.1	8.5
F	191.1	189.3	0.9	198.1	196.5	0.8	27.3	25.2	8.1
Scenario	ρ_l			ρ_1			W_{sl}^{DG}		
	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	85.3	84.5	0.9	12.8	12.8	0.0	25.1	23.2	8.5
B	82.4	82.2	0.2	12.0	12.0	0.0	22.9	21.3	7.6
C	83.5	83.3	0.2	11.8	11.8	0.0	27.2	25.4	7.4
D	85.3	84.6	0.9	11.7	11.5	1.5	25.2	23.2	8.6
E	82.4	82.2	0.2	10.4	10.4	0.1	22.9	21.3	7.4
F	83.5	83.3	0.2	9.7	9.7	0.0	27.3	25.4	7.2



(a) $\lambda_r = 100$ (b) $\lambda_r = 140$ (c) $\lambda_r = 170$

Fig. 5. Retrieval transaction throughput time; $C = 400$.



(a) $\lambda_r = 100$ (b) $\lambda_r = 140$ (c) $\lambda_r = 170$

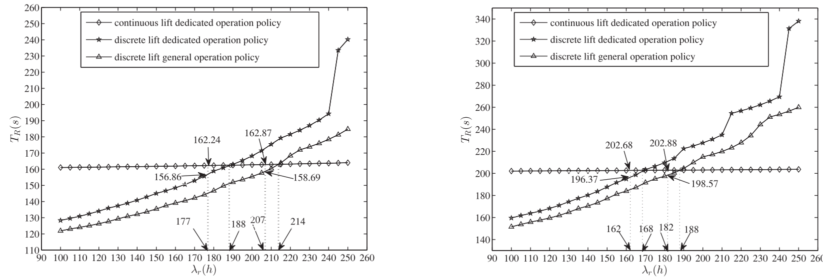
Fig. 6. Retrieval transaction throughput time; $C = 800$.

λ_r takes three values. We use the model to rapidly calculate the retrieval transaction throughput time for every feasible value of the number of tiers, which gives the optimum.

Figs. 5 and 6 depict the throughput time of the system with $C = 400$ and $C = 800$, respectively. Each system uses three lifts scenarios, i.e., continuous lifts, discrete lifts with the dedicated policy and discrete lifts with the general policy. The results show that we can find an optimal number of tiers that minimizes the retrieval transaction throughput time. Optimization of T entails trading-off the vertical and horizontal movement time. A large T increases the vertical movement time, while it leads to a small size tier that decreases the horizontal movement time. A small T produces a reverse situation. The figures also show that the transaction throughput time is quite insensitive to the number of tiers around the optimum. This implies that the system designer can randomly select a T optimal value, and it will not go wrong.

6.2. Comparison of different lift policies

This section compares different lift policies in terms of retrieval transaction throughput time. We consider the systems examined in Section 6.1. For each system, the retrieval transaction arrival rate λ_r varies from 100 to 250 per hour with a step size of 5, while other parameters are identical to those of Table 5. For each arrival rate λ_r, we first optimize the system structure to minimize the retrieval transaction throughput time. The results are presented in Fig. 7. First, in the CAP system with discrete lifts, the general lift policy always outperforms the dedicated lift policy in terms of retrieval transaction throughput time. This can be explained as sharing the lifts decreases the waiting time of cars for lifts. Second, it shows that the curves of the continuous (dedicated) lift and discrete lift (with both dedicated and general lifts policies) intersect. This implies that we can find a critical retrieval transaction arrival rate at



(a) $C = 400$ (b) $C = 800$

Fig. 7. Comparison of continuous and discrete lift operational policies in a small system and a large system, separately.

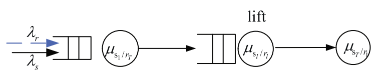


Fig. 8. CAP system with a rotating lift.

the intersection. When the retrieval transaction arrival rate is below this value, the discrete lift yields a shorter retrieval transaction throughput time. Otherwise, the continuous lift is preferred. This can be explained as that there is no waiting time of cars for the continuous lift, but each time the continuous lift loads or unloads a car, it will stop by a constant time. When the retrieval transaction arrival rate is low, the stopping time dominates the waiting time and a discrete lift is preferred. When the retrieval transaction arrival rate is large, the situation reverses and the continuous lift is preferred.

We conduct a sensitivity analysis in which we vary the lift speed and the speed of the rotating table. The base parameters are derived from Nctv7 (2012). The results can be found in Appendix F.14. Our conclusions appear to be quite insensitive to parameters changes.

6.3. General CAP systems

We first consider one rotating lift based on Nctv7 (2012). Then, we consider a general automated parking system with multiple lifts.

One rotating lift

The difference between CAP system and Nctv7 (2012) is the number of lifts and shuttles. There is only one rotating lift and no rotating shuttles in Nctv7 (2012). Storing or retrieving a car, the lift can rotate the car to the storage tier or the first tier. We can also model this system.

Fig. 8 shows the queuing network for the CAP system with a rotating lift. The storage transaction arrives at the first tier with arrival rate λ_s. The storage transaction requests a shuttle to get to the lift, and then the claimed shuttle moves from its dwell point to the storage car and transports the car to the lift. This operation is modeled as the service node μ_{s_1}. Then the car waits for the lift. Next, the transaction goes to the station μ_{s_l} representing the vertical transport of the storage car from the first tier to its designated tier. When the storage car reaches the designated tier, a shuttle moves from its dwell point to the lift and transports the car to the storage position, which is modeled as the service node μ_{s_T}. Finally, the transaction is stored at its designated tier, and then leaves the queuing system. The retrieval transaction process is the reverse of the storage process.

We can solve this model using the same approximation method in this paper. The automated parking system with one rotating lift is a simplified structure of CAP system. Since the turnover of one rotating lift is finite, it needs much time in waiting for the lift. Therefore, system performance of CAP system performs better.

