

- Find the order of the differential equation of the family of circles of radius 3 units.
- Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$.
- Find the co-ordinates of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.
- If $A = \begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$, then find $|\text{adj} A|$.
- If $y = 2\sqrt{\sec(e^{2x})}$, then find $\frac{dy}{dx}$.
- Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamonds.
- The probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time.
- Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are collinear.
- For any two vectors \vec{a} and \vec{b} , prove that
$$\left(\vec{a} \times \vec{b}\right)^2 = \vec{a}^2 \vec{b}^2 - \left(\vec{a} \cdot \vec{b}\right)^2$$
- Find :
$$\int \frac{x-1}{(x-2)(x-3)} dx$$
- Integrate :
$$\frac{e^x}{5-4e^x-e^{2x}}$$
 with respect to x.
- If $P(A) = 0.6$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find $P(A \cup B)$ and $P(A|B)$.
- Find the value of $(x - y)$ from matrix equation
$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

14. Find:

$$\int e^x (\cos x - \sin x) \operatorname{cosec}^2 x dx$$

15. Find the differential equation of the family of curves represented by $y^2 = a(b^2 - x^2)$.

16. Let an operation $*$ on the set of natural numbers N be defined by $a * b = a^b$. Find (i) whether $*$ is a binary or not, and (ii) if it is a binary, then is it commutative or not.

17. Find the particular solution of the differential equation :

$$(1 + e^{2x}) dy + (1 + y^2) e^x dx = 0, \text{ given that } y(0) = 1.$$

18. Find the particular solution of the differential equation :

$$x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0, \text{ given that } y(1) = \frac{\pi}{2}.$$

19. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

20. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1} .

21. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$; is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

22. Using vectors, find the value of x such that the four points $A(x, 5, -1)$, $B(3, 2, 1)$, $C(4, 5, 5)$ and $D(4, 2, -2)$ are coplanar.

23. Prove that :

$$\sin^{-1} \frac{4}{5} + \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{63}{65} = \frac{\pi}{2}$$

24. If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dx}{dy} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$.

25. Find :

$$\int (\sin x \cdot \sin 2x \cdot \sin 3x) dx$$

26. Differentiate $\tan^{-1} \frac{3x-x^3}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$ w.r.t. $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$.

27. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y), |x| < 1, |y| < 1$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

28. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

29. Evaluate :

$$\int_{-1}^2 |x^3 - x| dx$$

30. Find the equation of planes passing through the intersection of planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ and are at a unit distance from origin.

31. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit for type A souvenirs is ₹ 100 each and for type B souvenirs, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximise the profit ? Formulate the problem as a LLP and then solve it graphically.

32. Using integration, find the area of the following region :

$$\{(x, y) : x^2 + y^2 \leq 16a^2 \text{ and } y^2 \leq 6ax\}$$

33. Using integration, find the area of triangle ABC bounded by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.
34. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$. Also, find the distance between these two line Also, find the distance between these two lin Also, find the distance between these two lines.
35. Find the coordinates of the foot Q of the perpendicular drawn from the point P(1, 3, 4) to the plane $2x - y + z + 3 = 0$. Find the distance PQ and the image of P treating the plane as a mirror.

36. Using elementary row transformations, find the inverse of the matrix

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}.$$

37. Using matrices, solve the following system of linear equations :

$$2x + 3y + 10z = 4$$

$$4x - 6y + 5z = 1$$

$$6x + 9y - 20z = 2$$

38. The sum of the perimeters of circle and a square is K , where K is some constant. Prove that the sum of their areas is least when the side of the square is twice the radius of the circle.
39. A bag contains 5 red and 3 black balls and another bag contains 2 red and 6 black balls. Two balls are drawn at random (*without replacement*) from one of the bags and both are found to be red. Find the probability that balls are drawn from the first bag.