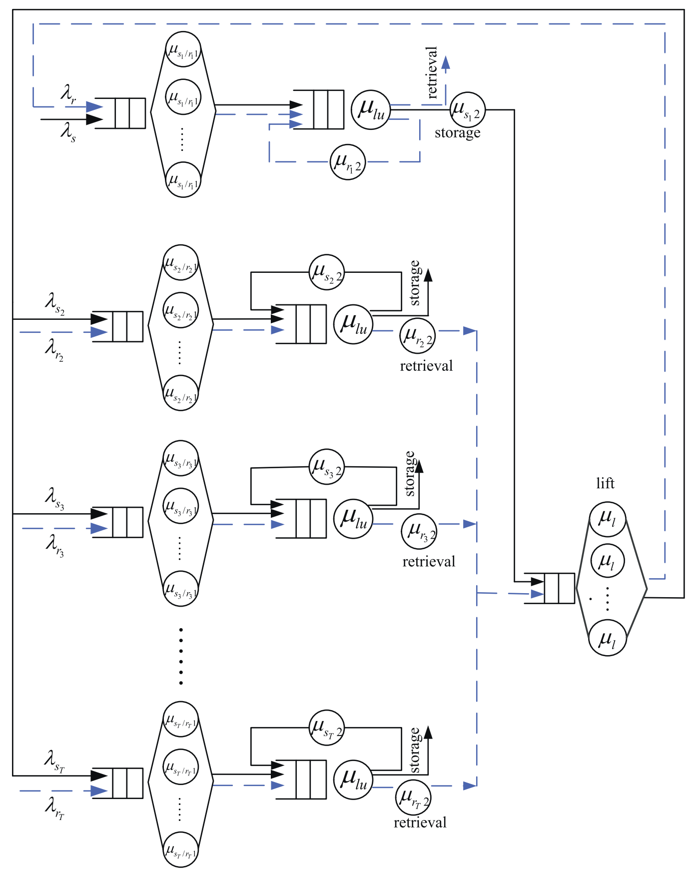


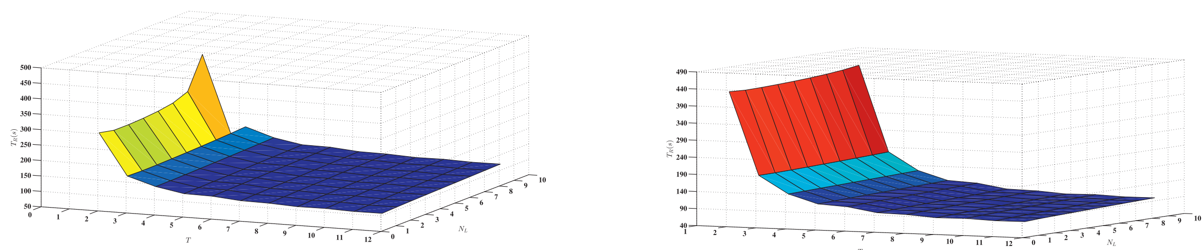
Fig. 9. CAP system with n lifts.

A general CAP system with multiple lifts

This kind of automated parking systems have found on the Internet in recent years (see Emirates, 2016; Liechtenstein, 2014; Nctv7, 2012 and Listopedia, 2016). These systems are similar to CAP system through varying the number of lifts and the number of shuttles. Below, we will study a more general automated parking system. Since the number of lifts dominate the system turnover, we investigate the number of the lifts range 1,2 to n. We build new models to explore the appropriate number of the lifts for CAP system and get the analytical results.

The n lifts process both storage and retrieval transactions (see Fig. 9). The lifts are modeled as a singer queuing system with n servers. We use the same the decomposition method (see Appendix D) for the waiting time estimation for the n lifts.

Fig. 10(a) shows that, for a given system capacity and the arrival rate, the optimal number of the lifts equals 4 for a general CAP systems with 2 tiers. When increasing the tiers ($t > 2$), the optimal



(a) $C = 400, \lambda_r = 200$ (b) $C = 800, \lambda_r = 150$

Fig. 10. Optimum the number of lifts.

number of the lifts is always equal to 2. The results show that the sever ability of two lifts is sufficient for a general CAP system. The results of Fig. 10 (b) are similar to Fig. 10 (a) except that the optimal number of the lifts equals 3 for a general CAP systems with 2 tiers. Therefore, the designer may choose two lifts for a general CAP system.

6.4. Cost analysis

We introduce operational and investment costs of CAP system. Combining time efficiency and investment cost, we calculate the minimum investment cost by enumerating all parameters over the feasible space. The minimum cost solution yields an attractive design from a practioner's perspective. The solution time is less than a second. At last, we compare cost with cubic parking system.

The operational costs contain labor, maintenance, and energy. Operational cost can easily be estimated, since this system is commonly used in practice. Therefore, we only estimate investment costs. The investment costs contain the cost of lifts, the cost of shuttles, the cost of floor space and the cost of the storage positions. Since building a CAP system with optimal structure that minimize the retrieval transaction time has not the minimum investment costs. So, we will first explore a trade-off point between the system retrieval transaction time and investment costs for multiple discrete lifts. Second, under the same capacity C, we compare the cost of CAP system with a live-cube parking system in Zaerpour et al. (2015b).

Combining time efficiency with investment cost, we first investigate an appropriate system layout for warehouse designer.

Below, we build the total minimum annual cost model.

$$min \quad TC(N_L, N_S, T, r) = C_L.N_L + C_S.N_S.T + C_{FS}.\pi.r^2 + C_{SP}.N_{SP}$$

$$s.t. \begin{cases} n_{SP}.(T-1) >= N_{SP}, \\ THT_{DG}(N_L, N_S, T, r, D, d) \leqslant THT_{DG_{max}}, \\ T <= \overline{T}, \\ r <= \overline{r}, r = D/2, \\ N_{SP} \quad \text{are given} \end{cases}$$

where C_L is the annual cost per lift, C_S is the annual cost per shuttle, C_{FS} is the annual cost per square meter floor space and C_{SP} is the annual cost per storage position. The first constraint is to make sure that the total number of storage positions is adequate for all customers arriving. The second constraint is to make sure that the retrieval throughput time of the CAP system is smaller than the maximum retrieval throughput time required, i.e., $THT_{DG}(N_L, N_S, T, r, D, d) \leqslant THT_{DG_{max}}$. The third and the forth constraint are the constraints on the system layout. The decision variables are the lifts N_L, the shuttles N_S, the tiers T and the system radius r.

We change the maximum retrieval throughput time from 60 to 250, with a step size of 10; we solve this model use the following search procedure:

a. We vary T from a lower bound 1 to an upper bound $\overline{T}$ with a stepsize of 1. We vary D from a lower bound $\underline{D}$ to an upper bound $\overline{D}$ with a stepsize of 0.1. For each combination of T and D, we calculate THT_{DG} by our queuing network models. If $THT_{DG} \leq THT_{DG_{max}}$, go to Step b.
b. We find the minimal number of lifts N_L and the minimum number of shuttles N_S that satisfies $THT_{DG}(N_L, N_S, T, r, D, d) \leq THT_{DG_{max}}$ by the queuing network, and then record the (N_L, N_S, T, r) as a feasible solution of this model.
c. We derive the total annual cost of all feasible solutions, and select the minimal total annual cost as the objective.

Table 10
Impact of system configuration on retrieval time and system cost; $C = 400$.

(a) $\lambda r = 200$.				(b) $\lambda r = 400$.			
T	N_L	$E[T]$	$TC(*10^6)$	T	N_L	$E[T]$	$TC(*10^6)$
2	4	249.31	80.80	2	3	344.96	80.78
4	2	98.82	15.00	4	4	105.19	15.10
6	2	80.76	9.63	6	3	85.56	9.69
8	2	71.49	8.26	8	2	75.13	8.26
10	2	66.95	7.90	10	2	69.95	7.90
12	2	88.61	7.40	12	2	116.58	7.40

Table 11
Impact of system configuration on retrieval time and system cost; $C = 800$.

(a) $\lambda r = 150$.				(b) $\lambda r = 200$.			
T	N_L	$E[T]$	$TC(*10^7)$	T	N_L	$E[T]$	$TC(*10^7)$
2	3	415.48	30.9	2	4	419.57	31
4	2	124.94	4.59	4	2	126.98	4.59
6	2	93.85	2.5	6	2	94.89	2.5
8	2	77.5	1.94	8	2	78.2	1.94
10	2	72.68	1.69	10	2	72.05	1.79
12	2	69.15	1.62	12	2	69.61	1.62
17	2	64.81	1.44	17	2	62.57	1.48
18	2	89.43	1.44	18	2	119.84	1.48

We consider this CAP system that needs to retrieve $N_{SP} = 400$ or $N_{SP} = 800$ cars corresponding to Figs. 5 and 6. The investment cost per lift is $60,000 (annualization in 12 years), the investment cost per shuttle is $50 (annualization in 10 years), the investment cost of floor space in a warehouse per square meter is $800 (annualization in 5 years) and the investment cost per storage position is $16,000 (annualization in 5 years). We consider an interest rate IR=1%. Then, the annual costs of a lift, a shuttle, a storage position and a square meter warehouse floor space are
$C_L = \sum_{t=1}^{12} \frac{60000(1+IR)^{t-1}}{12}$, $C_S = \sum_{t=1}^{10} \frac{50(1+IR)^{t-1}}{10}$, $C_{FL} = \sum_{t=1}^{5} \frac{800(1+IR)^{t-1}}{5}$, $C_{SP} = \sum_{t=1}^{5} \frac{16000(1+IR)^{t-1}}{5}$.

We derive optimal the number of the lifts, retrieval time and investment cost under different system configurations. In order to make comparison, we assume the system has the same storage capacity in Table 10 with a different number of tiers, following by Table 11. The car arrival rate are $\lambda r = 200$ in Table 10 (a)

Table 12
Comparison CAP system with live cubic parking system.

(a) Cubic parking system in Zaerpour et al. (2015b)			
T	C	$E[T]$	$TC_{Footprint}(*10^7)$
2	192(24 cars per level)	42.08	2496
4	192(24 cars per level)	32.08	1248
6	192(24 cars per level)	27.71	830
12	192(24 cars per level)	27.2	418
24	192(24 cars per level)	46.44	206
(b) Compact automatd parking system.			
$T-1$	C	$E[T]$	$TC_{Footprint}(*10^7)$
2	192(24 cars per level)	97.41	5231
4	192(24 cars per level)	74.82	1307
6	192(24 cars per level)	67.58	581
7	192(24 cars per level)	66.54	445
12	NAN	NAN	–

and $\lambda r = 400$ in Table 10 (b) with the same capacity $C = 400$ and the car arrival rate are $\lambda r = 150$ in Table 11 (a) and $\lambda r = 200$ in Table 11 (b) with the same capacity $C = 800$.

From Table 10, the systems with few height tiers (e.g., 2) both have longer retrieval times and require a larger investment cost. The systems with more height tiers (e.g., 10) have minimum retrieval time but the cost is not the minimum. The system with 12 tiers have minimum investment cost. The cost of the system with 12 tiers is about 6.3% lower than the cost of the system with 10 tiers. On the other hand, the annual retrieval time of the system with 12 tiers is 32.3% longer than the annual retrieval time of the system with 10 tiers. A system designer might prefer designing a system with 12 tiers instead of the system with 10 tiers because the decrease in absolute cost is more favorable than the slight increase in retrieval time. We can also find when the system with optimum tiers, the optimal number of the lifts is always equal to two. A system designer may design a system with only two lifts.

We increase the capacity C=800 in Table 11, and the conclusion we obtained is similar to Table 10.

For $C = 800, \lambda_r = 200$, we estimate the time that the parking tower can return the investment cost. Assume the hourly parking fee is 1.5$. Assume the cars remain parked for 8 hours per day. Then the annual parking service fees amount $200 \times 1.5 \times 8 \times 365 = 8.76 \times 10^5$\$. Based on Table 11(b), the minimum cost of the parking is 1.48×10^7\$. So, we calculate the investment the parking tower can return. The results show that 16 years are needed to return the investment cost.

Then, we compare the CAP system with a live-cube parking system.

Under the consideration of the land space, we get Table 12. To make fair comparison, we also consider a CAP system with the same capacity $C = 192$ that used in Zaerpour et al. (2015b).

Table 12 shows that, our retrieval time are always larger than Zaerpour et al. (2015b). The reasons are twofold: Compared with Zaerpour et al. (2015b), slowly the speeds of the lifts and shuttles are required by the CAP system with the security of this system and the waiting time for the lifts are considered. We can also derive the cost in footprint area of CAP system with few height tiers (e.g., 2 and 4) is larger than in the Zaerpour et al. (2015b). However, the cost in footprint area of CAP system with more than 4 tiers is lower than the cubic parking system.

7. Main insights

We optimize the system structure through numerical experiments. The results show that we can find an optimal number of tiers to minimize the retrieval transaction throughput time. Moreover, when the system designer selects a number of tiers around the optimal value, it will not go wrong.

Second, the comparison of different lift policies illustrates that the general lift policy always outperforms the dedicated lift policy in the system with discrete lifts. We find a critical retrieval transaction arrival rate: when the retrieval transaction arrival rate exceed this value, continuous lifts perform better in terms of retrieval transaction throughput time. Otherwise, the discrete lifts are preferred. We also find the optimum system parameters for small and large CAP systems depending on job arrival rates.

Third, we find the optimal number of lifts for a general parking system with n lifts. We find that the optimal number of lifts equals 2 for systems with more than 5 tiers. Under the optimum system configuration, the service capacity of the two lifts is adequate for this CAP system.

Forth, we also investigate system investment cost. It appears that a system with minimum investment cost also has acceptable retrieval times. Finally, we compare the CAP system with a live-cube parking system (Zaerpour et al., 2015b). The results show that the investment cost of a cylindrical CAP system is lower than a live-cube parking system when the system has more than 4 tiers.

8. Concluding remarks

This paper models and analyzes compact automated car parking and storage systems, which are increasingly used in major cities all around the world. The system we study uses a rotating ring equipped with multiple-shuttles at each tier for horizontal transport, and uses lifts for vertical transport. Due to multiple lifts and shuttles used in each tier, this system can process storage and retrieval transactions with high throughput capacity and short response times. We consider both continuous lifts and discrete lifts for the system, and study both a dedicated lift policy (one lift serves for storage and the other for retrieval) and a general lift policy (both lifts serve for both storage and retrieval). To estimate the system performance, we formulate travel time models to calculate the expected horizontal and vertical movement time, and formulate open queueing networks to estimate the expected waiting time. Simulation validation shows that our analytical models can accurately estimate the system performance.

CAP systems have hardly been explored in literature and much is left to be investigated. Researchers can study different layouts (like double deep storage for systems with large diameter, comparing the circular system with other rectangular systems described in literature), different material handling systems used, or different operational storage policies (like pre-positioning stored cars based on expected retrieval time). Next to a stochastic performance analysis, it would be interesting to investigate the CAP system performance for given arrival and departure time windows of each car, using e.g. a scheduling approach. We can also study other service rule of storage transaction, such as assigning priorities for storage cars based on their departure time.

Acknowledgments

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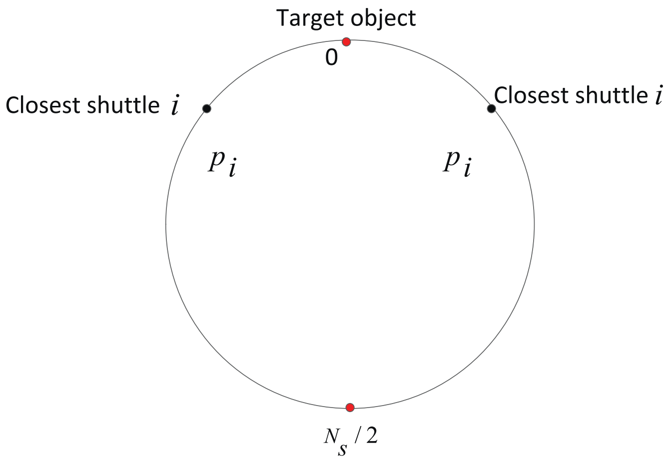


Fig. A1. Top view of a carousel ring about selecting the closest shuttle.

Appendix A. Selecting the closest shuttle and the closest storage position

Rather than selecting a random shuttle, we can also consider the closest shuttle for transport by substituting Eq. (3) by $D_{s_1,c} = \sum_{i=1}^{N_s/2} \frac{\pi d}{N_s} * p_i * i$, where $p_i = \frac{C_2^1 C_{N_s-2i}^{Na-1} + C_2^2 C_{N_s-2i}^{Na-2}}{C_{N_s}^{Na}}$. Here p_i is the possibility of the distance length from the closest shuttle dwell point to the target object, and i stands for the closet shuttle position points in the circle. That is to say the size of p_i depends on i. The label i is small, if it closes the target object, and p_i is large. As the carousel ring has the symmetry, i follows a discrete uniform distribution $[1, \frac{N_s}{2}]$ with a step size of one. N_a is the number of available shuttles (Fig. A1).

Rather than selecting a random storage position, we can also consider the closest storage position for transport by replacing Eq. (10) by $D_{s_t,s_p} = \sum_{i=1}^{n_{sp}/2} \frac{\pi D}{n_{sp}} * p_i * i$, where $p_i = \frac{C_2^1 C_{n_{sp}-2i}^{Nasp-1} + C_2^2 C_{n_{sp}-2i}^{Nasp-2}}{C_{n_{sp}}^{Nasp}}$. N_{asp} is the number of available storage positions. Our model can also process this situations by only substituting the distance D_{s_t,s_p}. Other steps are similar to our study.

Appendix B. Shuttle and lift acceleration and deceleration

We consider acceleration/deceleration of the rotating ring and the lift. We denote a as the acceleration/deceleration rate. The acceleration/deceleration rate of the rotating ring is a_s (m/s^2) and the lift is a_l (m/s^2). In the first case of Eq. (B.1), t_1 is the time for the shuttle or the lift reaching its maximum speed. $\left(D' - \frac{1}{2}at_1^2 * 2\right)/v$ is the constant travel time. In the second case, the shuttle or the lift cannot reach its maximum speed. It first accelerates and then decelerates, and the time is $\sqrt{D'/a}$. So, we derive (t_2) as Eq. (B.1)

$$t(t_2) = \begin{cases} \left(D' - \frac{1}{2}at_1^2 * 2\right)/v + 2t_1, & \text{if } t_2 > 2t_1 \\ 2\sqrt{D'/a}, & \text{otherwise } t_2 \le 2t_1 \end{cases} \tag{B.1}$$

In which $t_1 = v/a$, then, we can simplify Eq. (B.1) as Eq. (B.2) (Fig. B1).

$$t = \begin{cases} D'/v + v/a, & \text{if } t > 2t_1 \\ 2\sqrt{D'/a}, & \text{otherwise } t \le 2t_1 \end{cases} \tag{B.2}$$

where the first case means the shuttle or the lift can reach its maximum speed, the second case means it cannot. In the first case of Eq. (B.2), t_1 is the time for the shuttle or the lift to accelerate to its maximum speed. In the second case, the shuttle or the lift cannot reach its maximum speed. It first accelerates and then decelerates, the time is $\sqrt{D'/a}$. t is the same for CD, DD and DG.

Appendix C. Calculating mean waiting time for discrete lift with general lift policy

In step 1, we decompose the queuing network into -/G/1 queues that are mutual independence, and derive the arrival rates, the utilizations and the mean service time of each server. As it is random for each customer type visiting each server. So, the arrival rate of each server equals to the sum of the arrival rate of all customers. Although there are two lifts in this system, it only uses one lift after the customer chooses a lift. The customer chooses the lift randomly with the equal possibility 1/2. While the arrival rate of the each lift is equal to the sum of all arrival rate multiple the possibility 1/2.

$$\lambda_{si} = \lambda_{si,i} = \frac{\lambda_r}{T-1}, i = 2, 3 \ldots T, \lambda_{l_k} = \frac{\lambda_r}{2}, k = 1, 2$$

The utilization of each server equals to the ratio of the arrival rate to the service rate. The shuttle utilization is related to customer type i only, while the lift utilization is equal to the utilization levels of the whole customer types.

$$\rho_{si} = \frac{\lambda_{si,i}}{N_s * \mu_{si,i}}, i = 2, 3 \ldots T, \rho_{lk} = \frac{\lambda_{lk,1}}{\mu_{lk,1}} + \cdots + \frac{\lambda_{lk,T}}{\mu_{lk,T}}, k = 1, 2$$

We can derive the mean service rate of each server, which is the weighted average of the service rates of each customer type that visits this service node.

$$\mu_{si} = \frac{1}{\frac{\lambda_{si,i}}{\lambda_{si}} * \frac{1}{N_s * \mu_{si,i}}}, i = 2, 3 \ldots T,$$

$$\mu_{lk} = \frac{1}{\frac{\lambda_{lk,1}}{\lambda_{lk}} * \frac{1}{\mu_{lk,1}} + \cdots + \frac{\lambda_{lk,T}}{\lambda_{lk}} * \frac{1}{\mu_{lk,T}}}, k = 1, 2$$

In step 2, we calculate the square of the variation coefficient of the service times c_B^2 for each serves. The variation coefficient of the

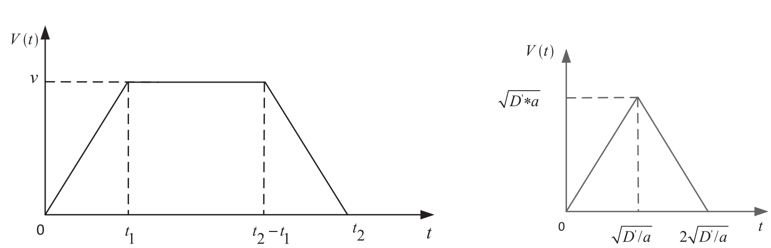


Fig. B1. Velocity-time relationship of the carousel ring and the lift travelling distance.

lift service time is null. Because it is constant if we fixed the tier about the lift.

$$c_{B,si}^2 = -1 + \frac{\lambda_{si,i}}{\lambda_{si}} * \left(\frac{\mu_{si}}{N_s * \mu_{si,i}}\right)^2 * \left(c_{B_{si,i}}^2 + 1\right), i = 2, 3 \ldots T,$$

$$c_{B,lk}^2 = -1 + \left[\frac{\lambda_{lk,1}}{\lambda_{lk}} * \left(\frac{\mu_{lk}}{\mu_{lk,1}}\right)^2 + \cdots + \frac{\lambda_{lk,T}}{\lambda_{lk}} * \left(\frac{\mu_{lk}}{\mu_{lk,T}}\right)^2\right], \quad k = 1, 2$$

In step 3, we calculate the square of the variation coefficient of the inter-arrival times for each server. The inter-arrival times is calculated through a three phases iteration (see Appendix D).

In step 4, we calculate the mean queue length and the mean waiting time of each server. We use the approximation method of Allen (1990).

$$\overline{Q}_{Si,i} = \overline{Q}_{Si,i,M/M/1} * \frac{\left(c_{Asi,i}^2 + c_{Bsi,i}^2\right)}{2}, i = 2, 3 \cdots T,$$

$$\overline{Q}_{lk,t} = \overline{Q}_{lk,t,M/M/1} * \frac{\left(c_{Alk,t}^2 + c_{Blk,t}^2\right)}{2}, k = 1, 2$$

The index i, k is referred to each server ($i = s_2, s_3, \ldots s_T, l_1, l_2$). Among the formulation above, the definition of $\overline{Q}_{i,t,M/M/1}$ is as follows:

$$\overline{Q}_{i,t,M/M/1} = \frac{\rho_{i,t}}{1-\rho_i} * P_{1,i}, P_{1,i} = \frac{\rho_i}{(1-\rho_i)} * P_{0,i},$$
$$P_{0,i} = \left(1 + \frac{\rho_i}{1-\rho_i}\right)^{-1}.$$

Then, the mean waiting time customer type t for service i, $\overline{W}_{i,t}$ can be calculated by Littles theorem (Little 1961).

$$\overline{W}_{i,t} = \overline{Q}_{i,t}/\lambda_{i,t}$$

Appendix D. Decomposition method for discrete lift with general policy

We first consider retrieval transactions, and it is reversible for storage transactions.

Phase 1: merging

Several arrival processes to each server are merged into a single arrival process in this stage. The arrival rate can be equal to the sum of the arrival rates of each arrival process. The variation coefficient of the inter-arrival times can through different methods to approximately calculate such as Pujolle, Chylla and Kuhn. Here we select the decomposition method of Pujolle that based on the paper of Marco Melacini et al.(2011).

$$c_{As_i,i}^2 = \frac{1}{\lambda_{s_i,i}}\left(\sum_j c_{js_i,i}^2 * \lambda_{j,i} * p_{js_i,i} + c_{0s_i,i}^2 * \lambda_{0,i} * p_{0s_i,i}\right), i = 2, 3 \ldots T,$$

$$c_{Al_k,r}^2 = \frac{1}{\lambda_{l_k,r}}\left(\sum_j c_{jl_k,r}^2 * \lambda_{j,r} * p_{jl_k,r} + c_{0l_k,r}^2 * \lambda_{0,r} * p_{0l_k,r}\right), k = 1, 2,$$

$$c_{As_i}^2 = \frac{1}{\lambda_{s_i}} * \sum_{t=2}^{T} c_{As_i,t}^2 * \lambda_{s_i,t}, i = 2, 3 \ldots T,$$

$$c_{Al_k}^2 = \frac{1}{\lambda_{l_k}} * \sum_{t=1}^{T} c_{Al_k,t}^2 * \lambda_{l_k,t}, k = 1, 2.$$

While i and j is the index of the server (i and $j = s_2, s_3, \ldots s_T, l_1, l_2$), $p_{ji,\,t}$ is the possibility that from the server i to the serve j of the customer type t, and $c_{0i,t}^2$ is the square of the variation coefficient of the inter-arrival time of customer type t at server i.

Phase 2: Flow

In this phase, we calculate the coefficient of variation c_D of the inter-departure times which is depend on not only the coefficient of variation of the inter-arrival times c_A but also the service times c_B. The coefficient of variation c_D of the inter-departure times can through vary methods to approximately calculate such as Pujolle, Whitt, Gelenbe, Chylla and Kuhn. Here we select the decomposition method of Whitt that based on the paper of Marco Melacini et al. (2011).

$$c_{Ds_i} = 1 + \frac{\rho_{s_i}^2 * \left(c_{B,s_i}^2 - 1\right)}{\sqrt{N_s}} + \left(1 - \rho_{s_i}^2\right) * \left(c_{As_i}^2 - 1\right), i = 2, 3 \cdots T$$

$$c_{D,l_k} = 1 + \rho_{l_k}^2 * \left(c_{B,l_k}^2 - 1\right) + \left(1 - \rho_{l_k}^2\right) * \left(c_{Al_k}^2 - 1\right), k = 1, 2$$

Phase 3: Splitting

In this phase, we use the following formulation that all authors used.

$$c_{ij,t}^2 = 1 + c_{Di}^2 - 1$$

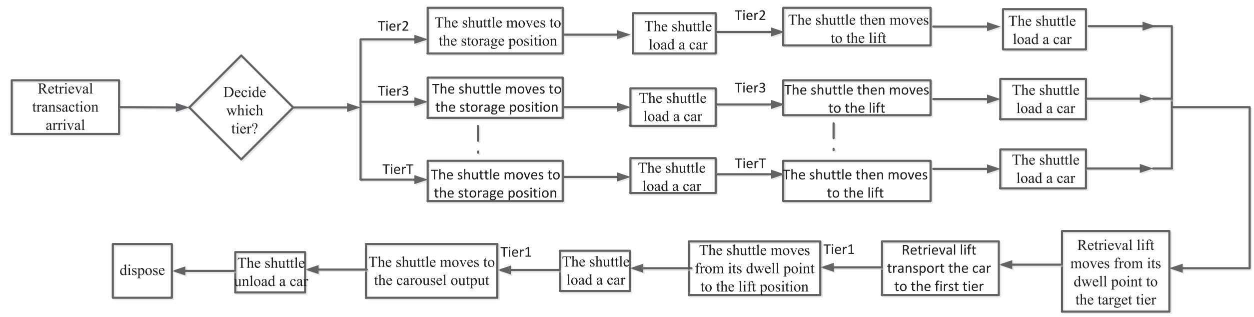
In three phases, it comes true through iterating once by once. The last value in the left of the following formulation is less than ε. Iteration k satisfies the following formulation:

$$\left(\frac{c_{ij,t}^2(k) - c_{ij,t}^2(k-1)}{c_{ij,t}^2(k)}\right) < \varepsilon$$

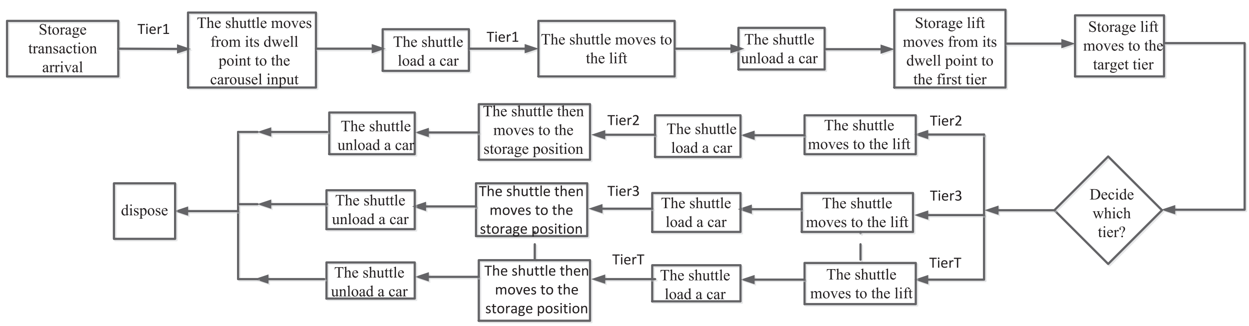
Appendix E. Simulation flow chart for retrieval and storage transaction

We explain the discrete-event simulation model in Appendix E. We give the simulation model based on the analysis model. We present the main events and processes in the simulation model as followings.

(1) Retrieval transaction arrival: The arrival process at each tier ($t > 1$) follows a Poisson distribution with the arrival rate $\lambda r/(T-1)$. For a given retrieval transaction, the distribution is equal possible at each tier.
(2) A retrieval transaction requires the target shuttle: The car to be retrieved requests a shuttle in the designated tier. The rotating ring rotates the assigned shuttle to the car. Then the shuttle loads the car. The service time depends on the shuttle position and the storage position of the retrieval load.
(3) A retrieval transaction moves from its storage position to the lift: The rotating ring rotates the shuttle with the car to the lift. The service time depends on the storage position of the retrieval load and the lift position.
(4) Waiting of the lift for the assigned shuttle: The assigned shuttle waits for the lift in one queue. After the lift arrives at the target tier, the shuttle unloads the car.
(5) Transportation of the lift from its dwell point to the first tier: The lift transports the assigned platform to the retrieval tier. Then the platform loads the car. The lift transports the car to the first tier. The service time depends on the dwell point of the lift position and the target tier.
(6) A retrieval transaction requires the target shuttle: The car requests a shuttle in the first tier. The rotating ring at the first tier rotates the assigned shuttle to the entrance of the lift. Then the shuttle loads the car. The service time depends on the shuttle position and the storage position of the lift.
(7) A retrieval transaction moves from lift position to the rotating table: The rotating ring rotates the shuttle with the car to the car rotating table. The shuttle unloads the car into the car rotating table. The service time depends on the lift position and the position of the rotating table.
(8) Departure of the retrieval transaction: The retrieval transaction is finished when the lift releases the load at the rotating table of the system.



(a) Retrieval transaction



(b) Storage transaction

Fig. E1. Simulation flow chart for retrieval and storage transaction.

Table F1
The speed of the lifts and the rotating.

v_s	a_s	v_l	a_l
$4(m/s)$	$2(m/s^2)$	$2(m/s)$	$1.5(m/s^2)$

Appendix F. sensitivity analysis and varied speeds of lifts and the rotating

We conduct sensitivity analysis and vary the speeds of the lifts and the rotating in Table F1. We drive and analyze the results.

Simulation results

We derive the results in Tables F2–F4. The relative errors of storage and retrieval throughput time are less than 5%. The relative error of expected waiting time of cars for lifts is less than 10%. The relative errors of the utilizations of both lifts and shuttles are less than 2.3%. Although we vary variables, our analytical models can accurately estimate the system performance of the CAP system, see Tables F2–F4.

Optimal system layout

Figs. F1 and F2 show that the retrieval transaction time is quite insensitive to the number of tiers around the optimum. The influence of Ton system performance is relatively robust in a scope near the optimal tiers. This implies our optimal decision in number of tiers is robust.

Comparison of different lift policies

From Fig. F3, we know that the results are insensitive to the speeds of the lifts and the shuttles. We find when the speeds of the lift and shuttles increase, the total storage and retrieval time decrease. However, we find the variation trend of the results are insensitive for two lifts, the continuous lifts are still smooth. Second, (we take Fig. F3(b) as an example) there exist a threshold point $\lambda r = 255$, when $\lambda r < 255$, discrete lift is better, and when $\lambda r > 255$, continuous lift is better.

Table F2
Simulation validation results of the CAP system with continuous lifts.

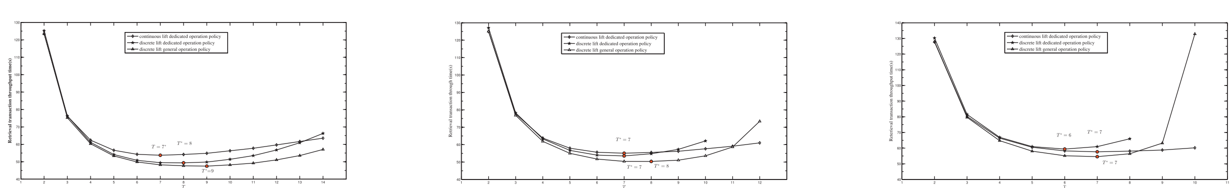
Scenario	T_R^{CD}			T_S^{CD}			ρ_1		
	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	54.6	54.8	0.3	56.1	56.2	0.1	0.0	0.0	0.1
B	56.3	55.9	0.7	58.0	58.3	0.5	0.1	0.1	2.3
C	67.6	67.3	0.4	69.4	69.7	0.4	0.104	0.1	0.8
D	765.0	76.2	0.2	77.5	77.6	0.1	0.0	0.0	0.2
E	75.4	75.4	0.1	77.0	77.4	0.5	0.1	0.1	0.0
F	84.3	84.3	0.1	86.2	86.6	0.5	0.1	0.1	0.1
Scenario	W_{R1}^{CD}			W_{S1}^{CD}					
	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$			
A	1.8	1.8	0.1	1.8	1.8	0.2			
B	3.1	3.1	0.2	3.1	3.1	0.2			
C	12.5	12.5	0.3	12.5	12.5	0.4			
D	1.8	1.8	0.2	1.8	1.8	0.3			
E	3.1	3.1	0.2	3.1	3.1	0.1			
F	12.5	12.5	0.1	12.5	12.5	0.1			

Table F3
Simulation validation results of the CAP system with discrete lift with dedicated lift policy.

Scenario	T_R^{DD}			T_S^{DD}			W_{Rl}^{DD}			W_{Sl}^{DD}		
	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	55.2	52.7	4.8	71.1	67.8	4.9	9.0	8.9	0.9	21.9	19.9	9.9
B	57.4	55.7	3.0	89.5	88.2	1.5	12.6	12.4	1.2	41.2	40.4	1.9
C	57.2	56.2	1.8	90.6	89.1	1.6	12.9	13.0	0.5	42.3	41.3	2.5
D	81.0	80.2	1.0	112.5	108.7	3.5	12.3	12.3	0.1	40.8	37.6	8.6
E	76.5	75.5	1.3	108.8	107.3	1.4	12.6	12.4	1.5	41.4	40.3	2.9
F	74.0	73.5	0.7	107.6	105.9	1.6	12.9	13.0	0.4	42.6	41.1	3.5
Scenario	ρ_{rl}			ρ_{sl}			ρ_1					
	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$			
A	0.6	0.6	0.8	0.8	0.8	0.4	0.1	0.1	2.1			
B	0.6	0.6	0.1	0.8	0.8	0.2	0.1	0.1	2.3			
C	0.6	0.6	0.9	0.8	0.8	0.0	0.1	0.1	0.8			
D	0.7	0.7	0.9	0.9	0.9	0.3	0.1	0.1	0.3			
E	0.7	0.6	0.1	0.8	0.8	0.2	0.1	0.1	0.0			
F	0.6	0.6	0.8	0.8	0.8	0.1	0.0	0.0	0.0			

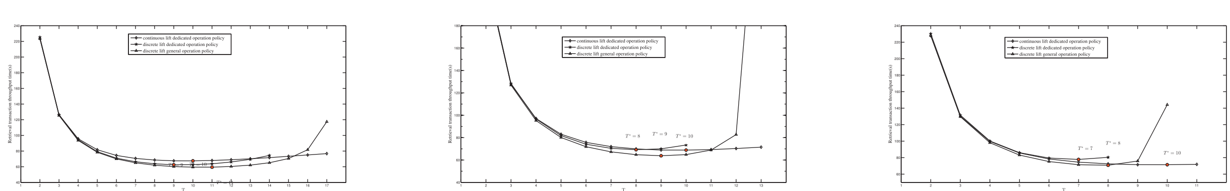
Table F4
Simulation validation results of the CAP system with discrete lift with general lift policy.

Scenario	T_R^{DG}			T_S^{DG}			W_{rl}^{DG}		
	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	56.8	55.9	1.7	61.0	60.3	1.3	9.2	9.5	3.2
B	54.8	54.3	0.9	59.5	59.8	0.5	10.1	11.1	8.8
C	54.1	53.7	0.7	59.3	59.4	0.2	10.3	11.0	5.7
D	78.2	79.3	1.3	82.4	81.9	0.6	9.2	9.4	2.9
E	73.9	74.2	0.4	78.6	79.2	0.7	10.1	11.2	9.0
F	70.9	71.0	0.1	76.1	76.4	0.5	10.4	11.0	5.9
Scenario	ρ_l			ρ_1			W_{sl}^{DG}		
	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(\%)$	$S(\%)$	$\delta(\%)$	$A(s)$	$S(s)$	$\delta(\%)$
A	0.8	0.8	0.7	0.1	0.1	0.1	9.2	9.2	0.2
B	0.8	0.8	0.3	0.1	0.1	2.3	10.1	10.9	7.4
C	0.8	0.8	0.4	0.1	0.1	0.8	10.3	10.8	4.7
D	0.8	0.8	0.7	0.1	0.1	0.2	9.2	9.2	0.1
E	0.8	0.8	0.3	0.1	0.1	0.0	10.1	11.0	7.7
F	0.8	0.8	0.4	0.1	0.1	0.0	10.4	10.9	4.9



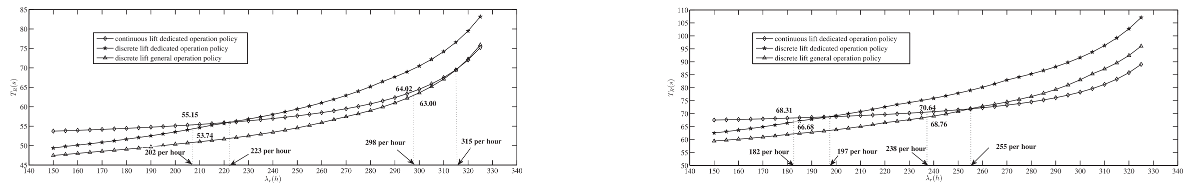
(a) $\lambda_r = 150$ (b) $\lambda_r = 200$ (c) $\lambda_r = 250$

Fig. F1. Retrieval transaction throughput time; $C = 400$.



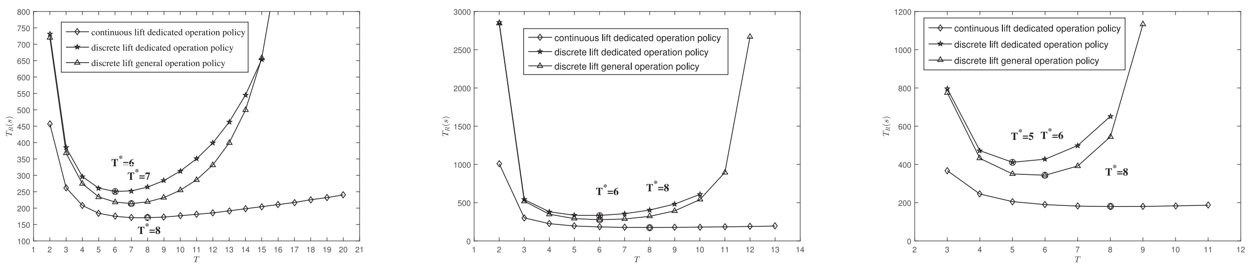
(a) $\lambda_r = 150$ (b) $\lambda_r = 200$ (c) $\lambda_r = 250$

Fig. F2. Retrieval transaction throughput time; $C = 800$.



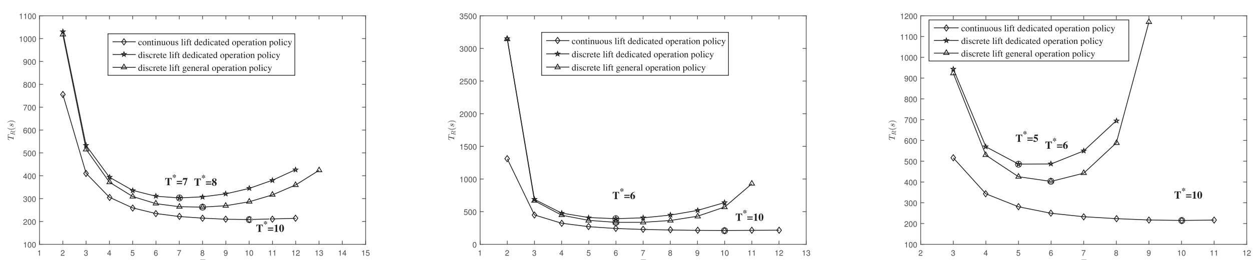
(a) $C = 400$ (b) $C = 800$

Fig. F3. Comparison of continuous and discrete lift operational policies in a system with small storage capacity and large storage capacity.



(a) $\lambda_r = 100$ (b) $\lambda_r = 140$ (c) $\lambda_r = 170$

Fig. G1. Retrieval transaction throughput time; $C = 400$.



(a) $\lambda_r = 100$ (b) $\lambda_r = 140$ (c) $\lambda_r = 170$

Fig. G2. Retrieval transaction throughput time; $C = 800$.

Appendix G. A model with non-Poisson process

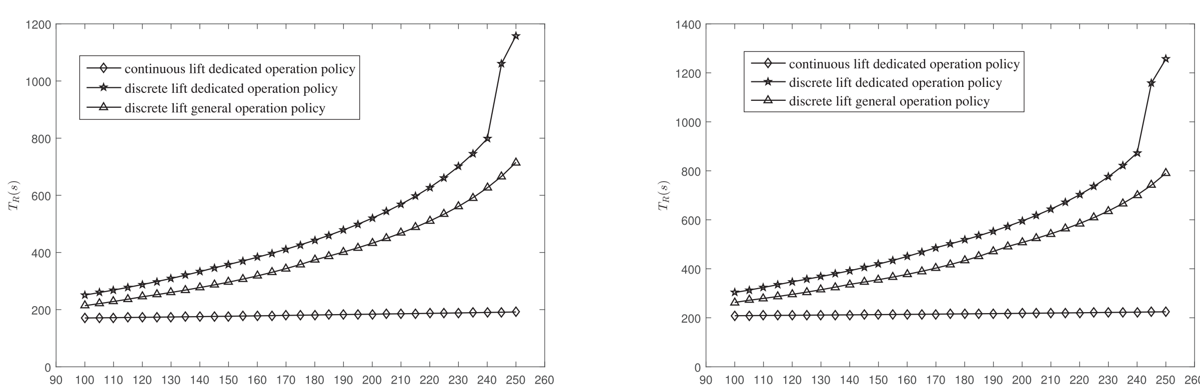
Optimal system layout

Figs. G1 and G2 show that the variation trend of the optimum tiers are insensitive to the arrival of customers. Furthermore, for the discrete lift, the retrieval time of general lift performs better than dedicated lift policy with the same optimal tiers.

However, the continuous lifts performs better than discrete lift with the same optimum tiers. This is different from Fig. 7. This can be explained as follows: when the initial interval time of customer arrival increase, the total time increase, and the continuous lifts have more advantages when the total retrieval time become longer.

Comparison of different lift policies

From Fig. G3, we find that the variation trend of the three lift operating policies are relatively insensitive to the second moment of the inter-arrival time of customers. For the discrete lift, the retrieval time of general lift performs better than dedicated lift policy. With the increase of retrieval time, the advantage of continuous lift is more significant.



(a) $C = 400$ (b) $C = 800$

Fig. G3. Comparison of continuous and discrete lift operational policies in two systems.

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